Integración básica

1 Resuelve las siguientes integrales indefinidas utilizando la propiedad de linealidad y la tabla de integrales inmediatas:

a) $\int (x-2)^2 dx$

b) $\int \frac{1}{x^4} dx$

c) $\int \frac{x^2 - x + 5}{x} dx$

d) $\int (3e^x - senx) dx$

e) $\int \sqrt[3]{x^2} dx$

f) $\int \frac{3}{5x^2+5} dx$

g) $\int \sqrt{\frac{4}{9-9x^2}} dx$

Solución

a) $\int (x-2)^2 dx = \int (x^2 - 4x + 4) dx = \int x^2 dx - 4\int x dx + 4\int dx =$

$$= \frac{x^3}{3} - 4\frac{x^2}{2} + 4x + C = \frac{x^3}{3} - 2x^2 + 4x + C$$

b) $\int \frac{1}{x^4} dx = \int x^{-4} dx = \frac{x^{-3}}{-3} + C = -\frac{1}{3}\frac{1}{x^3} + C$

c) $\int \frac{x^2 - x + 5}{x} dx = \int \left(x - 1 + \frac{5}{x} \right) dx = \int x dx - \int dx + 5\int \frac{1}{x} dx =$

$$= \frac{x^2}{2} - x + 5\ln|x| + C$$

d) $\int (3e^x - senx) dx = 3\int e^x dx - \int senx dx = 3e^x + \cos x + C$

e) $\int \sqrt[3]{x^2} dx = \int x^{2/3} dx = \frac{x^{5/3}}{5/3} + C = \frac{3}{5} x^{5/3} + C = \frac{3}{5}\sqrt[3]{x^5} + C = \frac{3}{5} x \sqrt[3]{x^2} + C$

f) $\int \frac{3}{5x^2+5} dx = \frac{3}{5}\int \frac{1}{x^2+1} dx = \frac{3}{5} arctagx + C$

g) $\int \sqrt{\frac{4}{9-9x^2}} dx = \int \sqrt{\frac{4}{9}} \sqrt{\frac{1}{1-x^2}} dx = \sqrt{\frac{4}{9}} \int \frac{1}{\sqrt{1-x^2}} dx = \frac{2}{3} arcsenx + C$

2 Resuelve las siguientes integrales indefinidas utilizando algún cambio de variable apropiado:

a) $\int x\sqrt{x-1}dx$

b) $\int \frac{senx}{\sqrt{\cos x}}dx$

c) $\int \frac{x^2}{x^3-2}dx$

d) $\int \left(e^x-3\right)^4 e^x dx$

e) $\int \frac{2x}{1+x^4}dx$

f) $\int \frac{\ln x}{x}dx$

g) $\int \frac{e^{tagx}}{\cos^2 x}dx$

Solución

a) $$\int x\sqrt{x-1}dx = \begin{pmatrix} t = x-1 \\ dt = dx \end{pmatrix} = \int (t+1)\sqrt{t}dt = \int t\sqrt{t}dt + \int \sqrt{t}dt =$$

$$= \int t\cdot t^{1/2}dt + \int t^{1/2}dt = \int t^{3/2}dt + \int t^{1/2}dt = \frac{t^{5/2}}{5/2} + \frac{t^{3/2}}{3/2} + C =$$

$$= \frac{2}{5}t^{5/2} + \frac{2}{3}t^{3/2} + C = \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$$

b) $$\int \frac{senx}{\sqrt{\cos x}}dx = \begin{pmatrix} t = \cos x \\ dt = -senxdx \end{pmatrix} = \int \frac{-dt}{\sqrt{t}} = -\int \frac{1}{t^{1/2}}dt = -\int t^{-1/2}dt =$$

$$= -\frac{t^{1/2}}{1/2} + C = -2t^{1/2} + C = -2\sqrt{t} + C = -2\sqrt{\cos x} + C$$

c) $$\int \frac{x^2}{x^3-2}dx = \begin{pmatrix} t = x^3-2 \\ dt = 3x^2dx \end{pmatrix} = \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\ln|t| + C = \frac{1}{3}\ln\left|x^3-2\right| + C$$

d) $$\int \left(e^x-3\right)^4 e^x dx = \begin{pmatrix} t = e^x-3 \\ dt = e^x dx \end{pmatrix} = \int t^4 dt = \frac{t^5}{5} + C = \frac{\left(e^x-3\right)^5}{5} + C$$

e) $$\int \frac{2x}{1+x^4}dx = \begin{pmatrix} t = x^2 \\ dt = 2xdx \end{pmatrix} = \int \frac{1}{1+t^2}dt = arctagt + C = arctag\left(x^2\right) + C$$

f) $$\int \frac{\ln x}{x}dx = \begin{pmatrix} t = \ln x \\ dt = \frac{1}{x}dx \end{pmatrix} = \int tdt = \frac{t^2}{2} + C = \frac{\left(\ln|x|\right)^2}{2} + C$$

g) $$\int \frac{e^{tagx}}{\cos^2 x}dx = \begin{pmatrix} t = tagx \\ dt = \frac{1}{\cos^2 x}dx \end{pmatrix} = \int e^t dt = e^t + C = e^{tagx} + C$$