

## CÁLCULO DE LÍMITES

1. Calcular los siguientes límites:

a)  $\lim_{x \rightarrow 0} \frac{x^3 - x^2 - 2x}{x^3 - x}$

b)  $\lim_{x \rightarrow -5} \frac{x^2 + 3x - 10}{x^3 + 8x^2 + 5x - 50}$

c)  $\lim_{x \rightarrow +\infty} \frac{(x^2 + 2)(x^2 - 2)}{4x^2 \cdot (x + 2)^2}$

d)  $\lim_{x \rightarrow 1} \frac{x^5 - 1}{x^4 + 2x^3 - x - 2}$

e)  $\lim_{x \rightarrow \infty} \frac{x(2x+1)^2 - 8x^3}{5x(x+2)^2}$

f)  $\lim_{x \rightarrow 2} \frac{x^4 - 6x^3 + 12x^2 - 8x}{x^3 - x^2 - 8x + 12}$

g)  $\lim_{x \rightarrow a} \frac{x^2 - ax}{x^2 + ax - 2a^2}$

h)  $\lim_{x \rightarrow 0} \frac{(a+x)^2 - a^2}{x}$

i)  $\lim_{x \rightarrow a} \frac{x^2 - (a+1)x + a}{x^2 - a^2}$

Solución:

a)  $\lim_{x \rightarrow 0} \frac{x^3 - x^2 - 2x}{x^3 - x} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{x(x^2 - x - 2)}{x(x^2 - 1)} = \lim_{x \rightarrow 0} \frac{x^2 - x - 2}{x^2 - 1} = 2$

b)  $\lim_{x \rightarrow -5} \frac{x^2 + 3x - 10}{x^3 + 8x^2 + 5x - 50} = \frac{0}{0} = \lim_{x \rightarrow -5} \frac{\cancel{(x+5)}(x-3)}{\cancel{(x+5)}^2(x-2)} = \lim_{x \rightarrow -5} \frac{x-3}{(x+5)(x-2)} = \frac{-8}{0} = +\infty$

c)  $\lim_{x \rightarrow +\infty} \frac{(x^2 + 2)(x^2 - 2)}{4x^2 \cdot (x + 2)^2} = \frac{\infty}{\infty}$  Dividimos por  $x^4$ :  $\lim_{x \rightarrow +\infty} \frac{\left(1 + \frac{2}{x^2}\right)\left(1 - \frac{2}{x^2}\right)}{4 \cdot \left(1 + \frac{2}{x}\right)^2} = \frac{1}{4}$

d)  $\lim_{x \rightarrow 1} \frac{x^5 - 1}{x^4 + 2x^3 - x - 2} = \frac{0}{0} = \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(x^4 + x^3 + x^2 + x + 1)}{\cancel{(x-1)}(x+2)(x^2 + x + 1)} = \lim_{x \rightarrow 1} \frac{x^4 + x^3 + x^2 + x + 1}{(x+2)(x^2 + x + 1)} = \frac{5}{9}$

e)  $\lim_{x \rightarrow \infty} \frac{x(2x+1)^2 - 8x^3}{5x(x+2)^2} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{(2x+1)^2 - 8x^2}{5(x+2)^2}$  Dividimos por  $x^2$ :

$$\lim_{x \rightarrow \infty} \frac{\left(2 + \frac{1}{x}\right)^2 - 8}{5\left(1 + \frac{2}{x}\right)^2} = \frac{4 - 8}{5} = -\frac{4}{5}$$

f)  $\lim_{x \rightarrow 2} \frac{x^4 - 6x^3 + 12x^2 - 8x}{x^3 - x^2 - 8x + 12} = \frac{0}{0} = \lim_{x \rightarrow 2} \frac{x(x-2)^3}{\cancel{(x-2)}^2(x+3)} = \lim_{x \rightarrow 2} \frac{x(x-2)}{x+3} = 0$

g)  $\lim_{x \rightarrow a} \frac{x^2 - ax}{x^2 + ax - 2a^2} = \frac{0}{0} = \lim_{x \rightarrow a} \frac{x(x-a)}{(x-a)(x+2a)} = \lim_{x \rightarrow a} \frac{x}{x+2a} = \frac{a}{3a} = \frac{1}{3}$

h)  $\lim_{x \rightarrow 0} \frac{(a+x)^2 - a^2}{x} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{a^2 + 2ax + x^2 - a^2}{x} = \lim_{x \rightarrow 0} \frac{x(2a+x)}{x} = \lim_{x \rightarrow 0} (2a+x) = 3a$

i)  $\lim_{x \rightarrow a} \frac{x^2 - (a+1)x + a}{x^2 - a^2} = \frac{0}{0} = \lim_{x \rightarrow a} \frac{(x-a)(x-1)}{(x-a)(x+a)} = \lim_{x \rightarrow a} \frac{x-1}{x+a} = \frac{a-1}{2a}$

2. Calcular los siguientes límites:

a)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 10x} - x)$

b)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 5x + 6} - x)$

c)  $\lim_{x \rightarrow +\infty} (\sqrt{4x^2 + 5x} - 2x)$

d)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - x - 6} - x)$

e)  $\lim_{x \rightarrow +\infty} (\sqrt{4x^2 + x} - \sqrt{4x^2 - 2x})$

f)  $\lim_{x \rightarrow +\infty} \frac{\sqrt{9x^2 + x} + 5x}{x + 3\sqrt{x^2 + x}}$

g)  $\lim_{x \rightarrow 3} \frac{2x - 6}{2 - \sqrt{x^2 - 5}}$

h)  $\lim_{x \rightarrow 3} \frac{x - \sqrt{3x}}{x - 3}$

i)  $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x^2 - 4x}$

j)  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{\sqrt{x} - 1}$

k)  $\lim_{x \rightarrow +\infty} \frac{(x-1)\sqrt{4x^3 + 3}}{\sqrt{x^5 + 1}}$

l)  $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 - 3x + 6} - \sqrt{x + 2}}{3x - 6}$

Solución:

a)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 10x} - x) = \infty - \infty$  Multiplicamos por el conjugado:

$$\lim_{x \rightarrow +\infty} \frac{x^2 + 10x - x^2}{\sqrt{x^2 + 10x} + x} = \lim_{x \rightarrow +\infty} \frac{10x}{\sqrt{x^2 + 10x} + x} = \lim_{x \rightarrow +\infty} \frac{10}{\sqrt{1 + \frac{10}{x}} + 1} = \frac{10}{2} = 5$$

b)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 5x + 6} - x) = \infty - \infty$  Multiplicamos por el conjugado:

$$\lim_{x \rightarrow +\infty} \frac{x^2 + 5x + 6 - x^2}{\sqrt{x^2 + 5x + 6} + x} = \lim_{x \rightarrow +\infty} \frac{5x + 6}{\sqrt{x^2 + 5x + 6} + x} = \lim_{x \rightarrow +\infty} \frac{5 + \frac{6}{x}}{\sqrt{1 + \frac{5}{x} + \frac{6}{x^2}} + 1} = \frac{5}{2}$$

c)  $\lim_{x \rightarrow +\infty} (\sqrt{4x^2 + 5x} - 2x) = \infty - \infty$  Multiplicamos por el conjugado:

$$\lim_{x \rightarrow +\infty} \frac{4x^2 + 5x - 4x^2}{\sqrt{4x^2 + 5x} + 2x} = \lim_{x \rightarrow +\infty} \frac{5x}{\sqrt{4x^2 + 5x} + 2x} = \lim_{x \rightarrow +\infty} \frac{5}{\sqrt{4 + \frac{5}{x}} + 2} = \frac{5}{4}$$

d)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - x - 6} - x) = \infty - \infty$  Multiplicamos por el conjugado:

$$\lim_{x \rightarrow +\infty} \frac{x^2 - x - 6 - x^2}{\sqrt{x^2 - x - 6} + x} = \lim_{x \rightarrow +\infty} \frac{-x - 6}{\sqrt{x^2 - x - 6} + x} = \lim_{x \rightarrow +\infty} \frac{-1 - \frac{6}{x}}{\sqrt{1 - \frac{1}{x} - \frac{6}{x^2}} + 1} = -\frac{1}{2}$$

e)  $\lim_{x \rightarrow +\infty} (\sqrt{4x^2 + x} - \sqrt{4x^2 - 2x}) = \infty - \infty$  Multiplicamos por el conjugado:

$$\lim_{x \rightarrow +\infty} \frac{4x^2 + x - 4x^2 + 2x}{\sqrt{4x^2 + x} + \sqrt{4x^2 - 2x}} = \lim_{x \rightarrow +\infty} \frac{3x}{\sqrt{4x^2 + x} + \sqrt{4x^2 - 2x}} = \lim_{x \rightarrow +\infty} \frac{3}{\sqrt{4 + \frac{1}{x}} + \sqrt{4 - \frac{2}{x}}} = \frac{3}{4}$$

f)  $\lim_{x \rightarrow +\infty} \frac{\sqrt{9x^2 + x + 5x}}{x + 3\sqrt{x^2 + x}} = \frac{\infty}{\infty}$  Dividimos por x:

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{9 + \frac{1}{x} + 5}}{1 + 3\sqrt{1 + \frac{1}{x}}} = \frac{8}{4} = 2$$

g)  $\lim_{x \rightarrow 3} \frac{2x - 6}{2 - \sqrt{x^2 - 5}} = \frac{0}{0} = \lim_{x \rightarrow 3} \frac{(2x - 6)(2 + \sqrt{x^2 - 5})}{4 - x^2 + 5} = \lim_{x \rightarrow 3} \frac{2(x - 3)(2 + \sqrt{x^2 - 5})}{(3 - x)(x + 3)} = \lim_{x \rightarrow 3} \frac{2(2 + \sqrt{x^2 - 5})}{-(x + 3)} = -\frac{4}{3}$

h)  $\lim_{x \rightarrow 3} \frac{x - \sqrt{3x}}{x - 3} = \frac{0}{0} = \lim_{x \rightarrow 3} \frac{x^2 - 3x}{(x - 3)(x + \sqrt{3x})} = \lim_{x \rightarrow 3} \frac{x(x - 3)}{(x - 3)(x + \sqrt{3x})} = \lim_{x \rightarrow 3} \frac{x}{x + \sqrt{3x}} = \frac{3}{6} = \frac{1}{2}$

i)  $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x^2 - 4x} = \frac{0}{0} = \lim_{x \rightarrow 4} \frac{x + 5 - 9}{(x^2 - 4x)(\sqrt{x+5} + 3)} = \lim_{x \rightarrow 4} \frac{x - 4}{x(x - 4)(\sqrt{x+5} + 3)} = \lim_{x \rightarrow 4} \frac{1}{x(\sqrt{x+5} + 3)} = \frac{1}{24}$

j)  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{\sqrt{x} - 1} = \frac{0}{0} = 0$

k)  $\lim_{x \rightarrow +\infty} \frac{(x-1)\sqrt{4x^3+3}}{\sqrt{x^5+1}} = \frac{\infty}{\infty} = \lim_{x \rightarrow +\infty} \frac{(x-1)\sqrt{4x^3}}{\sqrt{x^5}} = \lim_{x \rightarrow +\infty} \frac{(x-1)2x\sqrt{x}}{x^2\sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{2(x-1)}{x} = \lim_{x \rightarrow +\infty} \left(2 - \frac{2}{x}\right) = 2$

$$1) \lim_{x \rightarrow 2} \frac{\sqrt{x^2 - 3x + 6} - \sqrt{x + 2}}{3x - 6} = \frac{0}{0} =$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 3x + 6 - x - 2}{(3x - 6)(\sqrt{x^2 - 3x + 6} + \sqrt{x + 2})} = \lim_{x \rightarrow 2} \frac{x^2 - 2x + 4}{(3x - 6)(\sqrt{x^2 - 3x + 6} + \sqrt{x + 2})} =$$

$$= \lim_{x \rightarrow 2} \frac{(x - 2)^2}{3(x - 2)(\sqrt{x^2 - 3x + 6} + \sqrt{x + 2})} = \lim_{x \rightarrow 2} \frac{x - 2}{3(\sqrt{x^2 - 3x + 6} + \sqrt{x + 2})} = 0$$

3. Dada la función:

$$f(x) = \frac{x^3 - x^2 - 2x}{x^4 - x^2}$$

Hallar:

a)  $\lim_{x \rightarrow -1} f(x)$

b)  $\lim_{x \rightarrow 2} f(x)$

c)  $\lim_{x \rightarrow +\infty} f(x)$

d)  $\lim_{x \rightarrow 0} f(x)$

Solución:

$$a) \lim_{x \rightarrow -1} f(x) = \frac{0}{0} = \lim_{x \rightarrow -1} \frac{x^3 - x^2 - 2x}{x^4 - x^2} = \lim_{x \rightarrow -1} \frac{x(x-2)(x+1)}{x^2(x+1)(x-1)} = \lim_{x \rightarrow -1} \frac{x-2}{x(x-1)} = -\frac{3}{2}$$

$$b) \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x-2}{x(x-1)} = 0$$

$$c) \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{x-2}{x^2-x} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x} - \frac{2}{x^2}}{1 - \frac{1}{x}} = 0$$

$$d) \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{x-2}{x(x-1)} = \frac{-2}{0} = -\infty$$

4. Dada la función:

$$f(x) = \begin{cases} \frac{x^2 - 4x + 3}{x^2 - 1} & \text{si } x < 1 \\ \frac{1-x}{\sqrt{x}-1} & \text{si } x \geq 1 \end{cases}$$

a) Calcular  $\lim_{x \rightarrow -\infty} f(x)$ ,  $\lim_{x \rightarrow +\infty} f(x)$ ,  $\lim_{x \rightarrow 0} f(x)$

b) Demuestra que no existe  $\lim_{x \rightarrow 1} f(x)$

Solución:

$$a) \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x^2 - 4x + 3}{x^2 - 1} = \lim_{x \rightarrow -\infty} \frac{1 - \frac{4}{x} + \frac{3}{x^2}}{1 - \frac{1}{x^2}} = 1$$

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{1-x}{\sqrt{x}-1} = \lim_{x \rightarrow +\infty} -(\sqrt{x}+1) = -\infty$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{x^2 - 4x + 3}{x^2 - 1} = -3$$

$$b) \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{1-x}{\sqrt{x}-1} = \lim_{x \rightarrow 1^+} -(\sqrt{x}+1) = -2$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \frac{x^2 - 4x + 3}{x^2 - 1} = \lim_{x \rightarrow 1^-} \frac{(x-3)(x-1)}{(x-1)(x+1)} = \lim_{x \rightarrow 1^-} \frac{x-3}{x+1} = -1$$

Luego,  $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x) \rightarrow$  No existe  $\lim_{x \rightarrow 1} f(x)$

5. Dada la función:

$$f(x) = \frac{3x^3 - 7x^2 + 5x - 1}{mx^3 - 3x + 2}$$

Se sabe que  $\lim_{x \rightarrow 2} f(x) = \frac{5}{4}$ . Se pide:

a) Hallar  $m$

b)  $\lim_{x \rightarrow +\infty} f(x)$  y  $\lim_{x \rightarrow 1} f(x)$ .

Solución:

$$a) \lim_{x \rightarrow 2} f(x) = \frac{5}{4} \rightarrow \lim_{x \rightarrow 2} \frac{3x^3 - 7x^2 + 5x - 1}{mx^3 - 3x + 2} = \frac{5}{8m - 4} = \frac{5}{4} \rightarrow 8m - 4 = 4 \rightarrow m = 1$$

$$b) \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{3x^3 - 7x^2 + 5x - 1}{x^3 - 3x + 2} = \lim_{x \rightarrow +\infty} \frac{3 - \frac{7}{x} + \frac{5}{x^2} - \frac{1}{x^3}}{1 - \frac{3}{x^2} + \frac{2}{x^3}} = 3$$

$$\lim_{x \rightarrow 1} f(x) = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{3x^3 - 7x^2 + 5x - 1}{x^3 - 3x + 2} = \lim_{x \rightarrow 0} \frac{(x-1)^2(3x-1)}{(x-1)^2(x+2)} = \lim_{x \rightarrow 0} \frac{3x-1}{x+2} = -\frac{1}{2}$$