Examen de Matemáticas 2º de Bachillerato CS

Noviembre 2014

Problema 1 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{2x^2-3x+1}-\sqrt{2x^2+2x-1}\right)$

2. $\lim\limits_{x\longrightarrow 1}\dfrac{8x^4-5x^2-4x+1}{3x^5+x-4}$

3. $\lim\limits_{x\longrightarrow 5}\dfrac{\sqrt{3x^2-8}-\sqrt{12x+7}}{x-5}$

4. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{x^2+x-5}{x^2}\right)^{x-1}$

5. $\lim\limits_{x\longrightarrow\infty}\dfrac{\sqrt{5x^2+2x-1}}{-x+3}$

6. $\lim\limits_{x\longrightarrow 0}\dfrac{x^5-x^2+2x}{4x}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{2x^2-3x+1}-\sqrt{2x^2+2x-1}\right)=-\dfrac{5\sqrt{2}}{4}$

2. $\lim\limits_{x\longrightarrow 1}\dfrac{8x^4-5x^2-4x+1}{3x^5+x-4}=\dfrac{9}{8}$

3. $\lim\limits_{x\longrightarrow 5}\dfrac{\sqrt{3x^2-8}-\sqrt{12x+7}}{x-5}=\dfrac{9\sqrt{67}}{67}$

4. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{x^2+x-5}{x^2}\right)^{x-1}=e$

5. $\lim\limits_{x\longrightarrow\infty}\dfrac{\sqrt{3x^2-x+2}}{-x+8}=-\sqrt{5}$

6. $\lim\limits_{x\longrightarrow 0}\dfrac{x^5-x^2+2x}{4x}=\dfrac{1}{2}$

Problema 2 Calcular las siguientes derivadas:

1. $y=(3x^2+x-9)^{16}$

2. $y=\ln\left(\dfrac{\sqrt{x^2+1}}{x^3-1}\right)$

3. $y = \dfrac{e^{2x^2-1}}{\sqrt{x^2-1}}$

4. $y = e^{x^2+1}\ln(x+1)$

5. $y = (\ln(x^2+2))^{15}$

6. $y = xe^x$

Solución:

1. $y = (3x^2+x-9)^{16} \Longrightarrow y' = 16(3x^2+x-9)^{15}(6x+1)$

2. $y = \ln\left(\dfrac{\sqrt{x^2+1}}{x^3-1}\right) \Longrightarrow y' = \dfrac{2x}{2(x^2+1)} - \dfrac{3x^2}{x^3-1}$

3. $y = \dfrac{e^{2x^2-1}}{\sqrt{x^2-1}} \Longrightarrow y' = \dfrac{4xe^{2x^2-1}\sqrt{x^2-1} - e^{2x^2-1}\frac{2x}{2\sqrt{x^2-1}}}{x^2-1}$

4. $y = e^{x^2+1}\ln(x+1) \Longrightarrow y' = 2xe^{x^2+1}\ln(x+1) + e^{x^2+1}\frac{1}{x+1}$

5. $y = (\ln(x^2+2))^{15} \Longrightarrow y' = 15(\ln(x^2+2))^{14}\frac{2x}{x^2+2}$

6. $y = xe^x \Longrightarrow y' = e^x + xe^x$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{5x^2-1}{x^2+2}$ en el punto $x = 1$.

2. $f(x) = \dfrac{x^2+3}{2x-1}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = \dfrac{4}{3}$ e $y - b = m(x-a)$

$$f'(x) = -\frac{22x}{(x^2+2)^2} \Longrightarrow m = f'(1) = \frac{22}{9}$$

Recta Tangente: $y - \dfrac{4}{3} = \dfrac{22}{9}(x-1)$

Recta Normal: $y - \dfrac{4}{3} = -\dfrac{9}{22}(x-1)$

2. $b = f(a) \Longrightarrow b = f(0) = -3$ e $y - b = m(x-a)$

$$f'(x) = \frac{2(x^2-x-3)}{(2x-1)^2} \Longrightarrow m = f'(0) = -6$$

Recta Tangente: $y + 3 = -6x$

Recta Normal: $y + 3 = \dfrac{1}{6}x$