

1º- Calcular:

$$a) \int \left(\frac{\operatorname{arsen} x}{2\sqrt{1-x^2}} \right) dx$$

Solución:

$$\int \left(\frac{\operatorname{arsen} x}{2\sqrt{1-x^2}} \right) dx = \frac{1}{2} \int \left(\frac{\operatorname{arsen} x}{\sqrt{1-x^2}} \right) dx = \frac{1}{4} \operatorname{arsen}^2 x + k$$

$$b) \int \left(\frac{x^5 + x^4 - 8}{x^3 - 4x} \right) dx$$

Solución:

$$\begin{aligned} \int \left(\frac{x^5 + x^4 - 8}{x^3 - 4x} \right) dx &= \int \left(x^2 + x + 4 + \frac{2}{x} - \frac{3}{x+2} + \frac{5}{x-2} \right) dx = \\ &= \int x^2 dx + \int x dx + \int 4 dx + \int \frac{2}{x} dx - \int \frac{3}{x+2} dx + \int \frac{5}{x-2} dx = \\ &= \frac{x^3}{3} + \frac{x^2}{2} + 4x + 2 \ln|x| - 3 \ln|x+2| + 5 \ln|x-2| + k = \\ &= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \ln \left(k \frac{x^2(x-2)^5}{(x+2)^3} \right) \end{aligned}$$

2º.- Calcular:

$$a) \int_1^e \ln x dx$$

Solución:

$$\begin{aligned} \int_1^e \ln x dx &\Leftrightarrow \begin{cases} u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = dx \rightarrow v = \int dx = x \end{cases} \Rightarrow x \ln x - \int x \cdot \frac{1}{x} dx \Big|_1^e = x \ln x - x \Big|_1^e = \\ &= (e \cdot \ln(e) - e) - (1 \cdot \ln 1 - 1) = 1 \end{aligned}$$

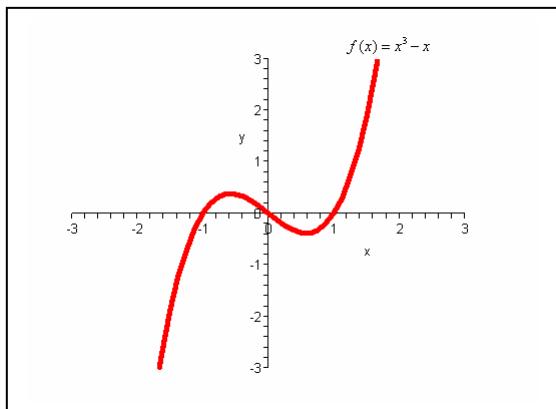
$$\text{b) } \int_0^1 \frac{2}{\sqrt{4-x^2}} dx$$

Solución:

Haciendo el cambio $x = 2\operatorname{sen}t$, $dx = 2\cos t$, luego

$$\begin{aligned}\int_0^1 \frac{2}{\sqrt{4-x^2}} dx &= \int_0^{\frac{\pi}{6}} \frac{2}{\sqrt{4-4\operatorname{sen}^2 t}} 2\cos t dt = \int_0^{\frac{\pi}{6}} \frac{2}{2\sqrt{1-\operatorname{sen}^2 t}} 2\cos t dt = \\ &= \int_0^{\frac{\pi}{6}} \frac{2}{2\cos t} 2\cos t dt = \int_0^{\frac{\pi}{6}} 2 dt = 2t \Big|_0^{\frac{\pi}{6}} = \frac{\pi}{3}\end{aligned}$$

3º.- Calcular el área limitada por la gráfica de la función $f(x) = x^3 - x$, el eje de abscisas y las rectas $x = -2$ y $x = 2$



Solución:

$$\begin{aligned}S &= - \int_{-2}^{-1} (x^3 - x) dx + \int_{-1}^0 (x^3 - x) dx - \\ &\quad - \int_0^1 (x^3 - x) dx + \int_1^2 (x^3 - x) dx = \\ &= \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{9}{4} = 5 \text{ un. de área}\end{aligned}$$