

## Integrales inmediatas de tipo potencial

$$\begin{aligned} \bullet \int x^2 (3x^3 + 14)^3 dx &= \frac{1}{9} \int 9x^2 (3x^3 + 14)^3 dx = \frac{1}{9} \frac{1}{4} \int 4 \cdot 9x^2 (3x^3 + 14)^3 dx = \\ &= \frac{1}{36} (3x^3 + 14)^4 + C \end{aligned}$$

$$\begin{aligned} \bullet \int \sqrt[5]{5x+6} dx &= \int (5x+6)^{1/5} dx = \frac{1}{5} \int 5(5x+6)^{1/5} dx = \\ &= \frac{1}{5} \frac{(5x+6)^{1/5+1}}{\frac{1}{5}+1} = \frac{1}{6} \sqrt[5]{(5x+6)^6} + C \end{aligned}$$

$$\begin{aligned} \bullet \int \frac{17x}{\sqrt[3]{6x^2+8}} dx &= 17 \int x (6x^2+8)^{-1/3} dx = 17 \frac{1}{12} \int 12x (6x^2+8)^{-1/3} dx = \\ &= \frac{17}{12} \frac{(6x^2+8)^{-1/3+1}}{-\frac{1}{3}+1} = \frac{51}{24} \sqrt[3]{(6x^2+8)^2} + C \end{aligned}$$

$$\bullet \int \frac{\operatorname{arctg} x}{1+x^2} dx = \int \frac{1}{1+x^2} \operatorname{arctg} x dx = \frac{1}{2} \int 2 \frac{1}{1+x^2} \operatorname{arctg} x dx = \frac{1}{2} (\operatorname{arctg} x)^2 + C$$

$$\bullet \int \sec^2 x \sqrt{\operatorname{tg} x} dx = \int \frac{1}{\cos^2 x} (\operatorname{tg} x)^{1/2} dx = \frac{(\operatorname{tg} x)^{1/2+1}}{\frac{1}{2}+1} = \frac{2}{3} (\operatorname{tg} x)^{3/2} + C$$

$$\begin{aligned} \bullet \int \frac{dx}{(3x+1)^4} &= \int (3x+1)^{-4} dx = \frac{1}{3} \int 3(3x+1)^{-4} dx = \frac{1}{3} \frac{(3x+1)^{-4+1}}{-4+1} = \\ &= -\frac{1}{9} (3x+1)^{-3} + C \end{aligned}$$

$$\bullet \int \cos x \cdot \operatorname{sen}^3 x dx = \frac{1}{4} \int 4 \cos x \operatorname{sen}^3 x dx = \frac{1}{4} \operatorname{sen}^4 x + C$$

$$\begin{aligned} \bullet \int \frac{(x+3)}{(x^2+6x)^{1/3}} dx &= \int (x+3)(x^2+6x)^{-1/3} dx = \frac{1}{2} \int 2(2x+6)(x^2+6x)^{-1/3} dx = \\ &= \frac{1}{2} \int (2x+6)(x^2+6x)^{-1/3} dx = \frac{1}{2} \frac{(x^2+6x)^{-1/3+1}}{-\frac{1}{3}+1} = \frac{3}{4} (x^2+6x)^{2/3} + C \end{aligned}$$

$$\begin{aligned} \bullet \int \sqrt{x^2-2x^4} dx &= \int \sqrt{x^2(1-2x^2)} dx = \int x \sqrt{1-2x^2} = \int x (1-2x^2)^{1/2} dx = \\ &= -\frac{1}{4} \int -4x (1-2x^2)^{1/2} dx = -\frac{1}{4} \frac{(1-2x^2)^{1/2+1}}{\frac{1}{2}+1} = -\frac{1}{6} (1-2x^2)^{3/2} + C \end{aligned}$$

## Integrales inmediatas de tipo exponencial

$$\bullet \int x^2 \cdot 7^{x^3+5} dx = \frac{1}{3} \int 3x^2 \cdot 7^{x^3+5} dx = \frac{1}{3 \ln 7} 7^{x^3+5} + C$$

$$\bullet \int \frac{5^{\sqrt{x}}}{\sqrt{x}} dx = \int \frac{1}{\sqrt{x}} 5^{\sqrt{x}} dx = 2 \int \frac{1}{2\sqrt{x}} 5^{\sqrt{x}} dx = \frac{2}{\ln 5} \int \frac{1}{2\sqrt{x}} 5^{\sqrt{x}} \ln 5 dx = \\ = \frac{2}{\ln 5} 5^{\sqrt{x}} + C$$

$$\bullet \int \frac{e^{\arcsen x}}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} e^{\arcsen x} dx = e^{\arcsen x} + C$$

$$\bullet \int \cos 3x \cdot e^{\sen 3x} dx = \frac{1}{3} \int 3 \cos 3x \cdot e^{\sen 3x} dx = \frac{1}{3} e^{\sen 3x} + C$$

$$\bullet \int \frac{6^{\ln x}}{x} dx = \int \frac{1}{x} 6^{\ln x} dx = \frac{1}{\ln 6} \int \frac{1}{x} 6^{\ln x} \ln 6 dx = \frac{1}{\ln 6} 6^{\ln x} + C$$

## Integrales inmediatas de tipo logarítmico

$$\bullet \int \frac{\sen x - \cos x}{\sen x + \cos x} dx = - \int \frac{\sen x - \cos x}{\sen x + \cos x} dx = - \int \frac{\cos x - \sen x}{\sen x + \cos x} dx = \\ = - \ln |\sen x + \cos x| + C$$

$$\bullet \int \frac{x}{1+x^2} dx = \frac{1}{2} \int \frac{2x}{1+x^2} dx = \frac{1}{2} \ln |x^2+1| + C$$

$$\bullet \int \frac{27x^2+30x+3}{3x^3+5x^2+x-1} dx = \int \frac{3(9x^2+10x+1)}{3x^3+5x^2+x-1} dx = 3 \int \frac{9x^2+10x+1}{3x^3+5x^2+x-1} dx = \\ = 3 \ln |3x^3+5x^2+x-1| + C$$

$$\bullet \int \frac{1}{\operatorname{tg} x} dx = \int \frac{1}{\frac{\sen x}{\cos x}} dx = \int \frac{\cos x}{\sen x} dx = \ln |\sen x| + C$$

$$\bullet \int \frac{7^{2x}}{7^{2x}+5} dx = \frac{1}{2} \int \frac{2 \cdot 7^{2x}}{7^{2x}+5} dx = \frac{1}{2} \frac{1}{\ln 7} \int \frac{2 \cdot 7^{2x} \cdot \ln 7}{7^{2x}+5} dx = \\ = \frac{1}{2 \ln 7} \ln |7^{2x}+5| + C$$

$$\bullet \int \frac{1}{x \ln x} dx = \int \frac{\frac{1}{x}}{\ln x} dx = \ln |\ln x| + C$$

$$\bullet \int \frac{1}{\cos^2 x \cdot \operatorname{tg} x} dx = \int \frac{\frac{1}{\cos^2 x}}{\operatorname{tg} x} dx = \ln |\operatorname{tg} x| + C$$

$$\bullet \int \frac{1}{\sqrt{x}(1-\sqrt{x})} dx = \int \frac{\frac{1}{\sqrt{x}}}{1-\sqrt{x}} dx = -2 \int \frac{\frac{1}{-2\sqrt{x}}}{1-\sqrt{x}} dx = -2 \ln |1-\sqrt{x}| + C$$

### Integración por partes

$$\bullet \int \operatorname{arctg} x dx = \left[ \begin{array}{l} u = \operatorname{arctg} x \longrightarrow du = \frac{1}{1+x^2} dx \\ dv = dx \longrightarrow v = x \end{array} \right] = x \cdot \operatorname{arctg} x -$$

$$- \int x \frac{1}{1+x^2} dx = x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx = x \operatorname{arctg} x - \frac{1}{2} \ln |1+x^2| + C$$

$$\bullet \int \operatorname{arcsen} x dx = \left[ \begin{array}{l} u = \operatorname{arcsen} x \longrightarrow du = \frac{1}{\sqrt{1-x^2}} dx \\ dv = dx \longrightarrow v = x \end{array} \right] =$$

$$= x \operatorname{arcsen} x - \int \frac{x}{\sqrt{1-x^2}} dx = x \operatorname{arcsen} x - \underbrace{\frac{1}{2} \int \frac{2x(1-x^2)^{-1/2}}{2} dx}_{\text{inmediata de tipo potencial}} =$$

$$= x \operatorname{arcsen} x - (1-x^2)^{1/2} = x \operatorname{arcsen} x - \sqrt{1-x^2} + C$$

$$\bullet \int \ln(x+1) dx = \left[ \begin{array}{l} u = \ln(x+1) \longrightarrow du = \frac{1}{x+1} dx \\ dv = dx \longrightarrow v = x \end{array} \right] =$$

$$= x \ln |x+1| - \int x \frac{1}{x+1} dx = x \ln |x+1| - x + \ln |x+1| + C$$

Calculamos  $\int \frac{x}{x+1} dx = \int 1 dx - \int \frac{1}{x+1} dx = x - \ln |x+1|$

$$\frac{x}{-x-1} \cdot \frac{1}{1} \quad \text{de donde} \quad \frac{x}{x+1} = 1 - \frac{1}{x+1}$$

$$\bullet \int x \operatorname{arctg} x dx = \left[ \begin{array}{l} u = \operatorname{arctg} x \longrightarrow du = \frac{1}{1+x^2} dx \\ dv = x dx \longrightarrow v = \frac{x^2}{2} \end{array} \right] =$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \int \frac{x^2}{2} \frac{1}{1+x^2} dx = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \quad \textcircled{4}$$



$$\text{Calculamos } \int \frac{x^2}{1+x^2} dx = \int 1 dx - \int \frac{1}{1+x^2} dx = x - \arctg x$$

$$\frac{x^2}{-x^2-1} \frac{|x^2+1|}{1} \text{ de donde } \frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$$

$$\stackrel{\textcircled{2}}{=} \frac{x^2}{2} \arctg x - \frac{1}{2} (x - \arctg x) + C$$

$$\bullet \int x \ln(x+1) dx = \left[ \begin{array}{l} u = \ln(x+1) \longrightarrow du = \frac{1}{x+1} dx \\ dv = x dx \longrightarrow v = \frac{x^2}{2} \end{array} \right] =$$

$$= \frac{x^2}{2} \ln|x+1| - \int \frac{x^2}{2} \frac{1}{x+1} dx = \frac{x^2}{2} \ln|x+1| - \frac{1}{2} \int \frac{x^2}{x+1} dx \stackrel{\textcircled{3}}{=}$$

$$\text{Calculamos } \int \frac{x^2}{1+x} dx = \int (x-1) dx + \int \frac{1}{x+1} dx = \frac{x^2}{2} - x + \ln|x+1|$$

$$\frac{x^2}{-x^2-x} \frac{|x+1|}{x-1} \text{ de donde } \frac{x^2}{x+1} = x-1 + \frac{1}{x+1}$$

$$\stackrel{\textcircled{4}}{=} \frac{x^2}{2} \ln|x+1| - \frac{1}{2} \left( \frac{x^2}{2} - x + \ln|x+1| \right) + C$$

$$\bullet \int (3x^2+2x-7) \cos x dx = \left[ \begin{array}{l} u = 3x^2+2x-7 \longrightarrow du = (6x+2) dx \\ dv = \cos x dx \longrightarrow v = \text{sen } x \end{array} \right] =$$

$$= (3x^2+2x-7) \text{sen } x - \underbrace{\int (6x+2) \text{sen } x dx}_{\text{por partes}} \stackrel{\textcircled{5}}{=}$$

$$\text{Calculamos } \int (6x+2) \text{sen } x dx = \left[ \begin{array}{l} u = 6x+2 \longrightarrow du = 6 dx \\ dv = \text{sen } x dx \longrightarrow v = -\cos x \end{array} \right] =$$

$$= -(6x+2) \cos x + 6 \int \cos x dx = -(6x+2) \cos x + 6 \text{sen } x$$

$$\stackrel{\textcircled{6}}{=} (3x^2+2x-7) \text{sen } x + (6x+2) \cos x - 6 \text{sen } x + C$$

$$\bullet \int (5x^2-3) 4^{3x+1} dx = \left[ \begin{array}{l} u = 5x^2-3 \longrightarrow du = 10x dx \\ dv = 4^{3x+1} dx \longrightarrow v = \int 4^{3x+1} dx = \frac{1}{4 \ln 4} 4^{3x+1} \end{array} \right] =$$

inmediata de tipo exponencial

$$= (5x^2-3) \frac{1}{4 \ln 4} 4^{3x+1} - \frac{1}{4 \ln 4} \underbrace{\int 10x \cdot 4^{3x+1} dx}_{\text{por partes}} \stackrel{\textcircled{7}}{=}$$

$$\text{Calculamos } \int 10x \cdot 4^{3x+1} dx = \left[ \begin{array}{l} u = 10x \rightarrow du = 10 dx \\ dv = 4^{3x+1} dx \rightarrow v = \frac{1}{3 \ln 4} 4^{3x+1} \end{array} \right] =$$

$$= 10x \frac{1}{3 \ln 4} 4^{3x+1} - \frac{1}{3 \ln 4} 10 \int 4^{3x+1} dx =$$

$$= 10x \frac{1}{3 \ln 4} 4^{3x+1} - \frac{1}{3 \ln 4} 10 \frac{1}{3 \ln 4} 4^{3x+1}$$

$$\stackrel{\textcircled{2}}{=} (5x^2 - 3) \frac{1}{3 \ln 4} 4^{3x+1} - \frac{1}{3 \ln 4} \left( 10x \frac{1}{3 \ln 4} 4^{3x+1} - \frac{1}{3 \ln 4} 10 \frac{4^{3x+1}}{3 \ln 4} \right) + C$$

$$\bullet \int \frac{x}{\operatorname{sen}^2 x} dx = \left[ \begin{array}{l} u = x \rightarrow du = dx \\ dv = \frac{1}{\operatorname{sen}^2 x} dx \rightarrow v = \int \frac{1}{\operatorname{sen}^2 x} dx = -\operatorname{cotg} x \end{array} \right] =$$

$$= -x \operatorname{cotg} x + \int \operatorname{cotg} x dx = -x \operatorname{cotg} x + \int \frac{\cos x}{\operatorname{sen} x} dx =$$

$$= -x \operatorname{cotg} x + \ln |\operatorname{sen} x| + C$$

$$\bullet \int \sec^4 x dx = \int \frac{1}{\cos^4 x} dx = \int \frac{1}{\cos^2 x \cdot \cos^2 x} dx = \int \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x} dx =$$

$$= \left[ \begin{array}{l} u = \frac{1}{\cos^2 x} \rightarrow du = \frac{2 \operatorname{sen} x}{\cos^3 x} dx \\ dv = \frac{1}{\cos^2 x} dx \rightarrow v = \operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x} \end{array} \right] = \frac{\operatorname{sen} x}{\cos^3 x} - 2 \int \frac{\operatorname{sen}^2 x}{\cos^4 x} dx =$$

$$= \frac{\operatorname{sen} x}{\cos^3 x} - 2 \int \frac{1 - \cos^2 x}{\cos^4 x} dx = \frac{\operatorname{sen} x}{\cos^3 x} - 2 \int \frac{1}{\cos^4 x} dx + 2 \int \frac{1}{\cos^2 x} dx =$$

$$= \frac{\operatorname{sen} x}{\cos^3 x} - 2 \int \frac{1}{\cos^4 x} dx + 2 \operatorname{tg} x \Rightarrow \left( \text{pasando } -2 \int \frac{1}{\cos^4 x} dx \text{ a la izqda.} \right)$$

Es la integral que estamos calculando

$$3 \int \frac{1}{\cos^4 x} dx = \frac{\operatorname{sen} x}{\cos^3 x} + 2 \operatorname{tg} x \Rightarrow \int \frac{1}{\cos^4 x} dx = \frac{1}{3} \left( \frac{\operatorname{sen} x}{\cos^3 x} + 2 \operatorname{tg} x \right) + C$$

$$\bullet \int \frac{\ln x}{\sqrt{x}} dx = \left[ \begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = \frac{1}{\sqrt{x}} dx \rightarrow v = \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} \end{array} \right] =$$

$$= 2\sqrt{x} \ln x - 2 \int \frac{\sqrt{x}}{x} dx = 2\sqrt{x} \ln x - 4\sqrt{x} + C$$

## Integración por cambio de variable

$$\begin{aligned} \bullet \int \frac{3x}{\sqrt{1+7x^2}} dx &= \left[ \begin{array}{l} t = \sqrt{1+7x^2} \rightarrow t^2 = 1+7x^2 \\ dx = \frac{t}{7x} dt \end{array} \right] = \int \frac{3x}{\sqrt{t^2}} \frac{t}{7x} dt = \\ &= \frac{3}{7} \int dt = \frac{3t}{7} = \frac{3}{7} \sqrt{1+7x^2} + C \end{aligned}$$

También se puede resolver como una integral inmediata

$$\begin{aligned} \bullet \int \frac{5}{x \ln x} dx &= \left[ \begin{array}{l} t = \ln x \rightarrow dt = \frac{1}{x} dx \\ dx = x dt \end{array} \right] = \int \frac{5}{x \cdot t} x dt = \\ &= 5 \int \frac{1}{t} dt = 5 \ln |t| = 5 \ln |\ln x| + C \end{aligned}$$

$$\begin{aligned} \bullet \int \frac{5}{\sqrt{9-4x^2}} dx &= \left[ \begin{array}{l} 2x = 3 \operatorname{sen} t \rightarrow x = \frac{3}{2} \operatorname{sen} t \\ dx = \frac{3}{2} \cos t dt \end{array} \right] = \int \frac{5}{\sqrt{9-9 \operatorname{sen}^2 t}} \frac{3}{2} \cos t dt = \\ &\text{En este caso el cambio de variable nos lo dan} \end{aligned}$$

$$= \frac{15}{2} \int \frac{\cos t}{\sqrt{9} \sqrt{1-\operatorname{sen}^2 t}} dt = \frac{5}{2} \int \frac{\cos t}{\cos t} dt = \frac{5}{2} \int dt = \frac{5}{2} t =$$

$$= \frac{5}{2} \operatorname{arc} \operatorname{sen} \left( \frac{2x}{3} \right) + C \quad \boxed{\operatorname{sen}^2 t + \cos^2 t = 1} \quad \text{Hay que sabérsela}$$

$$\begin{aligned} \text{ya que si } x = \frac{3}{2} \operatorname{sen} t &\rightarrow \frac{2x}{3} = \operatorname{sen} t \rightarrow \operatorname{arc} \operatorname{sen} \left( \frac{2x}{3} \right) = \operatorname{arc} \operatorname{sen} (\operatorname{sen} t) \rightarrow \\ &\rightarrow \operatorname{arc} \operatorname{sen} \left( \frac{2x}{3} \right) = t \end{aligned}$$

$$\bullet \int \sqrt{4-x^2} dx = \left[ \begin{array}{l} x = 2 \operatorname{sen} t \rightarrow dx = 2 \cos t dt \\ \text{El cambio de variable nos lo dan} \end{array} \right] = \int \sqrt{4-4 \operatorname{sen}^2 t} \cdot 2 \cos t dt =$$

$$= \int 2 \cos t \cdot 2 \cos t dt = 4 \int \cos^2 t dt = 4 \int \frac{1 + \cos 2t}{2} dt =$$

$$= 2 \int (1 + \cos 2t) dt = 2 \left( t + \frac{\operatorname{sen} 2t}{2} \right) = 2t + \operatorname{sen} 2t + C =$$

$$= 2 \operatorname{arc} \operatorname{sen} \left( \frac{x}{2} \right) + \frac{x \sqrt{4-x^2}}{2} + C$$

$$\text{ya que si } x = 2 \operatorname{sen} t \rightarrow \operatorname{sen} t = \frac{x}{2} \rightarrow t = \operatorname{arc} \operatorname{sen} \left( \frac{x}{2} \right)$$

$$\text{y } \boxed{\operatorname{sen}(2t) = 2 \operatorname{sen} t \cos t} = 2 \operatorname{sen} t \cdot \sqrt{1-\operatorname{sen}^2 t} = x \sqrt{1-\left(\frac{x}{2}\right)^2} = \frac{x \sqrt{4-x^2}}{2}$$

Hay que sabérsela

$$\bullet \int \frac{1}{3+2x} dx = \left[ \begin{array}{l} t = 3+2x \rightarrow dt = 2 dx \\ dx = \frac{1}{2} dt \end{array} \right] = \int \frac{1}{t} \cdot \frac{1}{2} dt =$$

$$= \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \ln |t| = \frac{1}{2} \ln |3+2x| + C$$

También se puede resolver como una integral inmediata

$$\bullet \int \frac{x}{2-x^2} dx = \left[ \begin{array}{l} t = 2-x^2 \rightarrow dt = -2x dx \\ dx = \frac{-1}{2x} dt \end{array} \right] = \int \frac{1}{t} \cdot \frac{-1}{2x} x dt =$$

$$= -\frac{1}{2} \int \frac{1}{t} dt = -\frac{1}{2} \ln |t| = -\frac{1}{2} \ln |2-x^2| + C$$

$$\bullet \int e^{\sin x} \cdot \cos x dx = \left[ \begin{array}{l} t = \sin x \rightarrow dt = \cos x dx \\ \text{es lo que} \\ \text{tenemos en la integral} \end{array} \right] = \int e^t dt =$$

$$= e^t = e^{\sin x} + C$$

$$\bullet \int \frac{x}{1+x^4} dx = \left[ \begin{array}{l} t = x^2 \rightarrow dt = 2x dx \\ \frac{dt}{2} = x dx \end{array} \right] = \int \frac{1}{1+t^2} \cdot \frac{1}{2} dt =$$

$$= \frac{1}{2} \int \frac{1}{1+t^2} dt = \frac{1}{2} \operatorname{arctg} t = \frac{1}{2} \operatorname{arctg} x^2 + C$$