EJERCICIOS Resueltos de REGLA DE L ' HÔPITAL

Indeterminaciones $\left[\dfrac{0}{0}\right]$

- $$\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\cos x}{1} = \frac{\cos 0}{1} = \frac{1}{1} = \mathbf{1}$$

- $$\lim_{x\to 0} \frac{e^x - 1}{x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{e^x}{1} = \frac{e^0}{1} = \frac{1}{1} = \mathbf{1}$$

- $$\lim_{x\to 1} \frac{\ln x}{x^2 + x - 2} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{\frac{1}{x}}{2x + 1} = \lim_{x\to 1} \frac{1}{2x^2 + x} = \frac{1}{2\cdot 1^2 + 1} = \mathbf{\frac{1}{3}}$$

- $$\lim_{x\to 0} \frac{x - \operatorname{sen} x}{x^3} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{1 - \cos x}{3x^2} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\operatorname{sen} x}{6x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\cos x}{6} = \mathbf{\frac{1}{6}}$$

- $$\lim_{x\to 0} \frac{\operatorname{tg} 3x}{\operatorname{tg} x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\frac{3}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \frac{\frac{3}{1}}{\frac{1}{1}} = \mathbf{3}$$

- $$\lim_{x\to 0} \frac{\operatorname{sen} 3x}{x - \frac{3}{2}\operatorname{sen} 2x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{3\cdot\cos 3x}{1 - \frac{3}{\cancel{2}}\cdot\cos 2x\cdot\cancel{2}} = \lim_{x\to 0} \frac{3\cdot\cos 3x}{1 - 3\cos 2x} = \frac{3\cdot 1}{1 - 3\cdot 1} = \mathbf{-\frac{3}{2}}$$

- $$\lim_{x\to 0} \frac{\operatorname{sen} x - x\cos x}{\operatorname{sen}^3 x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\cos x - (\cos x - x\operatorname{sen} x)}{3\operatorname{sen}^2 x\cos x} = \lim_{x\to 0} \frac{x\cancel{\operatorname{sen} x}}{3\operatorname{sen}^{\cancel{2}} x\cos x} =$$

$$= \lim_{x\to 0} \frac{x}{3\operatorname{sen} x\cos x} \overset{\operatorname{sen} 2x = 2\operatorname{sen} x\cos x}{=} \lim_{x\to 0} \frac{x}{\frac{3}{2}\operatorname{sen} 2x} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 0} \frac{1}{\frac{3}{\cancel{2}}\cdot 2\cos 2x} = \lim_{x\to 0} \frac{x}{3\cos 2x} = \frac{2}{3\cdot\cos 0} = \frac{2}{3\cdot 1} = \mathbf{\frac{2}{3}}$$

- $$\lim_{x\to 0} \frac{x\operatorname{arc\,sen} x}{\operatorname{sen} x\cos x} = \left[\frac{0}{0}\right] \overset{\operatorname{sen} 2x = 2\operatorname{sen} x\cos x}{=} \lim_{x\to 0} \frac{x\operatorname{arc\,sen} x}{\frac{1}{2}\operatorname{sen} 2x} = \lim_{x\to 0} \frac{2x\operatorname{arc\,sen} x}{\operatorname{sen} 2x} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 0} \frac{2\cdot\operatorname{arc\,sen} x + \frac{2x}{1 + x^2}}{2\cdot\cos 2x} = \frac{2\cdot\operatorname{arc\,sen} 0 + \frac{2\cdot 0}{1 + 0^2}}{2\cdot\cos 0} = \frac{2\cdot 0 + \frac{2\cdot 0}{1 + 0^2}}{2\cdot 1} = \mathbf{0}$$

- $$\lim_{x\to 0} \frac{a^x - b^x}{x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{a^x\cdot\ln a - b^x\cdot\ln b}{1} = \ln \mathbf{a} - \ln \mathbf{b}$$

- $$\lim_{x\to 0^+} \frac{x - 1}{x^n - 1} = \left[\frac{0}{0}\right] = \lim_{x\to 0^+} \frac{x - 1}{x^n - 1} = \lim_{x\to 0^+} \frac{1}{n\cdot x^{n-1}} = \left[\frac{1}{0^+}\right] = +\infty$$

Indeterminaciones $\left[\dfrac{\infty}{\infty}\right]$ **e Inteterminaciones** $\mathbf{0 \cdot \infty}$

- $$\lim_{x\to+\infty} \frac{3x^2 + 2x - 1}{4x^2 + 6} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty} \frac{6x + 2}{8x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty} \frac{6}{8} = \frac{3}{4}$$

- $$\lim_{x\to+\infty} \frac{e^x}{x^2} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty} \frac{e^x}{2x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty} \frac{e^x}{2} = \left[\frac{+\infty}{2}\right] = +\infty$$

- $$\lim_{x\to+\infty} \frac{2^x}{x} = \lim_{x\to+\infty} \frac{2^x \cdot \ln 2}{1} = \left[\frac{+\infty}{1}\right] = +\infty$$

- $$\lim_{x\to+\infty} \frac{\ln^2 x}{\sqrt{x}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty} \frac{2\cdot \ln x \cdot \frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to+\infty} \frac{4\cdot\sqrt{x}\cdot \ln x}{x} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to+\infty} \frac{4\cdot \frac{1}{2\sqrt{x}}\cdot \ln x + \frac{1}{x}\cdot\sqrt{x}}{1} = \lim_{x\to+\infty} \left(\frac{2\cdot\ln x}{x} + \frac{\sqrt{x}}{x}\right) \overset{\frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}}{=}$$

$$\overset{\frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}}{=} \lim_{x\to+\infty} \left(\frac{2\cdot\ln x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right) = \lim_{x\to+\infty} \frac{2\cdot\ln x + 1}{\sqrt{x}} = \left[\frac{+\infty}{\infty}\right] =$$

$$= \lim_{x\to+\infty} \frac{2\cdot\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to+\infty} \frac{4\cdot\sqrt{x}}{x} \overset{\frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}}{=} \lim_{x\to+\infty} \frac{4}{\sqrt{x}} = \left[\frac{4}{\infty}\right] = 0$$

- $$\lim_{x\to\pi} \left[(x - \pi)\right]\cdot \operatorname{tg}\frac{x}{2} = [\,0\cdot\infty\,] = \lim_{x\to\pi} \frac{\operatorname{tg}\frac{x}{2}}{\frac{1}{x-\pi}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\pi} \frac{\frac{1}{2}\cdot\frac{1}{\cos^2\frac{x}{2}}}{\frac{-1}{(x-\pi)^2}} =$$

$$= \lim_{x\to\pi} \frac{-(x-\pi)^2}{2\cdot\cos^2\frac{x}{2}} = \left[\frac{0}{0}\right] = \lim_{x\to\pi} \frac{-2\cdot(x-\pi)}{2\cdot \cancel{2}\cdot\cos\frac{x}{2}\cdot\left(-\operatorname{sen}\frac{x}{2}\right)\cdot\frac{1}{\cancel{2}}} \overset{2\operatorname{sen}\frac{x}{2}\cos\frac{x}{2} = \operatorname{sen} x}{=}$$

$$\overset{2\operatorname{sen}\frac{x}{2}\cos\frac{x}{2} = \operatorname{sen} x}{=} \lim_{x\to\pi} \frac{-2\cdot(x-\pi)}{-\operatorname{sen} x} = \left[\frac{0}{0}\right] = \lim_{x\to\pi} \frac{-2}{\cos x} = \frac{-2}{-\cos\pi} = \frac{-2}{-1} = -2$$

- $$\lim_{x\to\frac{\pi}{2}} (\sec x - \operatorname{tg} x) = [\,\infty - \infty\,] = \lim_{x\to\frac{\pi}{2}} \left(\frac{1}{\cos x} - \frac{\operatorname{sen} x}{\cos x}\right) = \lim_{x\to\frac{\pi}{2}} \frac{1 - \operatorname{sen} x}{\cos x} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{-\operatorname{sen} x} = \frac{-\cos\frac{\pi}{2}}{-\operatorname{sen}\frac{\pi}{2}} = \frac{0}{-1} = 0$$

- $$\lim_{x\to 1}\left(\frac{\mathbf{e}}{\mathbf{e^x - e}} - \frac{\mathbf{1}}{\mathbf{x - 1}}\right) = [\,\infty - \infty\,] = \lim_{x\to 1}\frac{e\cdot x \cancel{- e} - e^x \cancel{+ e}}{(e^x - e)\cdot(x - 1)} = \lim_{x\to 1}\frac{e\cdot x - e}{x\cdot e^x - e\cdot x + e^x + e} =$$

$$= \frac{e - e}{e + e - e - e} = \left[\frac{0}{0}\right] = \lim_{x\to 1}\frac{-\,e^x}{\cancel{e^x} + e^x + x\cdot e^x \cancel{- e^x}} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 1}\frac{e - e^x}{e^x + x\cdot e^x - e - \cancel{-e^x}} = \lim_{x\to 1}\frac{-\,e^x}{e^x + x\cdot e^x} = \frac{-\,e}{e + e} = \frac{-\,\cancel{e}}{2\cancel{e}} = \mathbf{-\frac{1}{2}}$$

Indeterminaciones $\mathbf{1^{\infty}}\ ,\ \mathbf{0^0}\ ,\ \mathbf{\infty^0}\ \cdots$

- $$\lim_{\mathbf{x\to 0}}\left(\frac{\mathbf{2^x + 3^x}}{\mathbf{2}}\right)^{\frac{1}{x}} = [\,\infty^0\,] = \mathbf{M}$$

Tomando logaritmo neperiano en ambos miembros:

$$\ln\mathbf{M} = \ln\left[\lim_{x\to\infty}\left(\frac{2^x + 3^x}{2}\right)^{\frac{1}{x}}\right] = \lim_{x\to 0}\ln\left[\left(\frac{2^x + 3^x}{2}\right)^{\frac{1}{x}}\right] = \lim_{x\to 0}\frac{1}{x}\cdot\ln\left(\frac{2^x + 3^x}{2}\right) =$$

$$= [\,0\cdot\infty\,] = \lim_{x\to 0}\frac{\ln\left(\frac{2^x + 3^x}{2}\right)}{x} = \left[\frac{0}{0}\right] = \lim_{x\to 0}\frac{\frac{\cancel{2}}{2^x + 3^x}\cdot\frac{2^x\cdot\ln 2 + 3^x\cdot\ln 3}{\cancel{2}}}{2} = \frac{\frac{\ln 2 + \ln 3}{2^0 + 3^0}}{2} =$$

$$= \frac{\ln 2 + \ln 3}{4} \Rightarrow \mathbf{M} = \mathbf{e^{\frac{\ln 2 + \ln 3}{4}}}$$

- $$\lim_{\mathbf{x\to+\infty}}\left(\mathbf{x + e^x + e^{2x}}\right)^{\frac{1}{x}} = [\,1^{\infty}\,] = \mathbf{M}$$

Tomando logaritmo neperiano en ambos miembros:

$$\ln\mathbf{M} = \ln\left(x + e^x + e^{2x}\right)^{\frac{1}{x}} = \lim_{x\to+\infty}\ln\left(x + e^x + e^{2x}\right)^{\frac{1}{x}} = \lim_{x\to+\infty}\frac{\ln\left(x + e^x + e^{2x}\right)}{x} =$$

$$= \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty}\frac{1 + e^x + 2\cdot e^x}{x + e^x + e^{2x}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty}\frac{e^x + 4\cdot e^x}{1 + e^x + 2\cdot e^{2x}} =$$

$$= \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty}\frac{e^x + 8\cdot e^x}{e^x + 4\cdot e^{2x}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty}\frac{\frac{e^x}{e^{2x}} + \frac{8\cdot e^{2x}}{e^{2x}}}{\frac{e^x}{e^{2x}} + \frac{4\cdot e^{2x}}{e^{2x}}} =$$

$$= \lim_{x\to+\infty}\frac{e^{-x} + 8}{e^{-x} + 4} = \frac{0 + 8}{0 + 4} = 2 \Rightarrow \mathbf{M} = \mathbf{e^2}$$

- $\lim_{\mathbf{x}\to\frac{\pi}{2}} \left(\operatorname{sen}\mathbf{x} \right)^{\operatorname{tg}\mathbf{x}} = \left[1^{\infty} \right] = \mathbf{M}$

Tomando logaritmo neperiano en ambos miembros:

$$\ln \mathbf{M} = \ln \lim_{x\to\frac{\pi}{2}} \left(\operatorname{sen} x \right)^{\operatorname{tg} x} = \lim_{x\to\frac{\pi}{2}} \ln \left(\operatorname{sen} x \right)^{\operatorname{tg} x} = \lim_{x\to\frac{\pi}{2}} \operatorname{tg} x \cdot \ln \left(\operatorname{sen} x \right) = \lim_{x\to\frac{\pi}{2}} \frac{\ln \left(\operatorname{sen} x \right)}{\frac{1}{\operatorname{tg} x}} =$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\ln \left(\operatorname{sen} x \right)}{\operatorname{cotg} x} = \left[\frac{0}{0} \right] = \lim_{x\to\frac{\pi}{2}} \frac{\frac{\cos x}{\operatorname{sen} x}}{\frac{-1}{\operatorname{sen}^2 x}} = \lim_{x\to\frac{\pi}{2}} \frac{\operatorname{sen}^{\cancel{2}} x \cdot \cos x}{-\cancel{\operatorname{sen} x}} =$$

$$= \lim_{x\to\frac{\pi}{2}} -\frac{1}{2} \cdot \operatorname{sen} 2x = 0 \quad \Rightarrow \quad \mathbf{M} = \mathbf{1}$$

- $\lim_{\mathbf{x}\to+\infty} \left(\mathbf{e}^{\mathbf{2x}} + \mathbf{1} \right)^{\frac{\mathbf{1}}{\mathbf{x}}} = \left[\infty^{0} \right] = \mathbf{M}$

Tomando logaritmo neperiano en ambos miembros:

$$\ln \mathbf{M} = \ln \lim_{x\to+\infty} \left(e^{2x} + 1 \right)^{\frac{1}{x}} = \lim_{x\to+\infty} \ln \left(e^{2x} + 1 \right)^{\frac{1}{x}} = \lim_{x\to+\infty} \frac{1}{x} \cdot \ln \left(e^{2x} + 1 \right) =$$

$$= \lim_{x\to+\infty} \frac{\ln \left(e^{2x} + 1 \right)}{x} = \left[\frac{\infty}{\infty} \right] = \lim_{x\to+\infty} \frac{2 \cdot e^{2x}}{e^{2x} + 1} = \left[\frac{\infty}{\infty} \right] = \lim_{x\to+\infty} \frac{4 \cdot \cancel{e^{2x}}}{2 \cdot \cancel{e^{2x}}} =$$

$$\frac{4}{2} = 2 \quad \Rightarrow \quad \mathbf{M} = \mathbf{e^2}$$