

Examen de Matemáticas 1º de Bachillerato CS
Mayo 2014

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \rightarrow \infty} \frac{-2x^3 + 2x - 1}{3x^3 - 5}$
2. $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x + 1}{x^2 + 3} \right)^{2x}$
3. $\lim_{x \rightarrow \infty} \frac{\sqrt{16x^2 - x + 1}}{-x^2 + 3}$
4. $\lim_{x \rightarrow 1} \frac{8x^4 + 2x^3 - 9x^2 - 2x + 1}{2x^3 + x^2 - 4x + 1}$
5. $\lim_{x \rightarrow 5} \frac{\sqrt{x^2 - 2} - \sqrt{4x + 3}}{x - 5}$

Solución:

1. $\lim_{x \rightarrow \infty} \frac{-2x^3 + 2x - 1}{3x^3 - 5} = -\frac{2}{3}$
2. $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x + 1}{x^2 + 3} \right)^{2x} = e^4$
3. $\lim_{x \rightarrow \infty} \frac{\sqrt{16x^2 - x + 1}}{-x^2 + 3} = 0$
4. $\lim_{x \rightarrow 1} \frac{8x^4 + 2x^3 - 9x^2 - 2x + 1}{2x^3 + x^2 - 4x + 1} = \frac{9}{2}$
5. $\lim_{x \rightarrow 5} \frac{\sqrt{x^2 - 2} - \sqrt{4x + 3}}{x - 5} = \frac{3\sqrt{23}}{23}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{5x^3 + 2x^2 - x + 3}$
2. $y = \ln(3x^5 + 2)$
3. $y = (3x^2 + 2)^{12}$
4. $y = (x^2 - x + 1)(2x^3 + 2x^2 + 1)$
5. $y = \frac{3x^2 - 2x + 1}{2x - 6}$

6. $y = x^7 \ln x$

Solución:

1. $y = e^{5x^3+2x^2-x+3} \implies y' = (15x^2 + 4x - 1)e^{5x^3+2x^2-x+3}$

2. $y = \ln(3x^5 + 2) \implies y' = \frac{15x^4}{3x^5 + 2}$

3. $y = (3x^2 + 2)^{12} \implies y' = 12(3x^2 + 2)^{11}(6x)$

4. $y = (x^2 - x + 1)(2x^3 + 2x^2 + 1) \implies y' = (2x - 1)(2x^3 + 2x^2 + 1) + (x^2 - x + 1)(6x^2 + 2x)$

5. $y = \frac{3x^2 - 2x + 1}{2x - 6} \implies y' = \frac{(6x - 2)(2x - 6) - (3x^2 - 2x + 1)2}{(2x - 6)^2}$

6. $y = x^7 \ln x \implies y' = 7x^6 \ln x + x^7 \frac{1}{x}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \frac{3x^2 - 5}{x^2 + 1}$ en el punto $x = 1$.

2. $f(x) = \frac{2x + 1}{x + 5}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \implies b = f(1) = -1$ e $y - b = m(x - a)$

$$f'(x) = -\frac{16x}{(x^2 + 1)^2} \implies m = f'(1) = 4$$

Recta Tangente: $y + 1 = 4(x - 1)$

Recta Normal: $y + 1 = -\frac{1}{4}(x - 1)$

2. $b = f(a) \implies b = f(0) = \frac{1}{5}$ e $y - b = m(x - a)$

$$f'(x) = \frac{9}{(x + 5)^2} \implies m = f'(0) = \frac{9}{25}$$

Recta Tangente: $y - \frac{1}{5} = \frac{9}{25}x$

Recta Normal: $y - \frac{1}{5} = -\frac{25}{9}x$

Problema 4 Calcular las siguientes integrales:

1. $\int (5x^2 - 3x + 1) dx$
2. $\int \left(\frac{3x^2 - 2\sqrt[4]{x} - 4}{x} - 5e^x \right) dx$
3. $\int (x^2 + 3x + 1) dx$ sabiendo que la primitiva $F(1) = 1$.
4. $\int 5xe^{3x^2-2} dx$
5. $\int \frac{5x}{7x^2 - 1} dx$

Solución:

1. $\int (5x^2 - 3x + 1) dx = \frac{5x^3}{3} - \frac{3x^2}{2} + x + C$
2. $\int \left(\frac{3x^2 - 2\sqrt[4]{x} - 4}{x} - 5e^x \right) dx = \frac{3x^2}{2} - 8x^{1/4} - 4 \ln|x| - 5e^x + C$

3.

$$F(x) = \int (x^2 + 3x + 1) dx = \frac{x^3}{3} + \frac{3x^2}{2} + x + C$$

como

$$F(1) = 1 \implies \frac{1}{3} + \frac{3}{2} + 1 + C = 1 \implies C = -\frac{11}{6}$$

$$F(x) = \frac{x^3}{3} + \frac{3x^2}{2} + x - \frac{11}{6}$$

4. $\int 5xe^{3x^2-2} dx = \frac{5}{6}e^{3x^2-2} + C$
5. $\int \frac{5x}{7x^2 - 1} dx = \frac{5}{14} \ln|7x^2 - 1| + C$