

RESOLUCIÓN DE SISTEMAS DE ECUACIONES NO LINEALES

Resuelve los siguientes sistemas de ecuaciones:

$$1) \begin{cases} x^2 - y^2 = 51 \\ x^2 + y^2 = 149 \end{cases}$$

$$2) \begin{cases} x + y = 3 \\ x^2 - y^2 = 21 \end{cases}$$

$$3) \begin{cases} x + y = 7 \\ x^2 - y^2 = -7 \end{cases}$$

$$4) \begin{cases} x^2 + y^2 = 9 \\ 2x + y = 3 \end{cases}$$

$$5) \begin{cases} x + y = 5 \\ (x+1)(y-1) = 4 \end{cases}$$

$$6) \begin{cases} xy + 2y = 4 \\ 3x - y = 5 \end{cases}$$

$$7) \begin{cases} 6x - 5y = 14 \\ xy = 72 \end{cases}$$

$$8) \begin{cases} y + 5 = 3x \\ x - 2y^2 = 0 \end{cases}$$

$$9) \begin{cases} 5x + 7y = 61 \\ xy = 8 \end{cases}$$

$$10) \begin{cases} 4x + 5y = 6 \\ xy = -2 \end{cases}$$

$$11) \begin{cases} 2x^2 - y^2 = 5 \\ 3x^2 + 4y^2 = 57 \end{cases}$$

$$12) \begin{cases} 4x^2 - y^2 = 8 \\ 2x^2 - 3y^2 = -6 \end{cases}$$

$$13) \begin{cases} x + 2y = 0 \\ x^2 + xy = 2 \end{cases}$$

$$14) \begin{cases} x + 3y = 2 \\ y^2 + 2xy = -1 \end{cases}$$

Soluciones:

$$1) \begin{cases} x^2 - y^2 = 51 \\ x^2 + y^2 = 149 \end{cases}$$

Solución: $\boxed{x_1 = 10, y_1 = \pm 7 \\ x_2 = -10, y_2 = \pm 7}$

$$\begin{array}{r} \begin{cases} x^2 - y^2 = 51 \\ x^2 + y^2 = 149 \end{cases} \Rightarrow x^2 = 100 \Rightarrow x = \pm 10 \Rightarrow 100 - y^2 = 51 \Rightarrow y^2 = 49 \Rightarrow y = \pm 7 \\ 2x^2 = 200 \end{array}$$

$$2) \begin{cases} x + y = 3 \\ x^2 - y^2 = 21 \end{cases}$$

Solución: $\boxed{x_1 = 5, y_1 = -2}$

$$\begin{aligned} y &= 3 - x \\ x^2 - (3 - x)^2 &= 21 \Rightarrow x^2 - x^2 + 6x - 9 - 21 = 0 \Rightarrow 6x = 30 \Rightarrow x = 5 \Rightarrow y = -2 \end{aligned}$$

$$3) \begin{cases} x + y = 7 \\ x^2 - y^2 = -7 \end{cases}$$

Solución: $\boxed{x_1 = 3, y_1 = 4}$

$$\begin{aligned} y &= 7 - x \\ x^2 - (7 - x)^2 &= -7 \Rightarrow x^2 - x^2 + 14x - 49 + 7 = 0 \Rightarrow 14x = 42 \Rightarrow x = 3 \Rightarrow y = 4 \end{aligned}$$

$$4) \begin{cases} x^2 + y^2 = 9 \\ 2x + y = 3 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 0, y_1 = 3 \\ x_2 = \frac{12}{5}, y_2 = \frac{9}{5} \end{array}}$

$$y = 3 - 2x$$

$$x^2 + (3 - 2x)^2 = 9 \Rightarrow x^2 + 4x^2 - 12x + 9 - 9 = 0 \Rightarrow 5x^2 - 12x = 0$$

$$x(5x - 12) = 0 \Rightarrow \begin{cases} x_1 = 0 \Rightarrow y_1 = 3 \\ 5x - 12 = 0 \Rightarrow x_2 = \frac{12}{5} \Rightarrow y_2 = 3 - \frac{24}{5} = -\frac{9}{5} \end{cases}$$

$$5) \begin{cases} x + y = 5 \\ (x+1)(y-1) = 4 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 0, y_1 = 5 \\ x_2 = 3, y_2 = 2 \end{array}}$

$$y = 5 - x$$

$$(x+1)(4-x) = 4 \Rightarrow 4x - x^2 + 4 - x = 4 \Rightarrow x^2 - 3x = 0$$

$$x(x-3) = 0 \Rightarrow \begin{cases} x_1 = 0 \Rightarrow y_1 = 5 \\ x - 3 = 0 \Rightarrow x_2 = 3 \Rightarrow y_2 = 5 - 3 = 2 \end{cases}$$

$$6) \begin{cases} yx + 2y = 4 \\ 3x - y = 5 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 2, y_1 = 1 \\ x_2 = -7, y_2 = -26 \end{array}}$

$$y = 3x - 5$$

$$yx + 2y = 4 \Rightarrow y(x + 2) = 4 \Rightarrow (3x - 5)(x + 2) = 4 \Rightarrow 3x^2 + 6x - 5x - 10 = 4 \Rightarrow 3x^2 + x - 14 = 0$$

$$x = \frac{-1 \pm \sqrt{(-1)^2 - 4 \cdot 3 \cdot (-14)}}{2 \cdot 3} = \frac{-1 \pm 13}{6} = \begin{cases} x_1 = \frac{-1+13}{6} = 2 \Rightarrow y_1 = 1 \\ x_2 = \frac{-1-13}{6} = -\frac{7}{3} \Rightarrow y_2 = -12 \end{cases}$$

$$7) \begin{cases} 6x - 5y = 14 \\ xy = 72 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 9, y_1 = 8 \\ x_2 = -\frac{20}{3}, y_2 = -\frac{54}{5} \end{array}}$

$$y = \frac{6x - 14}{5}$$

$$xy = 72 \Rightarrow \frac{x(6x - 14)}{5} = 72 \Rightarrow 6x^2 - 14x - 360 = 0 \Rightarrow 3x^2 - 7x - 180 = 0$$

$$x = \frac{7 \pm \sqrt{7^2 - 4 \cdot 3 \cdot (-180)}}{2 \cdot 3} = \frac{7 \pm 47}{6} = \begin{cases} x_1 = \frac{7+47}{6} = 9 \Rightarrow y_1 = 8 \\ x_2 = \frac{7-47}{6} = -\frac{20}{3} \Rightarrow y_2 = -\frac{54}{5} \end{cases}$$

$$8) \begin{cases} y + 5 = 3x \\ x - 2y^2 = 0 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 2, y_1 = 1 \\ x_2 = \frac{25}{18}, y_2 = -\frac{5}{6} \end{array}}$

$$x - 2(3x - 5)^2 = 0 \Rightarrow x - 18x^2 - 50 + 60x = 0 \Rightarrow 18x^2 - 61x + 50 = 0$$

$$x = \frac{61 \pm \sqrt{61^2 - 4 \cdot 18 \cdot 50}}{2 \cdot 18} = \frac{61 \pm 11}{36} = \begin{cases} x_1 = \frac{61+11}{36} = 2 \Rightarrow y_1 = 1 \\ x_2 = \frac{61-11}{36} = \frac{25}{18} \Rightarrow y_2 = -\frac{5}{6} \end{cases}$$

$$9) \begin{cases} 5x + 7y = 61 \\ xy = 8 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 1, y_1 = 8 \\ x_2 = \frac{56}{5}, y_2 = \frac{5}{7} \end{array}}$

$$y = \frac{61 - 5x}{7} \Rightarrow \frac{x(61 - 5x)}{7} = 8 \Rightarrow 61x - 5x^2 - 56 = 0 \Rightarrow 5x^2 - 61x + 56 = 0$$

$$x = \frac{61 \pm \sqrt{61^2 - 4 \cdot 5 \cdot 56}}{2 \cdot 5} = \frac{61 \pm 51}{10} = \begin{cases} x_1 = \frac{61+51}{10} = \frac{56}{5} \Rightarrow y_1 = \frac{5}{7} \\ x_2 = \frac{61-51}{10} = 1 \Rightarrow y_2 = 8 \end{cases}$$

$$10) \begin{cases} 4x + 5y = 6 \\ xy = -2 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = -1, y_1 = 2 \\ x_2 = \frac{5}{2}, y_2 = -\frac{4}{5} \end{array}}$

$$y = \frac{6 - 4x}{5}$$

$$xy = -2 \Rightarrow \frac{x(6 - 4x)}{5} = -2 \Rightarrow 6x - 4x^2 + 10 = 0 \Rightarrow 2x^2 - 3x - 5 = 0$$

$$x = \frac{3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-5)}}{2 \cdot 2} = \frac{3 \pm 7}{4} = \begin{cases} x_1 = \frac{3+7}{4} = \frac{5}{2} \Rightarrow y_1 = -\frac{4}{5} \\ x_2 = \frac{3-7}{4} = -1 \Rightarrow y_2 = 2 \end{cases}$$

$$11) \begin{cases} 2x^2 - y^2 = 5 \\ 3x^2 + 4y^2 = 57 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = \sqrt{7}, y_1 = \pm 3 \\ x_2 = -\sqrt{7}, y_2 = \pm 3 \end{array}}$

$$\begin{aligned} &\begin{cases} 8x^2 - 4y^2 = 20 \\ 3x^2 + 4y^2 = 57 \end{cases} \Rightarrow x^2 = 7 \Rightarrow x = \pm \sqrt{7} \Rightarrow 14 - y^2 = 5 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3 \\ &11x^2 = 77 \end{aligned}$$

$$12) \begin{cases} 4x^2 - y^2 = 8 \\ 2x^2 - 3y^2 = -6 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = \sqrt{3}, y_1 = \pm 2 \\ x_2 = -\sqrt{3}, y_2 = \pm 2 \end{array}}$

$$\begin{aligned} &\begin{cases} 4x^2 - y^2 = 8 \\ -4x^2 + 6y^2 = 12 \end{cases} \Rightarrow y^2 = 4 \Rightarrow y = \pm 2 \Rightarrow 4x^2 - 4 = 8 \Rightarrow 4x^2 = 12 \Rightarrow x^2 = 3 \Rightarrow x = \pm \sqrt{3} \\ &5y^2 = 20 \end{aligned}$$

$$13) \begin{cases} x + 2y = 0 \\ x^2 + xy = 2 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 2, y_1 = -1 \\ x_2 = -2, y_2 = 1 \end{array}}$

$$x = -2y \Rightarrow 4y^2 - 2y^2 = 2 \Rightarrow 2y^2 = 2 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1 \Rightarrow x = \mp 2$$

$$14) \begin{cases} x + 3y = 2 \\ y^2 + 2xy = -1 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = -1, y_1 = 1 \\ x_2 = \frac{13}{5}, y_2 = -\frac{1}{5} \end{array}}$

$$x = 2 - 3y$$

$$y^2 + 2y(2 - 3y) = -1 \Rightarrow y^2 + 4y - 6y^2 = -1 \Rightarrow 5y^2 - 4y - 1 = 0$$

$$y = \frac{4 \pm \sqrt{4^2 - 4 \cdot 5 \cdot (-1)}}{2 \cdot 5} = \frac{4 \pm 6}{10} = \begin{cases} y_1 = \frac{4+6}{10} = 1 \Rightarrow x_1 = -1 \\ y_2 = \frac{4-6}{10} = -\frac{1}{5} \Rightarrow x_2 = \frac{13}{5} \end{cases}$$