

RESOLUCIÓN DE SISTEMAS DE ECUACIONES NO LINEALES

Resuelve los siguientes sistemas de ecuaciones:

$$1) \begin{cases} x = 2y \\ x^2 + y^2 = 20 \end{cases}$$

$$2) \begin{cases} x + y = 7 \\ x^2 - y^2 = 7 \end{cases}$$

$$3) \begin{cases} x + y = 3 \\ x^2 - y^2 = 21 \end{cases}$$

$$4) \begin{cases} 3y - x = 3 \\ x^2 - 5 = y^2 \end{cases}$$

$$5) \begin{cases} x + y = 10 \\ x^2 + y^2 = 52 \end{cases}$$

$$6) \begin{cases} x + y = 8 \\ x^2 + y^2 = 34 \end{cases}$$

$$7) \begin{cases} x^2 - y^2 = 5 \\ x^2 + y^2 = 13 \end{cases}$$

$$8) \begin{cases} x^2 - 4y^2 = 5 \\ x^2 + y^2 = 10 \end{cases}$$

$$9) \begin{cases} x - y = 2 \\ xy = 48 \end{cases}$$

$$10) \begin{cases} x - y = 21 \\ xy = 100 \end{cases}$$

$$11) \begin{cases} x + y = 5 \\ (6+x)(7+y) = 80 \end{cases}$$

$$12) \begin{cases} x + y = 3 \\ (x-1)(y+1) = 2 \end{cases}$$

$$13) \begin{cases} x^2 - 2y^2 = 1 \\ x^2 + y^2 = 13 \end{cases}$$

$$14) \begin{cases} y^2 - 2x^2 = -14 \\ x - y = 5 \end{cases}$$

Soluciones:

$$1) \begin{cases} x = 2y \\ x^2 + y^2 = 20 \end{cases}$$

Solución:
$$\boxed{\begin{array}{l} x_1 = -4, y_1 = 2 \\ x_2 = 4, y_2 = -2 \end{array}}$$

$$4y^2 + y^2 = 20 \Rightarrow 5y^2 = 20 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2 \Rightarrow x = \pm 4$$

$$2) \begin{cases} x + y = 7 \\ x^2 - y^2 = 7 \end{cases}$$

Solución:
$$\boxed{x = 4, y = 3}$$

$$y = 7 - x$$

$$x^2 - (7 - x)^2 = 7 \Rightarrow x^2 - 49 + 14x - x^2 = 7 \Rightarrow 14x = 56 \Rightarrow x = 4 \Rightarrow y = 7 - 4 = 3$$

$$3) \begin{cases} x + y = 3 \\ x^2 - y^2 = 21 \end{cases}$$

Solución:
$$\boxed{x = 5, y = -2}$$

$$y = 3 - x$$

$$x^2 - (3 - x)^2 = 21 \Rightarrow x^2 - 9 + 6x - x^2 = 21 \Rightarrow 6x = 30 \Rightarrow x = 5 \Rightarrow y = 3 - 5 = -2$$

$$4) \begin{cases} 3y - x = 3 \\ x^2 - 5 = y^2 \end{cases}$$

Solución:
$$\boxed{\begin{array}{l} x_1 = 3, y_1 = 2 \\ x_2 = -\frac{9}{4}, y_2 = \frac{1}{4} \end{array}}$$

$$x = 3y - 3$$

$$(3y - 3)^2 - 5 = y^2 \Rightarrow 9y^2 + 9 - 18y - 5 - y^2 = 0 \Rightarrow 8y^2 - 18y + 4 = 0 \Rightarrow 4y^2 - 9y + 2 = 0$$

$$y = \frac{9 \pm \sqrt{9^2 - 4 \cdot 4 \cdot 2}}{2 \cdot 4} = \frac{9 \pm 7}{8} = \begin{cases} y_1 = \frac{9+7}{8} = 2 \Rightarrow x_1 = 3 \\ y_2 = \frac{9-7}{8} = \frac{1}{4} \Rightarrow x_2 = -\frac{9}{4} \end{cases}$$

$$5) \begin{cases} x + y = 10 \\ x^2 + y^2 = 52 \end{cases}$$

Solución:
$$\boxed{\begin{array}{l} x_1 = 6, y_1 = 4 \\ x_2 = 4, y_2 = 6 \end{array}}$$

$$y = 10 - x$$

$$x^2 + (10 - x)^2 = 52 \Rightarrow x^2 + 100 - 20x + x^2 - 52 = 0 \Rightarrow 2x^2 - 20x + 48 = 0 \Rightarrow x^2 - 10x + 24 = 0$$

$$x = \frac{10 \pm \sqrt{10^2 - 4 \cdot 24}}{2} = \frac{10 \pm 2}{2} = \begin{cases} x_1 = \frac{10+2}{2} = 6 \Rightarrow y_1 = 4 \\ x_2 = \frac{10-2}{2} = 4 \Rightarrow y_2 = 6 \end{cases}$$

$$6) \begin{cases} x + y = 8 \\ x^2 + y^2 = 34 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 5, y_1 = 3 \\ x_2 = 3, y_2 = 5 \end{array}}$

$$y = 8 - x$$

$$x^2 + (8 - x)^2 = 34 \Rightarrow x^2 + 64 - 16x + x^2 - 34 = 0 \Rightarrow 2x^2 - 16x + 30 = 0 \Rightarrow x^2 - 8x + 15 = 0$$

$$x = \frac{8 \pm \sqrt{8^2 - 4 \cdot 15}}{2} = \frac{8 \pm 2}{2} = \begin{cases} x_1 = \frac{8+2}{2} = 5 \Rightarrow y_1 = 3 \\ x_2 = \frac{8-2}{2} = 3 \Rightarrow y_2 = 5 \end{cases}$$

$$7) \begin{cases} x^2 - y^2 = 5 \\ x^2 + y^2 = 13 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 3, y_1 = \pm 2 \\ x_2 = -3, y_2 = \pm 2 \end{array}}$

$$\begin{array}{r} \begin{cases} x^2 - y^2 = 5 \\ x^2 + y^2 = 13 \end{cases} \Rightarrow x^2 = 9 \Rightarrow x = \pm 3 \Rightarrow 9 - y^2 = 5 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2 \\ 2x^2 = 18 \end{array}$$

$$8) \begin{cases} x^2 - 4y^2 = 5 \\ x^2 + y^2 = 10 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 3, y_1 = \pm 1 \\ x_2 = -3, y_2 = \pm 1 \end{array}}$

$$\begin{array}{r} \begin{cases} -x^2 + 4y^2 = -5 \\ x^2 + y^2 = 10 \end{cases} \Rightarrow y^2 = 1 \Rightarrow y = \pm 1 \Rightarrow 1 + x^2 = 10 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3 \\ 5y^2 = 5 \end{array}$$

$$9) \begin{cases} x - y = 2 \\ xy = 48 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 8, y_1 = -6 \\ x_2 = -6, y_2 = 8 \end{array}}$

$$y = x - 2$$

$$x(x - 2) = 48 \Rightarrow x^2 - 2x - 48 = 0$$

$$x = \frac{2 \pm \sqrt{2^2 - 4 \cdot (-48)}}{2} = \frac{2 \pm 14}{2} = \begin{cases} x_1 = \frac{2+14}{2} = 8 \Rightarrow y_1 = -6 \\ x_2 = \frac{2-14}{2} = -6 \Rightarrow y_2 = 8 \end{cases}$$

$$10) \begin{cases} x - y = 21 \\ xy = 100 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 25, y_1 = 4 \\ x_2 = -4, y_2 = -25 \end{array}}$

$$y = x - 21$$

$$x(x - 21) = 100 \Rightarrow x^2 - 21x - 100 = 0$$

$$x = \frac{21 \pm \sqrt{21^2 - 4 \cdot (-100)}}{2} = \frac{21 \pm 29}{2} = \begin{cases} x_1 = \frac{21+29}{2} = 25 \Rightarrow y_1 = 4 \\ x_2 = \frac{21-29}{2} = -4 \Rightarrow y_2 = -25 \end{cases}$$

$$11) \begin{cases} x + y = 5 \\ (6+x)(7+y) = 80 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 4, y_1 = 1 \\ x_2 = 2, y_2 = 3 \end{array}}$

$$y = 5 - x$$

$$(6+x)(12-x) = 80 \Rightarrow 72 - 6x + 12x - x^2 = 80 \Rightarrow x^2 - 6x + 8 = 0$$

$$x = \frac{6 \pm \sqrt{6^2 - 4 \cdot 8}}{2} = \frac{6 \pm 2}{2} = \begin{cases} x_1 = \frac{6+2}{2} = 4 \Rightarrow y_1 = 1 \\ x_2 = \frac{6-2}{2} = 2 \Rightarrow y_2 = 3 \end{cases}$$

$$12) \begin{cases} x + y = 3 \\ (x-1)(y+1) = 2 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 3, y_1 = 0 \\ x_2 = 2, y_2 = 1 \end{array}}$

$$y = 3 - x \Rightarrow (x-1)(4-x) = 2 \Rightarrow 4x - x^2 - 4 + x = 2 \Rightarrow x^2 - 5x + 6 = 0$$

$$x = \frac{5 \pm \sqrt{5^2 - 4 \cdot 6}}{2} = \frac{5 \pm 1}{2} = \begin{cases} x_1 = \frac{5+1}{2} = 3 \Rightarrow y_1 = 0 \\ x_2 = \frac{5-1}{2} = 2 \Rightarrow y_2 = 1 \end{cases}$$

$$13) \begin{cases} x^2 - 2y^2 = 1 \\ x^2 + y^2 = 13 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 3, y_1 = \pm 2 \\ x_2 = -3, y_2 = \pm 2 \end{array}}$

$$\begin{array}{r} -x^2 + 2y^2 = -1 \\ x^2 + y^2 = 13 \\ \hline 3y^2 = 12 \end{array} \Rightarrow y^2 = 4 \Rightarrow y = \pm 2 \Rightarrow x^2 + 4 = 13 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$$

$$14) \begin{cases} y^2 - 2x^2 = -14 \\ x - y = 5 \end{cases}$$

Solución: $\boxed{\begin{array}{l} x_1 = 3, y_1 = -2 \\ x_2 = -13, y_2 = -18 \end{array}}$

$$y = x - 5$$

$$(x-5)^2 - 2x^2 = -14 \Rightarrow x^2 - 10x + 25 - 2x^2 + 14 = 0 \Rightarrow x^2 + 10x - 39 = 0$$

$$x = \frac{-10 \pm \sqrt{(-10)^2 - 4 \cdot (-39)}}{2} = \frac{5 \pm 1}{2} = \begin{cases} x_1 = \frac{-10+16}{2} = 3 \Rightarrow y_1 = -2 \\ x_2 = \frac{-10-16}{2} = -13 \Rightarrow y_2 = -18 \end{cases}$$