Deriva las siguientes expresiones:

$a)\ y = sen2x;\quad b)\ y = \cos(x-1);\quad c)\ y = \cos 3x^2;$

$d)\ y = \dfrac{x}{\sqrt[3]{x^2+4}};\quad e)\ y = \left(\dfrac{3x-1}{x^2+3}\right)^2;\quad f)\ y = \sqrt{sen4x}$

Solución

$a)\ y = sen2x$. Derivada del seno por la derivada del ángulo:

$$y = sen2x \Rightarrow y' = \cos 2x \cdot (2x)' \Rightarrow y' = 2\cdot\cos 2x$$

$b)\ y = \cos(x-1)$. Derivada del coseno por la derivada del ángulo:

$$y = \cos(x-1) \Rightarrow y' = -sen(x-1)\cdot(x-1)' \Rightarrow y' = -sen(x-1)$$

$c)\ y = \cos 3x^2$. Derivada del coseno por la derivada del ángulo:

$$y = \cos 3x^2 \Rightarrow y' = -sen3x^2\cdot\left(3x^2\right)' \Rightarrow y' = -6x\cdot sen3x^2$$

$d)\ y = \dfrac{x}{\sqrt[3]{x^2+4}}$. Derivada de un cociente:

$$y = \frac{x}{\sqrt[3]{x^2+4}} = \frac{x}{\left(x^2+4\right)^{1/3}}$$

$$y' = \frac{\left(x^2+4\right)^{1/3} - \left(\left(x^2+4\right)^{1/3}\right)'\cdot x}{\left(\left(x^2+4\right)^{1/3}\right)^2} = \frac{\left(x^2+4\right)^{1/3} - \frac{1}{3}x\cdot\left(x^2+4\right)^{-2/3}\cdot 2x}{\left(x^2+4\right)^{2/3}} =$$

$$= \frac{1}{3}\left(x^2+4\right)^{-2/3}\left[\frac{3\left(x^2+4\right)-2x^2}{\left(x^2+4\right)^{2/3}}\right] = \frac{x^2+12}{3\left(x^2+4\right)^{4/3}}$$

$e)\ y = \left(\dfrac{3x-1}{x^2+3}\right)^2$. Derivada de una potencia:

$$y' = 2\cdot\frac{3x-1}{x^2+3}\cdot\left(\frac{3x-1}{x^2+3}\right)' =$$

$$= 2\cdot\frac{3x-1}{x^2+3}\cdot\frac{3\cdot\left(x^2+3\right)-2x\cdot(3x-1)}{\left(x^2+3\right)^2} = \frac{2(3x-1)\left(-3x^2+2x+9\right)}{\left(x^2+3\right)^3}$$

$f)\ y = \sqrt{sen4x}$. Derivada de una raíz cuadrada:

$$y = \sqrt{sen4x} \Rightarrow y = (sen4x)^{1/2}$$

$$y' = \frac{1}{2}\cdot(sen4x)^{-1/2}\cdot(sen4x)' = \frac{1}{2}(sen4x)^{-1/2}\cdot 4\cos 4x = \frac{2\cos 4x}{\sqrt{sen4x}}$$

Derivar:

$$y=\frac{2}{\sqrt{3}}arctag\left(\frac{tag\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-\frac{1}{\sqrt{2}}arctag\left(\frac{tag\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

Solución

$$y'=\frac{2}{\sqrt{3}}\frac{1}{1+\frac{tg^2\left(\frac{x}{2}\right)}{3}}\cdot\frac{1}{2\sqrt{3}\cos^2\left(\frac{x}{2}\right)}-\frac{1}{\sqrt{2}}\frac{1}{1+\frac{tg^2\left(\frac{x}{2}\right)}{2}}\cdot\frac{1}{2\sqrt{2}\cos^2\left(\frac{x}{2}\right)}=$$

$$=\frac{1}{\left(3+tg^2\left(\frac{x}{2}\right)\right)}\cdot\frac{1}{\cos^2\left(\frac{x}{2}\right)}-\frac{1}{\left(4+2tg^2\left(\frac{x}{2}\right)\right)}\cdot\frac{1}{\cos^2\left(\frac{x}{2}\right)}=$$

$$=\frac{1}{3\cos^2\left(\frac{x}{2}\right)+tg^2\left(\frac{x}{2}\right)\cdot\cos^2\left(\frac{x}{2}\right)}-\frac{1}{4\cos^2\left(\frac{x}{2}\right)+2tg^2\left(\frac{x}{2}\right)\cdot\cos^2\left(\frac{x}{2}\right)}$$

Hemos de tener en cuenta que:

$$tg^2\left(\frac{x}{2}\right)=\frac{sen^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}\Rightarrow tg^2\frac{x}{2}\cdot\cos^2\left(\frac{x}{2}\right)=\frac{sen^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}\cdot\cos^2\left(\frac{x}{2}\right)=sen^2\left(\frac{x}{2}\right)$$

Por lo tanto:

$$y'=\frac{1}{3\cos^2\left(\frac{x}{2}\right)+sen^2\left(\frac{x}{2}\right)}-\frac{1}{4\cos^2\left(\frac{x}{2}\right)+2sen^2\left(\frac{x}{2}\right)}=$$

$$=\left\{\cos^2\left(\frac{x}{2}\right)+sen^2\left(\frac{x}{2}\right)=1\right\}=$$

$$=\frac{1}{1+2\cos^2\left(\frac{x}{2}\right)}-\frac{1}{2+2\cos^2\left(\frac{x}{2}\right)}=\left\{\cos^2\left(\frac{x}{2}\right)=\frac{1+\cos x}{2}\right\}=$$

$$=\frac{1}{1+1+\cos x}-\frac{1}{2+1+\cos x}=\frac{3+\cos x-(2+\cos x)}{(2+\cos x)(3+\cos x)}=\frac{1}{(2+\cos x)(3+\cos x)}$$

Derivar:

$$y=arc.\cos\left(\frac{1-x^2}{1+x^2}\right)$$

Solución

Derivada del arco coseno:

$$y' = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-2x\left(1+x^2\right)-2x\left(1-x^2\right)}{\left(1+x^2\right)^2} = \frac{2x+2x^3+2x-2x^3}{\left(1+x^2\right)^2\sqrt{1-\frac{1-2x^2+x^4}{1+2x^2+x^4}}} =$$

$$= \frac{4x}{\left(1+x^2\right)^2\sqrt{\frac{4x^2}{\left(1+x^2\right)^2}}} = \frac{4x}{2x\left(1+x^2\right)} = \frac{2}{1+x^2}$$

Derivar:

$$y = \left(1+\sqrt{1+x}\right)^{3/2} - 3\left(1+\sqrt{1+x}\right)^{1/2}$$

solución

Derivada de potencias:

$$y' = \frac{3}{2}\left(1+\sqrt{1+x}\right)^{1/2} \cdot \frac{1}{2\sqrt{1+x}} - \frac{3}{2}\left(1+\sqrt{1+x}\right)^{-1/2} \cdot \frac{1}{2\sqrt{1+x}} =$$

$$= \frac{3}{4\sqrt{1+x}}\left(\sqrt{1+\sqrt{1+x}} - \frac{1}{\sqrt{1+\sqrt{1+x}}}\right) = \frac{3}{4\sqrt{1+x}}\left(\frac{1+\sqrt{1+x}-1}{\sqrt{1+\sqrt{1+x}}}\right) =$$

$$= \frac{3}{4\sqrt{1+\sqrt{1+x}}}$$

Derivar:

$$y = arctagx + arctag\left(\frac{1}{x}\right) + arcsenx + \mathbf{arccos}\sqrt{1-x^2}$$

Solución

$$y' = \frac{1}{1+x^2} + \frac{1}{1+\left(\frac{1}{x^2}\right)} \cdot \frac{-1}{x^2} + \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-\left(1-x^2\right)}} \cdot \frac{-2x}{2\sqrt{1-x^2}} =$$

$$= \frac{1}{1+x^2} - \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x^2}} \cdot \frac{x}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} =$$

$$= \frac{2}{\sqrt{1-x^2}}$$

Derivar:

$$y = \ln \sqrt[3]{1 - x^4}$$

Solución

$$y = \ln \sqrt[3]{1 - x^4} \Rightarrow y = \frac{1}{3} \ln\left(1 - x^4\right)$$

Derivamos esta última expresión:

$$y' = \frac{1}{3} \cdot \frac{1}{\left(1 - x^4\right)} \cdot \left(-4x^3\right) = -\frac{4x^3}{3\left(1 - x^4\right)}$$