Examen de Matemáticas I – 1º de Bachillerato

1. Sabiendo que $\cot\alpha = \frac{4}{3}$, hallar $\operatorname{sen} 2\alpha$, $\cos 2\alpha$ y $\operatorname{tg} 2\alpha$. **(2 puntos)**

2. Dos individuos A y B observan un globo que está situado en un plano vertical que pasa por ellos. La distancia entre los individuos es de 4 Km. Los ángulos de elevación del globo desde los observadores son 46º y 52º, respectivamente. Halla la altura del globo y su distancia a cada observador. **(2 puntos)**

3. Simplifica al máximo la expresión $\frac{\operatorname{sen}\alpha \cdot \cos\alpha \cdot (1-\operatorname{tg}^2\alpha)}{\cos^2\alpha - \operatorname{sen}^2\alpha}$. **(1 punto)**

4. Comprueba que la igualdad $\frac{\cos x + \operatorname{tg} x}{\cos x \cdot \operatorname{tg} x} = \operatorname{cotg} x + \sec x$, es cierta. **(1 punto)**

5. Resuelve la ecuación trigonométrica $\frac{\cot x + tg\, x}{\cot x - tg\, x} = 2$. **(2 puntos)**

6. Resuelve el sistema de ecuaciones trigonométricas $\begin{cases} \cos x + \cos y = 1 \\ x + y = 360^\circ \end{cases}$. **(2 puntos)**

1) $\cot\alpha = \frac{4}{3} \Rightarrow \frac{\cos\alpha}{\operatorname{sen}\alpha} = \frac{4}{3} \Rightarrow \cos\alpha = \frac{4}{3}\operatorname{sen}\alpha \Rightarrow$

$\Rightarrow \cos^2\alpha = \frac{16}{9}\operatorname{sen}^2\alpha$. Como $\operatorname{sen}^2\alpha + \cos^2\alpha = 1 \Rightarrow$

$\Rightarrow \operatorname{sen}^2\alpha + \frac{16}{9}\operatorname{sen}^2\alpha = 1 \Rightarrow 9\operatorname{sen}^2\alpha + 16\operatorname{sen}^2\alpha = 9$

$\Rightarrow 25\operatorname{sen}^2\alpha = 9 \Rightarrow \operatorname{sen}^2\alpha = \frac{9}{25} \Rightarrow \underline{\underline{\operatorname{sen}\alpha = \frac{3}{5}}}$

(α está en el primer cuadrante).

Entonces $\cos\alpha = \frac{4}{3}\cdot\frac{3}{5} \Rightarrow \underline{\underline{\cos\alpha = \frac{4}{5}}}$

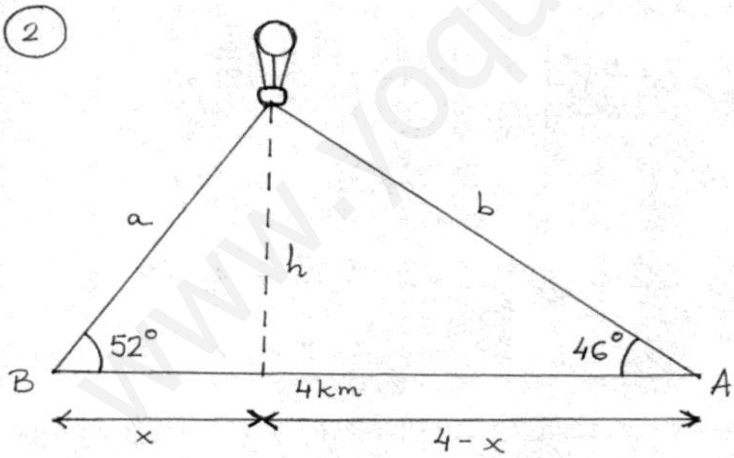
* $\operatorname{sen}2\alpha = 2\operatorname{sen}\alpha\cos\alpha = 2\cdot\frac{3}{5}\cdot\frac{4}{5} \Rightarrow \underline{\underline{\operatorname{sen}2\alpha = \frac{24}{25}}}$

* $\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25}$

$\Rightarrow \underline{\underline{\cos 2\alpha = \frac{7}{25}}}$

* $\operatorname{tg} 2\alpha = \frac{\operatorname{sen}2\alpha}{\cos 2\alpha} = \frac{24/25}{7/25} \Rightarrow \underline{\underline{\operatorname{tg} 2\alpha = \frac{24}{7}}}$

2)



$\left.\begin{array}{l} \operatorname{tg} 52 = \frac{h}{x} \\ \operatorname{tg} 46 = \frac{h}{4-x} \end{array}\right\} \Rightarrow$

$\left.\begin{array}{l} h = x\cdot\operatorname{tg} 52 \\ h = (4-x)\operatorname{tg} 46 \end{array}\right\} \Rightarrow$

$\Rightarrow x\operatorname{tg} 52 = (4-x)\operatorname{tg} 46 \Rightarrow x\operatorname{tg} 52 = 4\cdot\operatorname{tg} 46 - x\operatorname{tg} 46$

$\Rightarrow x\operatorname{tg} 52 + x\operatorname{tg} 46 = 4\operatorname{tg} 46 \Rightarrow x(\operatorname{tg} 52 + \operatorname{tg} 46) = 4\operatorname{tg} 46$

$\Rightarrow x = \frac{4\operatorname{tg} 46}{\operatorname{tg} 52 + \operatorname{tg} 46} \cong 1'79$; $h = x\operatorname{tg} 52 = 1'79\cdot\operatorname{tg} 52 \Rightarrow \underline{\underline{h \cong 2'29\text{ km}}}$

$\operatorname{sen} 52^\circ = \frac{h}{a} \Rightarrow a = \frac{h}{\operatorname{sen} 52^\circ} = \frac{2'29}{\operatorname{sen} 52^\circ} \Rightarrow \underline{\underline{a \cong 2'91\text{ km}}}$

$\operatorname{sen} 46^\circ = \frac{h}{b} \Rightarrow b = \frac{h}{\operatorname{sen} 46^\circ} = \frac{2'29}{\operatorname{sen} 46^\circ} \Rightarrow \underline{\underline{b \cong 3'18\text{ km}}}$

③ $$\frac{\operatorname{sen}\alpha\cos\alpha(1-\operatorname{tg}^2\alpha)}{\cos^2\alpha-\operatorname{sen}^2\alpha}=\frac{\operatorname{sen}\alpha\cos\alpha\left(1-\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}\right)}{\cos^2\alpha-\operatorname{sen}^2\alpha}=$$

$$=\frac{\operatorname{sen}\alpha\cos\alpha\,\frac{\cos^2\alpha-\operatorname{sen}^2\alpha}{\cos^2\alpha}}{\cos^2\alpha-\operatorname{sen}^2\alpha}=\frac{\operatorname{sen}\alpha\cos\alpha(\cos^2\alpha-\operatorname{sen}^2\alpha)}{\cos^2\alpha(\cos^2\alpha-\operatorname{sen}^2\alpha)}=$$

$$=\frac{\operatorname{sen}\alpha}{\cos\alpha}=\underline{\underline{\operatorname{tg}\alpha}}$$

④ $$\underline{\underline{\frac{\cos x+\operatorname{tg}x}{\cos x\cdot\operatorname{tg}x}}}=\frac{\cos x+\frac{\operatorname{sen}x}{\cos x}}{\cos x\cdot\frac{\operatorname{sen}x}{\cos x}}=\frac{\frac{\cos^2x+\operatorname{sen}x}{\cos x}}{\operatorname{sen}x}=$$

$$=\frac{\cos^2x+\operatorname{sen}x}{\cos x\cdot\operatorname{sen}x}=\frac{\cos^2x}{\cos x\cdot\operatorname{sen}x}+\frac{\operatorname{sen}x}{\cos x\operatorname{sen}x}=$$

$$=\frac{\cos x}{\operatorname{sen}x}+\frac{1}{\cos x}=\underline{\underline{\operatorname{cotg}x+\sec x}}$$

⑤ $$\frac{\operatorname{cotg}x+\operatorname{tg}x}{\operatorname{cotg}x-\operatorname{tg}x}=2\Rightarrow\frac{\frac{1}{\operatorname{tg}x}+\operatorname{tg}x}{\frac{1}{\operatorname{tg}x}-\operatorname{tg}x}=2\Rightarrow\frac{\frac{1+\operatorname{tg}^2x}{\operatorname{tg}x}}{\frac{1-\operatorname{tg}^2x}{\operatorname{tg}x}}=2$$

$$\Rightarrow\frac{1+\operatorname{tg}^2x}{1-\operatorname{tg}^2x}=2\Rightarrow 1+\operatorname{tg}^2x=2-2\operatorname{tg}^2x\Rightarrow 3\operatorname{tg}^2x=1$$

$$\Rightarrow\operatorname{tg}^2x=\frac{1}{3}\Rightarrow\operatorname{tg}x=\pm\sqrt{\frac{1}{3}}=\pm\frac{1}{\sqrt3}=\pm\frac{\sqrt3}{3}$$

$x_1=30^\circ+360^\circ k$ $\quad x_3=150^\circ+360^\circ k$

$x_2=210^\circ+360^\circ k$ $\quad x_4=330^\circ+360^\circ k$

⑥ $$\cos x+\cos y=1\Rightarrow 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}=1\Rightarrow$$

$$2\cos180^\circ\cos\frac{x-y}{2}=1\Rightarrow -2\cos\frac{x-y}{2}=1\Rightarrow$$

$$\cos\frac{x-y}{2}=-\frac{1}{2}\Rightarrow\frac{x-y}{2}=120\ (1)\ \text{ó}\ \frac{x-y}{2}=240\ (2)$$

En el caso (1) $\left.\begin{aligned}x-y&=240\\x+y&=360\end{aligned}\right\}\Rightarrow\boxed{\begin{aligned}x_1&=300^\circ+360k\\y_1&=60^\circ+360k\end{aligned}}$

En el caso (2) $\left.\begin{aligned}x-y&=480\\x+y&=360\end{aligned}\right\}\Rightarrow\boxed{\begin{aligned}x_2&=420^\circ+360k\\y_2&=-60^\circ+360k\end{aligned}}\Rightarrow$

soluciones equivalentes a $\boxed{\begin{aligned}x_2&=60^\circ+360^\circ k\\y_2&=300^\circ+360^\circ k\end{aligned}}$