

Problema 1 Discutir y resolver por el método de Gauss los siguientes sistemas:

$$\begin{cases} x+2y-z=1 \\ 2x-y+3z=2 \\ x+7y-6z=1 \end{cases}; \quad \begin{cases} x+y-3z=1 \\ 2x-y-z=-2 \\ 3x+y+2z=8 \end{cases}$$

Solución:

$$\begin{cases} x+2y-z=1 \\ 2x-y+3z=2 \\ x+7y-6z=1 \end{cases} \text{ Sistema Compatible Indeterminado} \implies \begin{cases} x=1-\lambda \\ y=\lambda \\ z=\lambda \end{cases}$$

$$\begin{cases} x+y-3z=1 \\ 2x-y-z=-2 \\ 3x+y+2z=8 \end{cases} \text{ Sistema Compatible Determinado} \implies \begin{cases} x=1 \\ y=3 \\ z=1 \end{cases}$$

Problema 2 Resolver las ecuaciones:

1. $\log(8-x) - \log(x+2) = 1$
2. $\log(3-x^2) - \log x = 1 + \log(x-2)$
3. $2\log(2-x) - 1 = \log x$
4. $3^{x^2+2} \cdot 9^{x-3} = 27^{x+1}$
5. $4^{x-1} + 2^{x+1} - 5 = 0$

Solución:

1. $\log(8-x) - \log(x+2) = 1 \implies \log \frac{8-x}{x+2} = \log 10 \implies 11x = -12 \implies x = -\frac{12}{11}$
2. $\log(3-x^2) - \log x = 1 + \log(x-2) \implies \log \frac{3-x^2}{x} = \log 10(x-2) \implies 11x^2 - 20x - 3 = 0 \implies x = 1,957505690 \text{ (nó vale)}, x = -0,1393238722 \text{ (no vale)}$
3. $2\log(2-x) - 1 = \log x \implies x^2 - 14x + 4 = 0 \implies x = 0,292, x = 13,708 \text{ (no vale)}$

4.

$$3^{x^2+2} \cdot 9^{x-3} = 27^{x+1} \implies x^2 - x - 7 = 0 \implies \begin{cases} x = 3, 192582403 \\ x = -2, 192582403 \end{cases}$$

5.

$$4^{x-1} + 2^{x+1} - 5 = 0 \implies t^2 + 8t - 20 = 0 \implies \begin{cases} t = 2 \implies x = 1 \\ t = -10 \text{ no vale} \end{cases}$$