Problema 1 Discutir y resolver por el método de Gauss los siguientes sistemas:

$$\begin{cases} x+ & 2y- & z = 1 \\ 2x- & y- & z = 0 \\ 3x+ & y+ & 2z = 2 \end{cases} ; \quad \begin{cases} x+ & y & + & z = 1 \\ x+ & 2y & - & z = 2 \\ 2x+ & 3y & & = 4 \end{cases}$$

Solución:

$$\begin{cases} x+ & 2y- & z = 1 \\ 2x- & y- & z = 0 \\ 3x+ & y+ & 2z = 2 \end{cases} \text{ Sistema Compatible Determinado} \Longrightarrow \begin{cases} x = 7/20 \\ y = 9/20 \\ z = 1/4 \end{cases}$$

$$\begin{cases} x+ & y & + & z = 1 \\ x+ & 2y & - & z = 2 \\ 2x+ & 3y & & = 4 \end{cases} \text{ Sistema Incompatible}$$

Problema 2 Resolver las ecuaciones:

1. $\ln x - \ln(1-x) = 1$

2. $\log(7 - x^2) - \log x = 1$

3. $\log(2x + 3) + \log x = -1$

Solución:

1. $\ln x - \ln(1-x) = 1 \Longrightarrow \ln \dfrac{x}{(1-x)} = \ln e \Longrightarrow$

   $x = e - ex \Longrightarrow x = \dfrac{e}{e+1} = 0,7310585786.$

2. $\log(7 - x^2) - \log x = 1 \Longrightarrow \log \dfrac{7 - x^2}{x} = \log 10 \Longrightarrow x^2 + 10x - 7 = 0 \Longrightarrow x = 0,6568542494, \ \ x = -10,65685424$ (no vale).

3. $\log(2x + 3) + \log x = -1 \Longrightarrow \log(2x^2 + 3x) = \log 10^{-1} \Longrightarrow 20x^2 + 30x - 1 = 0 \Longrightarrow x = 0,03262379212; \ x = -1,532623792$(no vale).

Problema 3 Resolver el siguiente sistema

$$\begin{cases} x^2 + y^2 = 1 \\ x + y = 0 \end{cases}$$

Solución:

$$\begin{cases} x^2 + y^2 = 1 \\ x + y = 0 \end{cases} \Longrightarrow \begin{cases} x = \frac{\sqrt{2}}{2}, \ y = -\frac{\sqrt{2}}{2} \\ x = -\frac{\sqrt{2}}{2}, \ y = \frac{\sqrt{2}}{2} \end{cases}$$

Problema 4 Resolver las inecuaciones siguientes:

1. $\dfrac{2x-1}{3} + \dfrac{x+1}{2} \geq 1 - \dfrac{x}{2}$

2. $\dfrac{x^2+x-2}{x^2+2x-15} \leq 0$

Solución:

1. $\dfrac{2x-1}{3} + \dfrac{x+1}{2} \geq 1 - \dfrac{x}{2} \implies \left[\dfrac{1}{2}, +\infty\right)$

2. $\dfrac{x^2+x-2}{x^2+2x-15} \leq 0 \implies (-5, -2] \cup [1, 3)$

Problema 5 Calcular los siguientes límites:

1. $\displaystyle\lim_{x \to \infty} \dfrac{2x^3+x-1}{3x^3+2}$

2. $\displaystyle\lim_{x \to \infty} \dfrac{3x^2+2x+1}{x^5+2}$

3. $\displaystyle\lim_{x \to \infty} \dfrac{x^6+x-1}{x^4+2}$

4. $\displaystyle\lim_{x \to \infty} \left(\dfrac{2x^2+x}{x^2-1}\right)^{3x^2-1}$

5. $\displaystyle\lim_{x \to \infty} \left(\dfrac{x^2-x-1}{2x^2}\right)^{2x-1}$

6. $\displaystyle\lim_{x \to \infty} \left(\dfrac{x+1}{x}\right)^{2x}$

Solución:

1. $\displaystyle\lim_{x \to \infty} \dfrac{2x^3+x-1}{3x^3+2} = \dfrac{2}{3}$

2. $\displaystyle\lim_{x \to \infty} \dfrac{3x^2+2x+1}{x^5+2} = 0$

3. $\displaystyle\lim_{x \to \infty} \dfrac{x^6+x-1}{x^4+2} = \infty$

4. $\displaystyle\lim_{x \to \infty} \left(\dfrac{2x^2+x}{x^2-1}\right)^{3x^2-1} = +\infty$

5. $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{x^2 - x - 1}{2x^2} \right)^{2x-1} = 0$

6. $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{x + 1}{x} \right)^{2x} = e^2$

Problema 6 Calcular los siguientes límites:

1. $\lim\limits_{x \longrightarrow 1} \dfrac{x^3 - 2x^2 + 2x - 1}{x^4 - x^3 + 2x - 2}$

2. $\lim\limits_{x \longrightarrow \infty} \left(\sqrt{x - 1} - \sqrt{x + 1} \right)$

3. $\lim\limits_{x \longrightarrow 2} \dfrac{\sqrt{2x^2 + 1} - 3}{x - 2}$

4. $\lim\limits_{x \longrightarrow -2} \dfrac{x^4 + 2x^3 - x^2 - x + 2}{x^3 + 2x^2 - x - 2}$

5. $\lim\limits_{x \longrightarrow \infty} \dfrac{\sqrt{2x^2 + x + 2} - x}{x - 1}$

6. $\lim\limits_{x \longrightarrow 5} \dfrac{2\sqrt{x - 1} - 2}{x - 2}$

Solución:

1. $\lim\limits_{x \longrightarrow 1} \dfrac{x^3 - 2x^2 + 2x - 1}{x^4 - x^3 + 2x - 2} = \dfrac{1}{3}$

2. $\lim\limits_{x \longrightarrow \infty} \left(\sqrt{x - 1} - \sqrt{x + 1} \right) = 0$

3. $\lim\limits_{x \longrightarrow 2} frac\sqrt{2x^2 + 1} - 3x - 2 = \dfrac{4}{3}$

4. $\lim\limits_{x \longrightarrow -2} \dfrac{x^4 + 2x^3 - x^2 - x + 2}{x^3 + 2x^2 - x - 2} = -\dfrac{5}{3}$

5. $\lim\limits_{x \longrightarrow \infty} \dfrac{\sqrt{2x^2 + x + 2} - x}{x - 1} = \sqrt{2} - 1$

6. $\lim\limits_{x \longrightarrow 2} \dfrac{2\sqrt{x - 1} - 2}{x - 2} = 0$