1. Dado el triángulo de vértices A(1,1), B(5,4) y C(-5,9), se pide:

a) Dibujarlo.

b) Demostrar que los vectores $\overrightarrow{AB}$ y $\overrightarrow{AC}$ son $\perp$

c) Hallar $|\overrightarrow{AB}|$ y $|\overrightarrow{AC}|$

d) Calcular su área. *(2 puntos)*

2. **a)** Hallar, en todas las formas conocidas, la ecuación de la recta **s** que tiene la misma pendiente que **r**: y=3x-1 y pasa por P(-1,2)

b) Hallar la distancia entre las dos rectas **r** y **s** anteriores.

c) Hallar el ángulo que forma **r** con la recta **t**: x-2y+4=0 *(2 puntos)*

3. Dadas las rectas $\left.\begin{array}{l} r: x+2y-3=0 \\ s: x-ky+4=0 \end{array}\right\}$ se pide:

a) Hallar **k** para que sean //

b) Hallar **k** para que sean $\perp$

c) Hallar la ecuación general de la recta $\perp$ a r que pasa por el origen. *(2 puntos)*

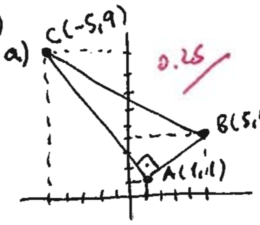
4. **a)** Operar $\dfrac{(3-2i)(3+i)-(2i-3)^2}{i^{23}-i^{-13}}$ en forma binómica.

b) Calcular $\dfrac{(-2\sqrt{3}-2i)^5}{(-4+4\sqrt{3}\,i)^3\,2i}$ en forma polar, y pasar el resultado a binómica. *(2 puntos)*

5. **a)** Calcular $\sqrt[4]{\dfrac{8\sqrt{3}+8i}{-\sqrt{3}+i}}$, dando el resultado en binómica.

b) Dibujar los afijos de las raíces anteriores. *(2 puntos)*

1

a) 

0.25

b) $\vec{AB} = B - A = (5,4) - (1,1) = (4,3)$; $\vec{AC} = C - A = (-5,9) - (1,1) = (-6,8)$ $\Big\}$ $\vec{AB}\cdot\vec{AC} = (4,3)\cdot(-6,8) = -24+24 = 0 \Rightarrow \vec{AB} \perp \vec{AC}$ 0.75

c) $|\vec{AB}| = \sqrt{16+9} = \boxed{5}$; $|\vec{AC}| = \sqrt{36+64} = \boxed{10}$ 0.25

d) $A = \frac{1}{2}\, \text{base} \times \text{altura} = \frac{1}{2}|\vec{AB}|\cdot|\vec{AC}| = \frac{1}{2}\cdot 5 \cdot 10 = \boxed{25\ u^2}$ 0.75

2

a) $m = 3 \Rightarrow \vec{u}_r = (1,3)$; pasa por $P(-1,2)$

$\boxed{\begin{cases} x = -1+\lambda \\ y = 2+3\lambda \end{cases}}$ PARAMÉTRICAS $\Rightarrow \boxed{\dfrac{x+1}{1} = \dfrac{y-2}{3}}$ CONTINUA $\Rightarrow 3x+3 = y-2$; $\boxed{3x - y + 5 = 0}$ GRAL. O IMPLÍCITA $\Rightarrow \boxed{y = 3x+5}$ EXPLÍCITA ; $\boxed{y-2 = 3(x+1)}$ PTO-PDTE. 0.1 cada uno

b) $r: 3x - y - 1 = 0$; $d(r,s) = d(P,r) = \dfrac{|-3-2-1|}{\sqrt{9+1}} = \dfrac{6}{\sqrt{10}} = \dfrac{6\sqrt{10}}{10} = \boxed{\dfrac{3\sqrt{10}}{5}\,u.}$ 0.75

c) $\left.\begin{array}{l} \vec{u}_r = (1,3) \\ \vec{u}_t = (2,1) \end{array}\right\}$ $\cos\alpha = \dfrac{|\vec{u}_r\cdot\vec{u}_t|}{\|\vec{u}_r\|\cdot\|\vec{u}_t\|} = \dfrac{|2+3|}{\sqrt{1+9}\cdot\sqrt{4+1}} = \dfrac{5}{\sqrt{10}\cdot\sqrt{5}} = \dfrac{5}{\sqrt{50}} = \dfrac{5\sqrt{50}}{50} = \dfrac{\sqrt{50}}{10} = \dfrac{5\sqrt{2}}{10} = \dfrac{\sqrt{2}}{2} \Rightarrow \boxed{\alpha = \arccos\dfrac{\sqrt{2}}{2} = 45^\circ}$ 0.75

3

$\left.\begin{array}{l} r: x+2y-3=0 \\ s: x-ky+4=0 \end{array}\right\}$

a) $\dfrac{1}{1} = \dfrac{2}{-k}$; $-k = 2$; $\boxed{k=-2}$ 0.5

b) $r \perp s \Rightarrow \vec{u}_r\cdot\vec{u}_s = 0 \Rightarrow (-2,1)\cdot(k,1) = -2k+1 = 0 \Rightarrow \boxed{k = 1/2}$ 0.5

c) $\vec{u}_r = (-2,1) \Rightarrow \vec{v} = (1,2) \Rightarrow \dfrac{x}{1} = \dfrac{y}{2} \to \boxed{2x - y = 0}$ 1

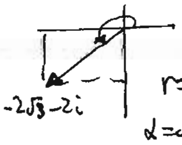
4

a) $\dfrac{(3-2i)(3+i) - (2i-3)^2}{i^{23} - i^{-13}} = \dfrac{9+3i-6i-2i^2 - (4i^2 - 12i + 9)}{i^{23} - \frac{1}{i^{13}}} = \dfrac{9-3i+2-(-4-12i+9)}{i^3 - \frac{1}{i}} = \dfrac{11-3i-5+12i}{-i - \frac{i}{-1}} = \dfrac{6+9i}{0} = \nexists$ 0.5 0.5

b) $\dfrac{(-2\sqrt{3}-2i)^5}{(-4+4\sqrt{3}i)^3\cdot 2i} = \dfrac{(4_{210^\circ})^5}{(8_{120^\circ})^3\cdot 2_{90^\circ}} = \dfrac{(4^5)_{1050^\circ}}{(8^3)_{360^\circ}\cdot 2_{90^\circ}} = \dfrac{(2^{10})_{1050^\circ}}{(2^9)_{360^\circ}\cdot 2_{90^\circ}} = \dfrac{(2^{10})_{1050^\circ}}{(2^{10})_{450^\circ}} = 1_{600^\circ} = \boxed{1_{240^\circ}} = \cos 240^\circ + i \,\text{sen}\, 240^\circ =$ 0.5

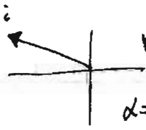
$\cos(180^\circ+60^\circ) = -\cos 60^\circ$; $\text{sen}(180+60) = -\text{sen}\, 60^\circ$

$= \boxed{-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i}$ 0.5



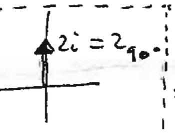
$r = \sqrt{12+4} = 4$

$\alpha = \text{arctg}\,\dfrac{-2}{-2\sqrt{3}} = \text{arctg}\,\dfrac{1}{\sqrt{3}} = 210^\circ$



$r = \sqrt{16+48} = 8$

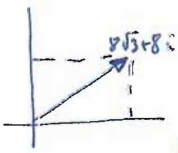
$\alpha = \text{arctg}\,\dfrac{4\sqrt{3}}{-4} = \text{arctg}(-\sqrt{3}) = 120^\circ$



5

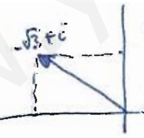
a) $\sqrt[4]{\dfrac{8\sqrt{3}+8i}{-\sqrt{3}+i}} = \sqrt[4]{\dfrac{16_{30^\circ}}{2_{150^\circ}}} = \sqrt[4]{8_{-120^\circ}} = \sqrt[4]{8_{240^\circ}} = R_\beta$ siendo $\begin{cases} R = \sqrt[4]{8} \\ \beta = \dfrac{240^\circ + k\cdot 360^\circ}{4} = 60^\circ + k\cdot 90^\circ \end{cases}$ donde $k = 0,1,2,3$

0,25



$r = \sqrt{192+64} = 16$

$\alpha = \text{arctg}\,\dfrac{8}{8\sqrt{3}} = 30^\circ$



$r = \sqrt{3+1} = 2$

$\alpha = \text{arctg}\,\dfrac{1}{-\sqrt{3}} = \text{arctg}\left(-\dfrac{\sqrt{3}}{3}\right) = 150^\circ$

0,75

$k=0 \to z_1 = \boxed{(\sqrt[4]{8})_{60^\circ}} = \sqrt[4]{8}\,(\cos 60^\circ + i\,\text{sen}\,60^\circ) = \sqrt[4]{8}\left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right) = \dfrac{\sqrt[4]{8}}{2} + \dfrac{\sqrt[4]{\ }\cdot\sqrt{3}}{2}i = \dfrac{\sqrt[4]{8}}{2} + \dfrac{\sqrt[4]{\ }\cdot\sqrt[4]{3^2}}{2}i = \boxed{\dfrac{\sqrt[4]{8}}{2} + \dfrac{\sqrt[4]{72}}{2}i}$ 0,25

$k=1 \to z_2 = \boxed{(\sqrt[4]{8})_{150^\circ}} = \sqrt[4]{8}\,(\cos 150^\circ + i\,\text{sen}\,150^\circ) = \sqrt[4]{8}\left(-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right) = \boxed{-\dfrac{\sqrt[4]{72}}{2} + \dfrac{\sqrt[4]{8}}{2}i}$ 0.25

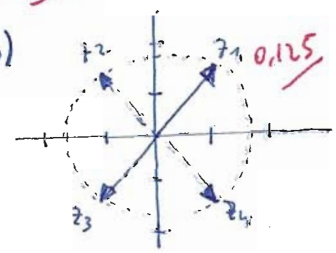
$\cos(180-30) = -\cos 30^\circ$; $\text{sen}(180-30) = \text{sen}\,30^\circ$

$k=2 \to z_3 = \boxed{(\sqrt[4]{8})_{240^\circ}} = \sqrt[4]{8}\,(\cos 240^\circ + i\,\text{sen}\,240^\circ) = \sqrt[4]{8}\left(-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i\right) = \boxed{-\dfrac{\sqrt[4]{8}}{2} - \dfrac{\sqrt[4]{72}}{2}i}$ 0.25

$\cos(180+60) = -\cos 60^\circ$; $\text{sen}(180+60) = -\text{sen}\,60^\circ$

$k=3 \to z_4 = \boxed{(\sqrt[4]{8})_{330^\circ}} = \sqrt[4]{8}\,(\cos 330^\circ + i\,\text{sen}\,330^\circ) = \sqrt[4]{8}\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i\right) = \boxed{\dfrac{\sqrt[4]{72}}{2} - \dfrac{\sqrt[4]{8}}{2}i}$ 0.25

$\cos(-30^\circ) = \cos 30^\circ$; $\text{sen}(-30^\circ) = -\text{sen}\,30^\circ$

b) 

0,125