1. Desarrollar y simplificar, dando el resultado racionalizado:

$$\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)^4 =$$ *(1,75 puntos)*

2. Dado $\alpha \in$ 4º cuadrante tal que $\operatorname{ctg}\alpha = -\frac{\sqrt{3}}{3}$, se pide: *(2 puntos)*

a) tg 2α

b) cos $\alpha/2$

c) cos $(\alpha+240º)$

d) sen $(\alpha-1920º)$

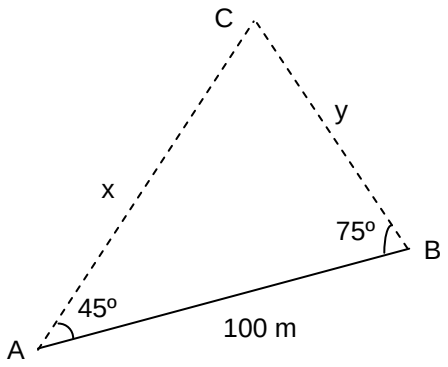
e) Razonar (sin calculadora) de qué α se trata.

3. Resolver: $\cos 2x + 3\operatorname{sen} x = 2$ (¡Comprobar las soluciones obtenidas!) *(2 puntos)*

4. **a)** Resolver el triángulo de datos a=4 m, b=3 m, $\hat{B} = 30º$

b) Hallar su área. *(2 puntos)*

5.



Dos observadores A y B, separados 100 m, ven el punto inaccesible C bajo los ángulos que indica la figura. Hallar a qué distancia se encuentran ambos observadores de dicho punto. *(2 puntos)*

NOTA: La ortografía, sintaxis, presentación cuidada (orden en el planteamiento, limpieza, caligrafía, etc.) y corrección en el lenguaje matemático se calificarán con un máximo de 0,25 puntos.

① $\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)^4 = \underbrace{\cancel{\binom{4}{0}}}_{1}\underbrace{(\sqrt{2})^4}_{4} - \underbrace{\cancel{\binom{4}{1}}}_{4}\underbrace{(\sqrt{2})^3}_{2\sqrt{2}}\cdot\underbrace{\frac{1}{\sqrt{3}}}_{\sqrt{3}/3} + \underbrace{\cancel{\binom{4}{2}}}_{6}\underbrace{(\sqrt{2})^2}_{2}\cdot\underbrace{\left(\frac{1}{\sqrt{3}}\right)^2}_{1/3} - \underbrace{\cancel{\binom{4}{3}}}_{4}\underbrace{\sqrt{2}}\cdot\underbrace{\left(\frac{1}{\sqrt{3}}\right)^3}_{\sqrt{3}/9} + \underbrace{\cancel{\binom{4}{4}}}_{1}\cdot\underbrace{\left(\frac{1}{\sqrt{3}}\right)^4}_{1/9} =$ ← 0.25

TOTAL: 1,75

0.25

$= 4 - 4\cdot 2\sqrt{2}\cdot\frac{\sqrt{3}}{3} + 6\cdot 2\cdot\frac{1}{3} - 4\sqrt{2}\cdot\frac{\sqrt{3}}{9} + \frac{1}{9} = 4 - \frac{8}{3}\sqrt{6} + 4 - \frac{4}{9}\sqrt{6} + \frac{1}{9} = \boxed{\frac{73}{9} - \frac{28}{9}\sqrt{6}}$ ← 1

0.25

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② $\operatorname{ctg}\alpha = -\frac{\sqrt{3}}{3} \Rightarrow \boxed{\operatorname{tg}\alpha = \frac{1}{\operatorname{ctg}\alpha} = -\frac{3}{\sqrt{3}} = -\frac{3\sqrt{3}}{3} = -\sqrt{3}}$ ← 0.1

a) $\boxed{\operatorname{tg} 2\alpha} = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha} = \frac{2\cdot(-\sqrt{3})}{1-(-\sqrt{3})^2} = \frac{-2\sqrt{3}}{1-3} = \frac{-2\sqrt{3}}{-2} = \boxed{\sqrt{3}}$ ← 0.3

b) $1+\operatorname{tg}^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 1+(-\sqrt{3})^2 = \frac{1}{\cos^2\alpha}$; $1+3 = \frac{1}{\cos^2\alpha}$; $4 = \frac{1}{\cos^2\alpha}$; $\cos^2\alpha = \frac{1}{4}$

→ $\cos\alpha = -\frac{1}{2}$ descartado pq $\alpha \in 4^{\circ}$ cuad.

→ $\boxed{\cos\alpha = \frac{1}{2}}$ ← 0.1

$\boxed{\cos\frac{\alpha}{2}} = \pm\sqrt{\frac{1+\cos\alpha}{2}} = -\sqrt{\frac{1+\frac{1}{2}}{2}} = -\sqrt{\frac{3}{4}} = \boxed{-\frac{\sqrt{3}}{2}}$ ← 0,4

$270 < \alpha < 360$

$135 < \alpha/2 < 180 \Rightarrow \frac{\alpha}{2} \in 2^{\circ}$ cuad.

c) $\boxed{\operatorname{sen}\alpha = \operatorname{tg}\alpha\cdot\cos\alpha = -\sqrt{3}\cdot\frac{1}{2} = -\frac{\sqrt{3}}{2}}$ ← 0.1

$\boxed{\cos(\alpha+240^{\circ})} = \cos\alpha\cos 240^{\circ} - \operatorname{sen}\alpha\operatorname{sen}240^{\circ} = \frac{1}{2}\cdot\left(-\frac{1}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\cdot\left(-\frac{\sqrt{3}}{2}\right) = -\frac{1}{4} - \frac{3}{4} = \boxed{-1}$ ← 0,4

$\cos(180^{\circ}+60) = -\cos 60^{\circ} = -\frac{1}{2}$ $\qquad$ $\operatorname{sen}(180+60) = -\operatorname{sen}60^{\circ} = -\sqrt{3}/2$

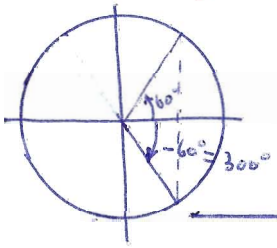
d) $\boxed{\operatorname{sen}(\alpha-1920^{\circ})} = \operatorname{sen}\alpha\cdot\cos 1920^{\circ} - \cos\alpha\cdot\operatorname{sen}1920^{\circ} = -\frac{\sqrt{3}}{2}\cdot\left(-\frac{1}{2}\right) - \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \boxed{0}$ ← 0,4

$1920^{\circ} \,|\, 360^{\circ}$
-120° 5 vueltas ⇒

$\cos 120^{\circ} = \cos(180-60) = -\cos 60^{\circ} = -\frac{1}{2}$

$\operatorname{sen}120^{\circ} = \operatorname{sen}(180-60) = \operatorname{sen}60 = \frac{\sqrt{3}}{2}$

e)



$\operatorname{tg}\alpha = -\sqrt{3} \Rightarrow \boxed{\alpha = \operatorname{arctg}(-\sqrt{3}) = -60^{\circ} = 300^{\circ}}$ ← 0.2

TOTAL: 2

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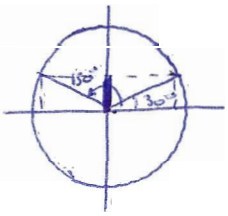
③ $\cos 2x + 3\operatorname{sen}x = 2$; $\cos^2 x - \operatorname{sen}^2 x + 3\operatorname{sen}x = 2$; $1 - \operatorname{sen}^2 x - \operatorname{sen}^2 x + 3\operatorname{sen}x = 2$; $-2\operatorname{sen}^2 x + 3\operatorname{sen}x - 1 = 0$

$\Rightarrow 2\operatorname{sen}^2 x - 3\operatorname{sen}x + 1 = 0 \rightarrow \operatorname{sen}x = \frac{3\pm\sqrt{9-8}}{4} = \frac{3\pm 1}{4}$

↗ $\operatorname{sen}x = 1 \Rightarrow x = 90^{\circ} + K\cdot 360^{\circ}$

↘ $\operatorname{sen}x = \frac{1}{2} \Rightarrow \begin{cases} x = 30^{\circ} + K\cdot 360^{\circ} \\ x = 150^{\circ} + K\cdot 360^{\circ} \end{cases}$ ← 0.75

0.5



comprobación:

$x = 90^{\circ} \rightarrow \cos 180^{\circ} + 3\operatorname{sen}90^{\circ} \overset{?}{=} 2$

$-1+3 = 2$ o.k. $\Rightarrow \boxed{x = 90^{\circ} + K\cdot 360^{\circ}}$ es solución ← 0.25

$x = 30^{\circ} \rightarrow \cos 60^{\circ} + 3\operatorname{sen}30^{\circ} \overset{?}{=} 2$

$\frac{1}{2} + 3\cdot\frac{1}{2} \overset{?}{=} 2$

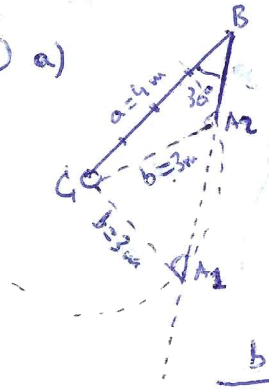
$\frac{4}{2} = 2$ o.k. $\Rightarrow \boxed{x = 30^{\circ} + K\cdot 360^{\circ}}$ es solu. ← 0.25

TOTAL: 2

$x = 150^{\circ} \rightarrow \cos 300^{\circ} + 3\operatorname{sen}150^{\circ} \overset{?}{=} 2$; $\frac{1}{2} + 3\cdot\frac{1}{2} = 2 \Rightarrow \boxed{x = 150^{\circ} + K\cdot 360^{\circ}}$ es solu. ← 0.25

$\cos 300^{\circ} = \cos(-60^{\circ}) = \cos 60^{\circ} = \frac{1}{2}$ $\qquad$ $\operatorname{sen}150^{\circ} = \operatorname{sen}(180-30^{\circ}) = \operatorname{sen}30^{\circ} = 1/2$

④ a)



$$\frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} \Rightarrow \frac{4}{\operatorname{sen} A} = \frac{3}{\operatorname{sen} 30^\circ} \Rightarrow \operatorname{sen} A = \frac{4 \operatorname{sen} 30^\circ}{3} \simeq \frac{2}{3} \Rightarrow$$

$$\Rightarrow \hat{A} = \operatorname{arcsen} \frac{2}{3} \begin{cases} \boxed{\hat{A}_1 \simeq 41^\circ 48' 37''} \Rightarrow \boxed{\hat{C}_1 = 180^\circ - (\hat{A}_1 + \hat{B}) \simeq 108^\circ 11' 23''} \\ \boxed{\hat{A}_2 \simeq 138^\circ 11' 23''} \Rightarrow \boxed{\hat{C}_2 = 180^\circ - (\hat{A}_2 + \hat{B}) \simeq 11^\circ 48' 37''} \end{cases}$$

0.8

$$\frac{b}{\operatorname{sen} B} = \frac{c_1}{\operatorname{sen} C_1} \Rightarrow \frac{3}{\operatorname{sen} 30^\circ} = \frac{c_1}{\operatorname{sen} 108^\circ 11' 23''} \Rightarrow \boxed{c_1 = \frac{3 \operatorname{sen} 108^\circ 11' 23''}{\operatorname{sen} 30^\circ} \simeq 5{,}7\,m} \leftarrow 0.3$$

$$\frac{b}{\operatorname{sen} B} = \frac{c_2}{\operatorname{sen} C_2} \Rightarrow \frac{3}{\operatorname{sen} 30^\circ} = \frac{c_2}{\operatorname{sen} 11^\circ 48' 37''} \Rightarrow \boxed{c_2 = \frac{3 \operatorname{sen} 11^\circ 48' 37''}{\operatorname{sen} 30^\circ} \simeq 1{,}23\,m} \leftarrow 0.3$$

b)

$$\boxed{S_{A_1BC_1} = \frac{1}{2} a b \operatorname{sen} \hat{C}_1 = \frac{1}{2} 4 \cdot 3 \cdot \operatorname{sen} 108^\circ 11' 23'' \simeq 5{,}70\,m^2} \leftarrow 0.3$$

$$\boxed{S_{A_2BC_2} = \frac{1}{2} a b \operatorname{sen} \hat{C}_2 = \frac{1}{2} 4 \cdot 3 \cdot \operatorname{sen} 11^\circ 48' 37'' \simeq 1{,}23\,m^2} \leftarrow 0.3$$

TOTAL: 2

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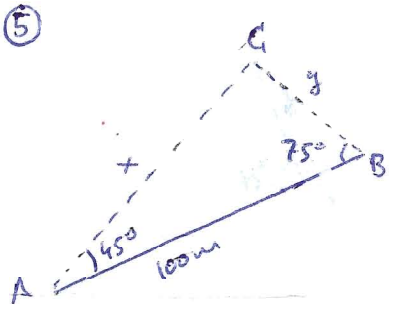
⑤



$$\hat{C} = 180 - (\hat{A} + \hat{B}) = 60^\circ$$

$$\frac{x}{\operatorname{sen} 75^\circ} = \frac{100}{\operatorname{sen} 60^\circ} \Rightarrow \boxed{x = \frac{100 \operatorname{sen} 75^\circ}{\operatorname{sen} 60^\circ} \simeq 111{,}54\,m} \quad 1$$

TOTAL: 2

$$\frac{y}{\operatorname{sen} 45^\circ} = \frac{100}{\operatorname{sen} 60^\circ} \Rightarrow \boxed{y = \frac{100 \operatorname{sen} 45^\circ}{\operatorname{sen} 60^\circ} \simeq 81{,}65\,m} \quad 1$$

LIMPIEZA ---------- 0.10
CORRECCIÓN LENGUAJE MATEMÁTICO ---- 0.05
ORDEN EN LA EXPOSICIÓN ---- 0.05
CALIGRAFÍA, ORTOGRAFÍA, SINTAXIS ---- 0.05