

## SOLUCIONES DEL EJERCICIO 7 DE LA RELACIÓN DE SUCESIONES

### COCIENTE DE POLINOMIOS

$$\lim_{n \rightarrow \infty} \frac{(n^2+2)(n^2-2)}{(n+2)^2(2n-1)^2} = \left(\frac{+\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{2}{n^2})n^2(1-\frac{2}{n^2})}{\left(n(1+\frac{2}{n})\right)^2\left(n(2-\frac{1}{n})\right)^2} = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{2}{n^2})n^2(1-\frac{2}{n^2})}{n^2(1+\frac{2}{n})^2 n^2(2-\frac{1}{n})^2} = \lim_{n \rightarrow \infty} \frac{(1+\frac{2}{n^2})(1-\frac{2}{n^2})}{(1+\frac{2}{n})^2(2-\frac{1}{n})^2} = \frac{1}{4}$$

$$\lim_{n \rightarrow \infty} \frac{3n^2 - n + 1}{n + 5} = \left(\frac{+\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{n^2(3 - \frac{1}{n} + \frac{1}{n^2})}{n(1 + \frac{5}{n})} = \lim_{n \rightarrow \infty} \frac{n(3 - \frac{1}{n} + \frac{1}{n^2})}{(1 + \frac{5}{n})} = +\infty$$

$$\lim_{n \rightarrow \infty} \frac{-n^3 + 2n - 1}{n^2 + n + 2} = \left(\frac{-\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{n^3(-1 + \frac{2}{n^2} - \frac{1}{n^3})}{n^2(1 + \frac{1}{n} + \frac{2}{n^2})} = \lim_{n \rightarrow \infty} \frac{n(-1 + \frac{2}{n^2} - \frac{1}{n^3})}{(1 + \frac{1}{n} + \frac{2}{n^2})} = -\infty$$

### COCIENTE DE POLINOMIOS CON RAICES

$$\lim_{n \rightarrow \infty} \frac{n^2 + 2n - 1}{\sqrt{n^2 - n + 4}} = \left(\frac{+\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot (1 + \frac{2}{n} - \frac{1}{n^2})}{\sqrt{n^2 \cdot (1 - \frac{1}{n} + \frac{4}{n^2})}} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot (1 + \frac{2}{n} - \frac{1}{n^2})}{n \cdot \sqrt{1 - \frac{1}{n} + \frac{4}{n^2}}} = \lim_{n \rightarrow \infty} \frac{n \cdot (1 + \frac{2}{n} - \frac{1}{n^2})}{\sqrt{1 - \frac{1}{n} + \frac{4}{n^2}}} = +\infty \cdot 1 = +\infty$$

$$\lim_{n \rightarrow \infty} \frac{3n^5 + 2n^3 - 1}{\sqrt{n^6 - n^4 + 4}} = \left(\frac{+\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{n^5 \cdot (3 + \frac{2}{n^2} - \frac{1}{n^5})}{\sqrt{n^6 \cdot (1 - \frac{1}{n^2} + \frac{4}{n^6})}} = \lim_{n \rightarrow \infty} \frac{n^5 \cdot (3 + \frac{2}{n^2} - \frac{1}{n^5})}{n^3 \cdot \sqrt{1 - \frac{1}{n^2} + \frac{4}{n^6}}} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot (3 + \frac{2}{n^2} - \frac{1}{n^5})}{\sqrt{1 - \frac{1}{n^2} + \frac{4}{n^6}}} = +\infty \cdot 3 = +\infty$$

$$\lim_{n \rightarrow \infty} \frac{3n^2 - 5n}{\sqrt{n^4 - 2n + 3}} = \left(\frac{+\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot (3 - \frac{5}{n})}{\sqrt{n^4 \cdot (1 - \frac{2}{n^3} + \frac{3}{n^4})}} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot (3 - \frac{5}{n})}{n^2 \cdot \sqrt{(1 - \frac{2}{n^3} + \frac{3}{n^4})}} = \lim_{n \rightarrow \infty} \frac{(3 - \frac{5}{n})}{\sqrt{(1 - \frac{2}{n^3} + \frac{3}{n^4})}} = 3$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{n^4 - 2n - 3}{\sqrt{2n^5 - 1}} &= \left(\frac{+\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{n^4 \cdot (1 - \frac{2}{n^3} - \frac{3}{n^4})}{\sqrt{n^5 \cdot (2 - \frac{1}{n^5})}} = \lim_{n \rightarrow \infty} \frac{n^4 \cdot (1 - \frac{2}{n^3} - \frac{3}{n^4})}{n^2 \cdot \sqrt{n \cdot (2 - \frac{1}{n^5})}} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot (1 - \frac{2}{n^3} - \frac{3}{n^4})}{\sqrt{n \cdot (2 - \frac{1}{n^5})}} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot (1 - \frac{2}{n^3} - \frac{3}{n^4})}{\sqrt{n} \cdot \sqrt{(2 - \frac{1}{n^5})}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \cdot (1 - \frac{2}{n^3} - \frac{3}{n^4})}{\sqrt{(2 - \frac{1}{n^5})}} = +\infty \cdot 1 = +\infty \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{-n^4 + 3}{\sqrt{3n^4 - 2n}} = \left(\frac{-\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{n^4 \cdot (-1 + \frac{3}{n^4})}{\sqrt{n^4 \cdot (3 - \frac{2}{n^3})}} = \lim_{n \rightarrow \infty} \frac{n^4 \cdot (-1 + \frac{3}{n^4})}{n^2 \cdot \sqrt{(3 - \frac{2}{n^3})}} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot (-1 + \frac{3}{n^4})}{\sqrt{(3 - \frac{2}{n^3})}} = +\infty \cdot \left(\frac{-1}{\sqrt{3}}\right) = -\infty$$

$$\lim_{n \rightarrow \infty} \frac{5n^2 - n + 2}{\sqrt{7n^6 + 3n^3}} = \left(\frac{+\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot (5 - \frac{1}{n} + \frac{2}{n^2})}{\sqrt{n^6 \cdot (7 + \frac{3}{n^3})}} = \lim_{n \rightarrow \infty} \frac{n^2 \cdot (5 - \frac{1}{n} + \frac{2}{n^2})}{n^3 \cdot \sqrt{(7 + \frac{3}{n^3})}} = \lim_{n \rightarrow \infty} \frac{(5 - \frac{1}{n} + \frac{2}{n^2})}{n \cdot \sqrt{(7 + \frac{3}{n^3})}} = \frac{5}{+\infty} = 0$$

$$\lim_{n \rightarrow \infty} \frac{-n^3 + 3n}{\sqrt{9n^4 + 4}} = \left(\frac{-\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{-n^3 \cdot (1 + \frac{3}{n})}{\sqrt{n^4 \cdot (9 + \frac{4}{n^4})}} = \lim_{n \rightarrow \infty} \frac{-n^3 \cdot (1 + \frac{3}{n})}{n^2 \cdot \sqrt{(9 + \frac{4}{n^4})}} = \lim_{n \rightarrow \infty} \frac{-n \cdot (1 + \frac{3}{n})}{\sqrt{(9 + \frac{4}{n^4})}} = -\infty \cdot \frac{1}{3} = -\infty$$

$$\lim_{n \rightarrow \infty} \frac{7n^3 - n + 2}{\sqrt{7n^6 + 3n^3}} = \left(\frac{+\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{n^3(7 - \frac{1}{n^2} + \frac{2}{n^3})}{\sqrt{n^6 \cdot (7 + \frac{3}{n^3})}} = \lim_{n \rightarrow \infty} \frac{n^3(7 - \frac{1}{n^2} + \frac{2}{n^3})}{n^3 \cdot \sqrt{(7 + \frac{3}{n^3})}} = \lim_{n \rightarrow \infty} \frac{(7 - \frac{1}{n^2} + \frac{2}{n^3})}{\sqrt{(7 + \frac{3}{n^3})}} = \frac{7}{\sqrt{7}} = \frac{7}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \sqrt{7}$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{8n^8 - 4n^2}}{2n^3 - 4n^4} = \left(\frac{+\infty}{+\infty}\right)_{ind} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^8 \cdot (8 - \frac{4}{n^6})}}{n^4 \cdot (\frac{2}{n} - 4)} = \lim_{n \rightarrow \infty} \frac{n^4 \cdot \sqrt{(8 - \frac{4}{n^6})}}{n^4 \cdot (\frac{2}{n} - 4)} = \lim_{n \rightarrow \infty} \frac{\sqrt{(8 - \frac{4}{n^6})}}{(\frac{2}{n} - 4)} = \frac{\sqrt{8}}{-4} = \frac{2\sqrt{2}}{-4} = \frac{\sqrt{2}}{-2}$$

### DIFERENCIA DE POLINOMIOS

$$\lim_{n \rightarrow \infty} \left( \frac{6n - 5}{3n^2 - 2} - \frac{5n - 3}{4n^2 + 1} \right) = 0 - 0 = 0$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \left( \frac{n^2}{2n - 1} - \frac{n^2 + 1}{2n + 1} \right) &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} \left( \frac{n^2(2n + 1) - (n^2 + 1)(2n - 1)}{(2n - 1)(2n + 1)} \right) \\ &= \lim_{n \rightarrow \infty} \left( \frac{2n^3 + n^2 - 2n^3 + n^2 - 2n + 1}{(2n - 1)(2n + 1)} \right) = \lim_{n \rightarrow \infty} \left( \frac{2n^2 - 2n + 1}{(2n - 1)(2n + 1)} \right) = \frac{2}{4} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \left( \frac{n^2 + 1}{3n + 1} - \frac{n^2}{3n - 1} \right) &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} \left( \frac{(n^2 + 1)(3n - 1) - n^2(3n + 1)}{(3n + 1)(3n - 1)} \right) \\ &= \lim_{n \rightarrow \infty} \left( \frac{3n^3 - n^2 + 3n - 1 - 3n^3 - n^2}{(3n + 1)(3n - 1)} \right) = \lim_{n \rightarrow \infty} \left( \frac{-2n^2 + 3n - 1}{(3n + 1)(3n - 1)} \right) = \frac{-2}{9} \end{aligned}$$

$$\begin{aligned}\lim_{n \rightarrow \infty} \left( \frac{n^3 + 2n^2}{3n^2 - n} - \frac{2n^3 - 1}{3n^2 - 3} \right) &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} \left( \frac{(n^3 + 2n^2)(3n^2 - 3) - (3n^2 - n)(2n^3 - 1)}{(3n^2 - n)(3n^2 - 3)} \right) \\ &= \lim_{n \rightarrow \infty} \left( \frac{3n^5 - 3n^3 + 6n^4 - 6n^2 - 6n^5 - 3n^2 + 2n^4 - n}{(3n^2 - n)(3n^2 - 3)} \right) = \lim_{n \rightarrow \infty} \left( \frac{-2n^5 + 8n^4 - 3n^3 - 9n^2 - n}{(3n^2 - n)(3n^2 - 3)} \right) = -\infty\end{aligned}$$

$$\begin{aligned}\lim_{n \rightarrow \infty} \left( 2n^2 - 1 - \frac{2n^4 + 5n^3 - n^2}{n^2 + 2} \right) &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} \frac{(2n^2 - 1)(n^2 + 2) - (2n^4 + 5n^3 - n^2)}{n^2 + 2} \\ &= \lim_{n \rightarrow \infty} \frac{2n^4 + 4n^2 - n^2 - 2 - 2n^4 - 5n^3 + n^2}{n^2 + 2} = \lim_{n \rightarrow \infty} \frac{-5n^3 + 4n^2 - 2}{n^2 + 2} = -\infty\end{aligned}$$

$$\lim_{n \rightarrow \infty} \left( 3n + 2 - \frac{7 + 3n^2}{n + 1} \right) = (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} \frac{(3n + 2)(n + 1) - (7 + 3n^2)}{n + 1} = \lim_{n \rightarrow \infty} \frac{3n^2 + 3n + 2n + 2 - 7 - 3n^2}{n + 1} = \lim_{n \rightarrow \infty} \frac{5n - 5}{n + 1} = 5$$

#### DIFERENCIA DE POLINOMIOS CON RAICES

$$\begin{aligned}\lim_{n \rightarrow \infty} \sqrt{2n^2 + 3n - 2} - \sqrt{2n^2 + 2} &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} \left( \sqrt{2n^2 + 3n - 2} - \sqrt{2n^2 + 2} \right) \cdot \frac{(\sqrt{2n^2 + 3n - 2} + \sqrt{2n^2 + 2})}{(\sqrt{2n^2 + 3n - 2} + \sqrt{2n^2 + 2})} \\ &= \lim_{n \rightarrow \infty} \frac{(\sqrt{2n^2 + 3n - 2})^2 - (\sqrt{2n^2 + 2})^2}{\sqrt{2n^2 + 3n - 2} + \sqrt{2n^2 + 2}} = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n - 2 - (2n^2 + 2)}{\sqrt{2n^2 + 3n - 2} + \sqrt{2n^2 + 2}} =\end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n - 2 - 2n^2 - 2}{\sqrt{2n^2 + 3n - 2} + \sqrt{2n^2 + 2}} = \lim_{n \rightarrow \infty} \frac{3n - 4}{\sqrt{2n^2 + 3n - 2} + \sqrt{2n^2 + 2}} = \left( \frac{+\infty}{+\infty} \right)_{ind} =$$

$$\lim_{n \rightarrow \infty} \frac{n \cdot \left( 3 - \frac{4}{n} \right)}{\sqrt{n^2 \cdot \left( 2 + \frac{3}{n} - \frac{2}{n^2} \right)} + \sqrt{n^2 \cdot \left( 2 + \frac{2}{n^2} \right)}} = \lim_{n \rightarrow \infty} \frac{n \cdot \left( 3 - \frac{4}{n} \right)}{n \cdot \left( \sqrt{2 + \frac{3}{n} - \frac{2}{n^2}} + \sqrt{2 + \frac{2}{n^2}} \right)} =$$

$$\lim_{n \rightarrow \infty} \frac{n \cdot \left( 3 - \frac{4}{n} \right)}{n \cdot \left[ \sqrt{2 + \frac{3}{n} - \frac{2}{n^2}} + \sqrt{2 + \frac{2}{n^2}} \right]} = \lim_{n \rightarrow \infty} \frac{3 - \frac{4}{n}}{\sqrt{2 + \frac{3}{n} - \frac{2}{n^2}} + \sqrt{2 + \frac{2}{n^2}}} = \frac{3}{2\sqrt{2}} = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\begin{aligned}\lim_{n \rightarrow \infty} \sqrt{n^2 + 4n + 1} - \sqrt{n^2 + 8n + 1} &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} \left( \sqrt{n^2 + 4n + 1} - \sqrt{n^2 + 8n + 1} \right) \cdot \frac{(\sqrt{n^2 + 4n + 1} + \sqrt{n^2 + 8n + 1})}{(\sqrt{n^2 + 4n + 1} + \sqrt{n^2 + 8n + 1})} \\ &= \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2 + 4n + 1})^2 - (\sqrt{n^2 + 8n + 1})^2}{\sqrt{n^2 + 4n + 1} + \sqrt{n^2 + 8n + 1}} = \lim_{n \rightarrow \infty} \frac{n^2 + 4n + 1 - (n^2 + 8n + 1)}{\sqrt{n^2 + 4n + 1} + \sqrt{n^2 + 8n + 1}} =\end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + 4n + 1 - n^2 - 8n - 1}{\sqrt{n^2 + 4n + 1} + \sqrt{n^2 + 8n + 1}} = \lim_{n \rightarrow \infty} \frac{-4n}{\sqrt{n^2 + 4n + 1} + \sqrt{n^2 + 8n + 1}} = \left( \frac{-\infty}{+\infty} \right)_{ind}$$

$$\lim_{n \rightarrow \infty} \frac{-4n}{\sqrt{n^2 \left( 1 + \frac{4}{n} + \frac{1}{n^2} \right)} + \sqrt{n^2 \left( 1 + \frac{8}{n} + \frac{1}{n^2} \right)}} = \lim_{n \rightarrow \infty} \frac{-4n}{n \left( \sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + \sqrt{1 + \frac{8}{n} + \frac{1}{n^2}} \right)} =$$

$$\lim_{n \rightarrow \infty} \frac{-4n}{n \left[ \sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + \sqrt{1 + \frac{8}{n} + \frac{1}{n^2}} \right]} = \lim_{n \rightarrow \infty} \frac{-4}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + \sqrt{1 + \frac{8}{n} + \frac{1}{n^2}}} = \frac{-4}{2} = -2$$

$$\begin{aligned}\lim_{n \rightarrow \infty} \sqrt{3 + 4n^2} - \sqrt{4n^2 + n} &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} \left( \sqrt{3 + 4n^2} - \sqrt{4n^2 + n} \right) \cdot \frac{(\sqrt{3 + 4n^2} + \sqrt{4n^2 + n})}{(\sqrt{3 + 4n^2} + \sqrt{4n^2 + n})} = \lim_{n \rightarrow \infty} \frac{(\sqrt{3 + 4n^2})^2 - (\sqrt{4n^2 + n})^2}{\sqrt{3 + 4n^2} + \sqrt{4n^2 + n}} \\ &= \lim_{n \rightarrow \infty} \frac{3 + 4n^2 - (4n^2 + n)}{\sqrt{3 + 4n^2} + \sqrt{4n^2 + n}} = \lim_{n \rightarrow \infty} \frac{3 + 4n^2 - 4n^2 - n}{\sqrt{3 + 4n^2} + \sqrt{4n^2 + n}} = \lim_{n \rightarrow \infty} \frac{3 - n}{\sqrt{3 + 4n^2} + \sqrt{4n^2 + n}} = \left( \frac{-\infty}{+\infty} \right)_{ind} \\ &= \lim_{n \rightarrow \infty} \frac{n \left( \frac{3}{n} - 1 \right)}{\sqrt{n^2 \left( \frac{3}{n^2} + 4 \right)} + \sqrt{n^2 \left( 4 + \frac{1}{n} \right)}} \\ &= \lim_{n \rightarrow \infty} \frac{n \left( \frac{3}{n} - 1 \right)}{n \left[ \sqrt{\left( \frac{3}{n^2} + 4 \right)} + \sqrt{\left( 4 + \frac{1}{n} \right)} \right]} = \lim_{n \rightarrow \infty} \frac{\frac{3}{n} - 1}{\sqrt{\left( \frac{3}{n^2} + 4 \right)} + \sqrt{\left( 4 + \frac{1}{n} \right)}} = \frac{-1}{4}\end{aligned}$$

$$\begin{aligned}
\lim_{n \rightarrow \infty} \sqrt{n^2 - 3n + 2} - n &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} (\sqrt{n^2 - 3n + 2} - n) \cdot \frac{(\sqrt{n^2 - 3n + 2} + n)}{(\sqrt{n^2 - 3n + 2} + n)} = \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2 - 3n + 2})^2 - n^2}{\sqrt{n^2 - 3n + 2} + n} \\
&= \lim_{n \rightarrow \infty} \frac{n^2 - 3n + 2 - n^2}{\sqrt{n^2 - 3n + 2} + n} = \lim_{n \rightarrow \infty} \frac{-3n + 2}{\sqrt{n^2 - 3n + 2} + n} = \left(\frac{-\infty}{+\infty}\right) = \lim_{n \rightarrow \infty} \frac{n(-3 + \frac{2}{n})}{\sqrt{n^2(1 - \frac{3}{n} + \frac{2}{n^2})} + n} = \lim_{n \rightarrow \infty} \frac{n(-3 + \frac{2}{n})}{n\sqrt{(1 - \frac{3}{n} + \frac{2}{n^2})} + 1} \\
&= \lim_{n \rightarrow \infty} \frac{n(-3 + \frac{2}{n})}{n\sqrt{(1 - \frac{3}{n} + \frac{2}{n^2})} + 1} = \lim_{n \rightarrow \infty} \frac{-3 + \frac{2}{n}}{\sqrt{(1 - \frac{3}{n} + \frac{2}{n^2})} + 1} = \frac{-3}{2}
\end{aligned}$$

$$\begin{aligned}
\lim_{n \rightarrow \infty} \sqrt{n^2 + 2n} - \sqrt{n^2 + n} &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} (\sqrt{n^2 + 2n} - \sqrt{n^2 + n}) \cdot \frac{(\sqrt{n^2 + 2n} + \sqrt{n^2 + n})}{(\sqrt{n^2 + 2n} + \sqrt{n^2 + n})} = \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2 + 2n})^2 - (\sqrt{n^2 + n})^2}{\sqrt{n^2 + 2n} + \sqrt{n^2 + n}} \\
&= \lim_{n \rightarrow \infty} \frac{n^2 + 2n - n^2 - n}{\sqrt{n^2 + 2n} + \sqrt{n^2 + n}} = \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 + 2n} + \sqrt{n^2 + n}} = \left(\frac{+\infty}{+\infty}\right) = \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2(1 + \frac{2}{n})} + \sqrt{n^2(1 + \frac{1}{n})}} \\
&= \lim_{n \rightarrow \infty} \frac{n}{n\sqrt{(1 + \frac{2}{n})} + n\sqrt{(1 + \frac{1}{n})}} = \lim_{n \rightarrow \infty} \frac{n}{n\left[\sqrt{(1 + \frac{2}{n})} + \sqrt{(1 + \frac{1}{n})}\right]} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{(1 + \frac{2}{n})} + \sqrt{(1 + \frac{1}{n})}} = \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\lim_{n \rightarrow \infty} \sqrt{4n^2 - 1} - \sqrt{4n^2 + 2n} &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} (\sqrt{4n^2 - 1} - \sqrt{4n^2 + 2n}) \cdot \frac{(\sqrt{4n^2 - 1} + \sqrt{4n^2 + 2n})}{(\sqrt{4n^2 - 1} + \sqrt{4n^2 + 2n})} \\
&= \lim_{n \rightarrow \infty} \frac{(\sqrt{4n^2 - 1})^2 - (\sqrt{4n^2 + 2n})^2}{\sqrt{4n^2 - 1} + \sqrt{4n^2 + 2n}} = \lim_{n \rightarrow \infty} \frac{4n^2 - 1 - 4n^2 - 2n}{\sqrt{4n^2 - 1} + \sqrt{4n^2 + 2n}} = \lim_{n \rightarrow \infty} \frac{-1 - 2n}{\sqrt{4n^2 - 1} + \sqrt{4n^2 + 2n}} = \left(\frac{-\infty}{+\infty}\right) \\
&= \lim_{n \rightarrow \infty} \frac{n(-\frac{1}{n} - 2)}{\sqrt{n^2(4 - \frac{1}{n^2})} + \sqrt{n^2(4 + \frac{2}{n})}} = \lim_{n \rightarrow \infty} \frac{n(-\frac{1}{n} - 2)}{n\sqrt{(4 - \frac{1}{n^2})} + n\sqrt{(4 + \frac{2}{n})}} = \lim_{n \rightarrow \infty} \frac{n(-\frac{1}{n} - 2)}{n\left[\sqrt{(4 - \frac{1}{n^2})} + \sqrt{(4 + \frac{2}{n})}\right]} \\
&= \lim_{n \rightarrow \infty} \frac{-\frac{1}{n} - 2}{\sqrt{(4 - \frac{1}{n^2})} + \sqrt{(4 + \frac{2}{n})}} = \frac{-2}{4} = -\frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\lim_{n \rightarrow \infty} \sqrt{2n^4 - 1} - 2n^2 &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} (\sqrt{2n^4 - 1} - 2n^2) \cdot \frac{(\sqrt{2n^4 - 1} + 2n^2)}{(\sqrt{2n^4 - 1} + 2n^2)} = \lim_{n \rightarrow \infty} \frac{(\sqrt{2n^4 - 1})^2 - (2n^2)^2}{\sqrt{2n^4 - 1} + 2n^2} = \lim_{n \rightarrow \infty} \frac{2n^4 - 1 - 4n^4}{\sqrt{2n^4 - 1} + 2n^2} = \lim_{n \rightarrow \infty} \frac{-1 - 2n^4}{\sqrt{2n^4 - 1} + 2n^2} \\
\left(\frac{-\infty}{+\infty}\right) &= \lim_{n \rightarrow \infty} \frac{n^4(-\frac{1}{n^4} - 2)}{\sqrt{n^4(2 - \frac{1}{n^4})} + 2n^2} = \lim_{n \rightarrow \infty} \frac{n^4(-\frac{1}{n^4} - 2)}{n^2\sqrt{(2 - \frac{1}{n^4})} + 2n^2} = \lim_{n \rightarrow \infty} \frac{n^4(-\frac{1}{n^4} - 2)}{n^2\left[\sqrt{(2 - \frac{1}{n^4})} + 2\right]} = \lim_{n \rightarrow \infty} \frac{n^2(-\frac{1}{n^2} - 2)}{\sqrt{(2 - \frac{1}{n^4})} + 2} = \frac{-\infty}{\sqrt{2} + 2} = -\infty
\end{aligned}$$

$$\begin{aligned}
\lim_{n \rightarrow \infty} n - \sqrt{n^2 + 10n} &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} n - \sqrt{n^2 + 10n} \cdot \frac{n + \sqrt{n^2 + 10n}}{n + \sqrt{n^2 + 10n}} = \lim_{n \rightarrow \infty} \frac{n^2 - (n^2 + 10n)}{n + \sqrt{n^2 + 10n}} = \lim_{n \rightarrow \infty} \frac{n^2 - n^2 - 10n}{n + \sqrt{n^2 + 10n}} \\
&= \lim_{n \rightarrow \infty} \frac{-10n}{n + \sqrt{n^2 + 10n}} = \left(\frac{-\infty}{+\infty}\right) = \lim_{n \rightarrow \infty} \frac{-10n}{n + \sqrt{n^2(1 + \frac{10}{n})}} \\
&= \lim_{n \rightarrow \infty} \frac{-10n}{n + n\sqrt{(1 + \frac{10}{n})}} = \lim_{n \rightarrow \infty} \frac{-10n}{n\left[1 + \sqrt{(1 + \frac{10}{n})}\right]} = \lim_{n \rightarrow \infty} \frac{-10}{1 + \sqrt{(1 + \frac{10}{n})}} = \frac{-10}{2} = -5
\end{aligned}$$

$$\begin{aligned}
\lim_{n \rightarrow \infty} \sqrt{2n^2 - 1} - \sqrt{2n^2 + 2n} &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} (\sqrt{2n^2 - 1} - \sqrt{2n^2 + 2n}) \cdot \frac{(\sqrt{2n^2 - 1} + \sqrt{2n^2 + 2n})}{(\sqrt{2n^2 - 1} + \sqrt{2n^2 + 2n})} = \lim_{n \rightarrow \infty} \frac{(\sqrt{2n^2 - 1})^2 - (\sqrt{2n^2 + 2n})^2}{\sqrt{2n^2 - 1} + \sqrt{2n^2 + 2n}} \\
&= \lim_{n \rightarrow \infty} \frac{2n^2 - 1 - (2n^2 + 2n)}{\sqrt{2n^2 - 1} + \sqrt{2n^2 + 2n}} = \lim_{n \rightarrow \infty} \frac{-1 - 2n}{\sqrt{2n^2 - 1} + \sqrt{2n^2 + 2n}} = \left(\frac{-\infty}{+\infty}\right) = \lim_{n \rightarrow \infty} \frac{n(-\frac{1}{n} - 2)}{\sqrt{n^2(2 - \frac{1}{n^2})} + \sqrt{n^2(2 + \frac{2}{n})}} \\
&= \lim_{n \rightarrow \infty} \frac{n(-\frac{1}{n} - 2)}{n\sqrt{(2 - \frac{1}{n^2})} + n\sqrt{(2 + \frac{2}{n})}} = \lim_{n \rightarrow \infty} \frac{n(-\frac{1}{n} - 2)}{n\left[\sqrt{(2 - \frac{1}{n^2})} + \sqrt{(2 + \frac{2}{n})}\right]} = \lim_{n \rightarrow \infty} \frac{-\frac{1}{n} - 2}{\sqrt{(2 - \frac{1}{n^2})} + \sqrt{(2 + \frac{2}{n})}} = \frac{-2}{\sqrt{2} + 1}
\end{aligned}$$

$$\begin{aligned}
\lim_{n \rightarrow \infty} \sqrt{n^2 - 10n + 8} - (n - 3) &= (+\infty) - (+\infty)_{ind} = \lim_{n \rightarrow \infty} [\sqrt{n^2 - 10n + 8} - (n - 3)] \cdot \frac{[\sqrt{n^2 - 10n + 8} + (n - 3)]}{[\sqrt{n^2 - 10n + 8} + (n - 3)]} = \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2 - 10n + 8})^2 - (n - 3)^2}{\sqrt{n^2 - 10n + 8} + (n - 3)} \\
&= \lim_{n \rightarrow \infty} \frac{n^2 - 10n + 8 - (n^2 - 6n + 9)}{\sqrt{n^2 - 10n + 8} + (n - 3)} = \lim_{n \rightarrow \infty} \frac{-4n - 1}{\sqrt{n^2 - 10n + 8} + (n - 3)} = \left(\frac{-\infty}{+\infty}\right)_{ind} = \\
&= \lim_{n \rightarrow \infty} \frac{n(-4 - \frac{1}{n})}{\sqrt{n^2(1 - \frac{10}{n} + \frac{8}{n^2})} + n(1 - \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n(-4 - \frac{1}{n})}{n\sqrt{(1 - \frac{10}{n} + \frac{8}{n^2})} + n(1 - \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{-4 - \frac{1}{n}}{\sqrt{(1 - \frac{10}{n} + \frac{8}{n^2})} + (1 - \frac{3}{n})} = \frac{-4}{2} = -2
\end{aligned}$$

### EL NÚMERO e

$$\lim_{n \rightarrow \infty} \left( \frac{4n-2}{4n-3} \right)^{4n+3} = (1^+)^{\infty}_{ind} = e^{\lim_{n \rightarrow \infty} (4n+3) \left( \frac{4n-2}{4n-3} - 1 \right)} = e^{\lim_{n \rightarrow \infty} (4n+3) \left( \frac{4n-2-4n+3}{4n-3} \right)} = e^{\lim_{n \rightarrow \infty} (4n+3) \left( \frac{1}{4n-3} \right)} = e^{\lim_{n \rightarrow \infty} \left( \frac{4n+3}{4n-3} \right)} = e^1 = e$$

$$\lim_{n \rightarrow \infty} \left( \frac{n^2+5}{n^2} \right)^{2n^2} = (1^+)^{\infty}_{ind} = e^{\lim_{n \rightarrow \infty} 2n^2 \left( \frac{n^2+5}{n^2} - 1 \right)} = e^{\lim_{n \rightarrow \infty} 2n^2 \left( \frac{n^2+5-n^2}{n^2} \right)} = e^{\lim_{n \rightarrow \infty} 2n^2 \left( \frac{5}{n^2} \right)} = e^{\lim_{n \rightarrow \infty} \left( \frac{10n^2}{n^2} \right)} = e^{10}$$

$$\lim_{n \rightarrow \infty} \left( \frac{3n^2+1}{3n^2-1} \right)^{\frac{n^2}{n+1}} = (1^+)^{\infty}_{ind} = e^{\lim_{n \rightarrow \infty} \frac{n^2}{n+1} \left( \frac{3n^2+1}{3n^2-1} - 1 \right)} = e^{\lim_{n \rightarrow \infty} \frac{n^2}{n+1} \left( \frac{3n^2+1-3n^2+1}{3n^2-1} \right)} = e^{\lim_{n \rightarrow \infty} \frac{n^2}{n+1} \left( \frac{2}{3n^2-1} \right)} = e^0 = 1$$

$$\lim_{n \rightarrow \infty} \left( \frac{n-1}{n-2} \right)^{n^2-3} = (1^+)^{\infty}_{ind} = e^{\lim_{n \rightarrow \infty} (n^2-3) \left( \frac{n-1}{n-2} - 1 \right)} = e^{\lim_{n \rightarrow \infty} (n^2-3) \left( \frac{n-1-n+2}{n-2} \right)} = e^{\lim_{n \rightarrow \infty} (n^2-3) \left( \frac{1}{n-2} \right)} = e^{+\infty} = +\infty$$

$$\lim_{n \rightarrow \infty} \left( \frac{4n-3}{4n} \right)^{\frac{2n^2-1}{2n}} = (1^+)^{\infty}_{ind} = e^{\lim_{n \rightarrow \infty} \frac{(2n^2-1)(4n-3)}{2n \cdot 4n} - 1} = e^{\lim_{n \rightarrow \infty} \frac{(2n^2-1)(4n-3-4n)}{2n \cdot 4n}} = e^{\lim_{n \rightarrow \infty} \frac{(2n^2-1)(-3)}{2n \cdot 4n}} = e^{\frac{-6}{8}} = e^{-\frac{3}{4}}$$

$$\lim_{n \rightarrow \infty} \left( \frac{2n+7}{2n-3} \right)^{\frac{n^4}{2n^3+1}} = (1^+)^{\infty}_{ind} = e^{\lim_{n \rightarrow \infty} \frac{n^4}{n^3+1} \left( \frac{2n+7}{2n-3} - 1 \right)} = e^{\lim_{n \rightarrow \infty} \frac{n^4}{n^3+1} \left( \frac{2n+7-2n+3}{2n-3} \right)} = e^{\lim_{n \rightarrow \infty} \frac{n^4}{n^3+1} \left( \frac{10}{2n-3} \right)} = e^{\frac{10}{2}} = e^5$$

$$\lim_{n \rightarrow \infty} \left( \frac{3n^2+5}{3n^2-n} \right)^{n^2-2} = (1^+)^{\infty}_{ind} = e^{\lim_{n \rightarrow \infty} (n^2-2) \left( \frac{3n^2+5}{3n^2-n} - 1 \right)} = e^{\lim_{n \rightarrow \infty} (n^2-2) \left( \frac{3n^2+5-3n^2+n}{3n^2-n} \right)} = e^{\lim_{n \rightarrow \infty} (n^2-2) \left( \frac{5+n}{3n^2-n} \right)} = e^{+\infty} = +\infty$$

$$\lim_{n \rightarrow \infty} \left( \frac{3n-2}{3n+2} \right)^{3n} = (1^+)^{\infty}_{ind} = e^{\lim_{n \rightarrow \infty} 3n \left( \frac{3n-2}{3n+2} - 1 \right)} = e^{\lim_{n \rightarrow \infty} 3n \left( \frac{3n-2-3n-2}{3n+2} \right)} = e^{\lim_{n \rightarrow \infty} 3n \left( \frac{-4}{3n+2} \right)} = e^{-4}$$

$$\lim_{n \rightarrow \infty} \left( 1 - \frac{1}{n} \right)^n = (1^+)^{\infty}_{ind} = e^{\lim_{n \rightarrow \infty} n \left( 1 - \frac{1}{n} - 1 \right)} = e^{\lim_{n \rightarrow \infty} n \left( -\frac{1}{n} \right)} = e^{\lim_{n \rightarrow \infty} -1} = e^{-1}$$

### EXPO-POTENCIALES INMEDIATOS

$$\lim_{n \rightarrow \infty} \left( \frac{2n^2+2}{n^2+n+1} \right)^{-n^2-n+1} = 2^{-\infty} = 0$$

$$\lim_{n \rightarrow \infty} \left( \frac{2n^4-7n}{5n^4-11} \right)^{n-2n^3+1} = \left( \frac{2}{5} \right)^{-\infty} = \frac{1}{\left( \frac{2}{5} \right)^{+\infty}} = \frac{1}{0^+} = +\infty$$

$$\lim_{n \rightarrow \infty} \left( \frac{9n^2-n+1}{3n^2+2n-3} \right)^{-n^3+2n^2-n} = 3^{-\infty} = 0$$

$$\lim_{n \rightarrow \infty} (5+2n^3-3n)^{\frac{4n^2-4n^3}{1+2n^3}} = (+\infty)^{-2} = \frac{1}{(+\infty)^2} = \frac{1}{+\infty} = 0$$

$$\lim_{n \rightarrow \infty} \left( \frac{2n^2-n+2}{4n^2+2n-3} \right)^{n^3+2n^2-n} = \left( \frac{2}{4} \right)^{+\infty} = 0$$

$$\lim_{n \rightarrow \infty} \left( \frac{4n^2-2n}{3n^2+6} \right)^{\frac{-2n^2+3}{3n-1}} = \left( \frac{4}{3} \right)^{-\infty} = \frac{1}{\left( \frac{4}{3} \right)^{+\infty}} = \frac{1}{+\infty} = 0$$