

SISTEMAS DE TRES ECUACIONES CON TRES INCÓGNITAS

Sistemas de tres ecuaciones con tres incógnitas.

Consideremos el siguiente sistema de tres ecuaciones con tres incógnitas:

$$\left. \begin{aligned} x + y + z &= 2 \\ 3x - 2y - z &= 4 \\ -2x + y + 2z &= 2 \end{aligned} \right\}$$

Para resolver este sistema, un procedimiento podría ser despejar, por ejemplo, de la primera ecuación la incógnita "z", y sustituirla en las otras dos ecuaciones, con lo cual se obtendría un sistema de dos ecuaciones con dos incógnitas, que ya sabría resolver.

$$\text{Ej: } \left. \begin{aligned} z &= 2 - x - y \\ 3x - 2y - (2 - x - y) &= 4 \\ -2x + y + 2(2 - x - y) &= 2 \end{aligned} \right\} \dots$$

Consideremos este otro sistema de ecuaciones, muy particular:

$$\left. \begin{aligned} x + 2y + z &= -1 \\ -3y - z &= 3 \\ z &= -2 \end{aligned} \right\} \quad \begin{aligned} y &= -\frac{1}{3} \\ z &= -2 \end{aligned}$$

Este sistema se denomina TRIANGULAR o ESCALONADO, su resolución es muy sencilla, basta con ir despejando las incógnitas de abajo a arriba.

$$-3y = 3 - z$$

$$y = -\frac{1}{3}$$

$$x = -1 + \frac{2}{3} + 2$$

$$x = \frac{2}{3} + 1$$

$$(x, y, z) = \left(\frac{5}{3}, -\frac{1}{3}, -2 \right)$$

$$x = \frac{5}{3}$$

Un sistema triangular como vemos es muy fácil de resolver. Con el método de Gauss convertiremos el sistema inicial en uno equivalente pero que es triangular

Ej: Resolvamos el sistema anterior por el método de Gauss

$$\left. \begin{array}{l} x + y + z = 2 \\ 3x - 2y - z = 4 \\ -2x + y + 2z = 2 \end{array} \right\} \text{GAUSS}$$

1º. Trabajamos sólo con los coeficientes de las incógnitas y los términos independientes, lo ponemos así:

$$\left(\begin{array}{ccc|c} x & y & z & TI \\ 1 & 1 & 1 & 2 \\ 1^{\circ} \textcircled{3} & -2 & -1 & 4 \\ 2^{\circ} \textcircled{-2} & 3^{\circ} \textcircled{1} & 2 & 2 \end{array} \right)$$

2º Para conseguir el 0 en la posición 1ª, lo haremos utilizando la ecuación 1ª.

$$\left(\begin{array}{ccc|c} x & y & z & TI \\ 1 & 1 & 1 & 2 \\ 3 & -2 & -1 & 4 \\ -2 & 1 & 2 & 2 \end{array} \right) \approx E_2 - 3E_1, \approx E_3 + 2E_1$$

$$\approx \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -5 & -4 & -2 \\ 0 & 3 & 4 & 6 \end{array} \right) \approx 5E_3 + 3E_2 \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -5 & -4 & -2 \\ 0 & 0 & 8 & 24 \end{array} \right)$$

$$8z = 24 \Rightarrow \underline{z = 3}$$

$$-5y - 4z = -2 \Rightarrow -5y - 12 = -2 \Rightarrow -5y = 10$$

$$\underline{y = -2}$$

$$x + y + z = 2 \Rightarrow x = 2 - y - z \Rightarrow x = 2 + 2 - 3 \Rightarrow \underline{x = 1}$$

Sistema Compatible Determinado.

Terna Soluc $(x, y, z) = (1, -2, 3)$

$$\left. \begin{array}{l} x + 5y - 3z = 7 \\ 2x - y + z = 11 \\ 4x + 3y - 4z = 3 \end{array} \right\} \left(\begin{array}{ccc|c} x & y & z & T_i \\ 1 & 5 & -3 & 7 \\ 2 & -1 & 1 & 11 \\ 4 & 3 & -4 & 3 \end{array} \right) \approx$$

$$\begin{array}{l} \approx \\ E_2 - 2E_1 \\ E_3 - 4E_1 \end{array} \left(\begin{array}{ccc|c} 1 & 5 & -3 & 7 \\ 0 & -11 & 7 & -3 \\ 0 & -17 & 8 & -25 \end{array} \right) \approx \left(\begin{array}{ccc|c} x & z & y & T_i \\ 1 & -3 & 5 & 7 \\ 0 & 7 & -11 & -3 \\ 0 & 8 & -17 & -25 \end{array} \right) \approx$$

$$\approx \begin{array}{l} 7E_3 - 8E_2 \end{array} \left(\begin{array}{ccc|c} 1 & -3 & 5 & 7 \\ 0 & 7 & -11 & -3 \\ 0 & 0 & -31 & -151 \end{array} \right)$$

$$-31y = -151 \Rightarrow \underline{y = \frac{151}{31}}$$

$$7z - 11y = -3$$

$$7z - 11 \cdot \left(\frac{151}{31} \right) = -3$$

$$7z = -3 + \frac{1661}{31}$$

$$\underline{\underline{z = \frac{224}{31}}}$$

$$x - 3z + 5y = 7$$

Sistema COMPATIBLE
DETERMINADO.

$$x - 3 \cdot \frac{224}{31} + 5 \cdot \frac{151}{31} = 7$$

$$x = \frac{134}{31}$$

$$\text{Terna Soluc } (x, y, z) = \left(\frac{134}{31}, \frac{151}{31}, \frac{224}{31} \right)$$

$$\left. \begin{array}{l} x - 2y + z = 3 \\ 2x - 5y + 3z = 4 \\ 5x + y + 7z = 11 \end{array} \right\} \left(\begin{array}{ccc|c} x & y & z & T_i \\ 1 & -2 & 1 & 3 \\ 2 & -5 & 3 & 4 \\ 5 & 1 & 7 & 11 \end{array} \right) \approx \begin{array}{l} E_2 - 2E_1 \\ E_3 - 5E_1 \end{array}$$

$$\approx \left(\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & -1 & 1 & -2 \\ 0 & 11 & 2 & -4 \end{array} \right) \xrightarrow{E_3 + 11E_2} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & 13 & -26 \end{array} \right)$$

$$13z = -26$$

$$\underline{\underline{z = -2}}$$

$$-y + z = -2$$

$$-y = -2 + 2$$

$$-y = 0$$

$$\underline{\underline{y = 0}}$$

$$x - 2y + z = 3$$

$$x = 3 + 2y - z$$

$$x = 3 + 2 \cdot 0 + 2$$

$$\underline{\underline{x = 5}}$$

SISTEMA COMPATIBLE DETERMINADO.

Terna solución:

$$(x, y, z) = (5, 0, -2)$$

$$\left. \begin{aligned} x - 3y + z &= -1 \\ -3x + 2y - z &= 0 \\ -x - 5y + 3z &= 1 \end{aligned} \right\} \left(\begin{array}{ccc|c} x & y & z & T_i \\ 1 & -3 & 1 & -1 \\ -3 & 2 & -1 & 0 \\ -1 & -5 & 3 & 1 \end{array} \right) \approx \begin{matrix} \\ \epsilon_2 + 3\epsilon_1 \\ \epsilon_3 - \epsilon_1 \end{matrix}$$

$$\approx \left(\begin{array}{ccc|c} 1 & -3 & 1 & -1 \\ 0 & -7 & 2 & -3 \\ 0 & -8 & 4 & 0 \end{array} \right) \approx \left(\begin{array}{ccc|c} x & z & y & T_i \\ 1 & 1 & -3 & -1 \\ 0 & 2 & -7 & -3 \\ 0 & 4 & -8 & 0 \end{array} \right) \approx \begin{matrix} \\ \\ \epsilon_3 - 2\epsilon_2 \end{matrix}$$

$$\approx \left(\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & 2 & -7 & -3 \\ 0 & 0 & 6 & 6 \end{array} \right)$$

$$6y = 6 \Rightarrow \underline{y = 1}$$

$$2z - 7y = -3$$

$$2z - 7 = -3$$

$$2z = 4$$

$$\underline{z = 2}$$

$$x + z - 3y = -1$$

$$x = -1 - z + 3y$$

$$x = -1 - 2 + 3$$

$$\underline{x = 0}$$

Terna soluc. $(x, y, z) = (0, 1, 2)$.

SISTEMA COMPATIBLE DETERMINADO.

$$\left. \begin{aligned} x - 2y - 3z &= 2 \\ 2x + y + 5z &= -1 \\ x + 3y + 8z &= -3 \end{aligned} \right\} \left(\begin{array}{ccc|c} x & y & z & T_i \\ 1 & -2 & -3 & 2 \\ 2 & 1 & 5 & -1 \\ 1 & 3 & 8 & -3 \end{array} \right) \approx$$

$$\begin{array}{l} \approx \\ E_2 - 2E_1 \\ E_3 - E_1 \end{array} \left(\begin{array}{ccc|c} 1 & -2 & -3 & 2 \\ 0 & 5 & 11 & -5 \\ 0 & 5 & 11 & -5 \end{array} \right) \xrightarrow{E_3 - E_2} \left(\begin{array}{ccc|c} 1 & -2 & -3 & 2 \\ 0 & 5 & 11 & -5 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$5y + 11z = -5$$

$$5y = -5 - 11z$$

$$y = \frac{-5 - 11z}{5}$$

$$x - 2y - 3z = 2$$

$$x - 2\left(\frac{-5 - 11z}{5}\right) - 3z = 2$$

$$x + \frac{10 + 22z}{5} - 3z = 2$$

$$x + \frac{10 + 22z - 15z}{5} = 2$$

$$x + \frac{10 + 7z}{5} = 2$$

$$x = -\frac{7z}{5} \quad \leftarrow \quad x = 2 - \frac{10 + 7z}{5}$$

Conclusión:

SISTEMA COMPATIBLE INDETERMINADO.

$$(x, y, z) = \left(-\frac{7\lambda}{5}, \frac{-5 - 11\lambda}{5}, \lambda \right) \text{ con } \lambda \in \mathbb{R}$$

$$\left. \begin{array}{l} 2x - y + z = -1 \\ x + 3y + z = 2 \\ 3x + 2y + 2z = 5 \end{array} \right\} \left(\begin{array}{ccc|c} x & y & z & T_i \\ 1 & 3 & 1 & 2 \\ 2 & -1 & 1 & -1 \\ 3 & 2 & 2 & 5 \end{array} \right) \xrightarrow{\begin{array}{l} E_2 - 2E_1 \\ E_3 - 3E_1 \end{array}} \approx$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -7 & -1 & -5 \\ 0 & -7 & -1 & -1 \end{array} \right) \xrightarrow{E_3 - E_2} \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -7 & -1 & -5 \\ 0 & 0 & 0 & 4 \end{array} \right) \quad 0 = 4$$



SISTEMA INCOMPATIBLE

Pon un ejemplo tal que sin necesidad de resolverlo el sistema sea compatible indeterminado.

$$\left. \begin{array}{l} x + 2y - 3z = 4 \\ 2x - 3y + 2z = -5 \\ 3x - y - z = -1 \end{array} \right\} \begin{array}{l} \text{COMPATIBLE} \\ \text{INDETERMINADO.} \end{array}$$

Incompatible

$$\left. \begin{array}{l} x + 2y - 3z = 4 \\ 2x - 3y + 2z = -5 \\ 3x - y - z = 3 \end{array} \right\} \text{INCOMPATIBLE.}$$

1º Resuelve estos sistemas de ecuaciones mediante el método de Gauss.

$$\textcircled{1} \left. \begin{array}{l} x - y + z = -1 \\ -3x + 2y + z = 3 \\ 2x + y + 2z = 0 \end{array} \right\} \left(\begin{array}{ccc|c} x & y & z & T_i \\ 1 & -1 & 1 & -1 \\ -3 & 2 & 1 & 3 \\ 2 & 1 & 2 & 0 \end{array} \right) \approx \begin{array}{l} E_2 + 3E_1 \\ E_3 - 2E_1 \end{array}$$

$$\approx \left(\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & -1 & 4 & 0 \\ 0 & 3 & 0 & 2 \end{array} \right) \xrightarrow{E_3 + 3E_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 12 & 2 \end{array} \right)$$

$$12z = 2$$

$$z = \frac{1}{6}$$

$$-y + 4z = 0$$

$$-y = -\frac{4}{6}$$

$$y = \frac{2}{3}$$

$$x - y + z = -1$$

$$x = -1 + \frac{2}{3} - \frac{1}{6}$$

$$x = \frac{-6 + 4 - 1}{6}$$

$$x = -\frac{3}{6} = -\frac{1}{2}$$

Sistema COMPATIBLE
DETERMINADO

Terna solución $(x, y, z) = \left(-\frac{1}{2}, \frac{2}{3}, \frac{1}{6}\right)$

$$\textcircled{2} \left. \begin{array}{l} 3x - 4y + 2z = 1 \\ -2x - 3y + z = 2 \\ 5x - y + z = 5 \end{array} \right\} \left(\begin{array}{ccc|c} x & y & z & T_i \\ 3 & -4 & 2 & 1 \\ -2 & -3 & 1 & 2 \\ 5 & -1 & 1 & 5 \end{array} \right) \approx$$

$$\approx \left(\begin{array}{ccc|c} z & x & y & T_i \\ 1 & -2 & -3 & 2 \\ 2 & 3 & -4 & 1 \\ 1 & 5 & -1 & 5 \end{array} \right) \xrightarrow{\substack{E_2 - 2E_1 \\ E_3 - E_1}} \left(\begin{array}{ccc|c} z & x & y & T_i \\ 1 & -2 & -3 & 2 \\ 0 & 7 & 2 & -3 \\ 0 & 7 & 2 & 3 \end{array} \right) \xrightarrow{E_3 - E_2}$$

$$\approx \left(\begin{array}{ccc|c} z & x & y & T_i \\ 1 & -2 & -3 & 2 \\ 0 & 7 & 2 & -3 \\ 0 & 0 & 0 & 6 \end{array} \right)$$

$$0 = 6$$



SISTEMA
INCOMPATIBLE

$$③ \quad \left. \begin{aligned} x - 2y &= -3 \\ -2x + 3y + z &= 4 \\ 2x + y - 5z &= 4 \end{aligned} \right\} \begin{pmatrix} x & y & z & \tau_i \\ 1 & -2 & 0 & -3 \\ -2 & 3 & 1 & 4 \\ 2 & 1 & -5 & 4 \end{pmatrix} \begin{array}{l} \\ E_2 + 2E_1 \\ E_3 - 2E_1 \end{array} \approx$$

$$\approx \begin{pmatrix} 1 & -2 & 0 & -3 \\ 0 & -1 & 1 & -2 \\ 0 & 5 & -5 & 10 \end{pmatrix} \begin{array}{l} \\ \\ E_3 + 5E_2 \end{array} \approx \begin{pmatrix} 1 & -2 & 0 & -3 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$-y + z = -2 \quad x - 2y = -3 \quad z = \lambda$$

$$-y = -2 - z$$

$$x = -3 + 2y$$

$$y = 2 + z$$

$$x = -3 + 2(2 + z)$$

$$\text{SISTEMA COMPATIBLE} \quad x = -3 + 4 + 2z$$

$$\text{INDETERMINADO} \quad x = 1 + 2z$$

$$(x, y, z) = (1 + 2\lambda, 2 + \lambda, \lambda) \text{ con } \lambda \in \mathbb{R}$$

$$④ \quad \left. \begin{aligned} x - 3y + 7z &= 10 \\ 5x - y + z &= 8 \\ x + 4y - 10z &= -11 \end{aligned} \right\} \begin{pmatrix} x & y & z & \tau_i \\ 1 & -3 & 7 & 10 \\ 5 & -1 & 1 & 8 \\ 1 & 4 & -10 & -11 \end{pmatrix} \begin{array}{l} \\ E_2 - 5E_1 \\ E_3 - E_1 \end{array} \approx$$

$$\approx \begin{pmatrix} 1 & -3 & 7 & 10 \\ 0 & 14 & -34 & -42 \\ 0 & 7 & -17 & -21 \end{pmatrix} \begin{array}{l} \\ \\ -2E_3 + E_2 \end{array} \approx \begin{pmatrix} 1 & -3 & 7 & 10 \\ 0 & 14 & -34 & -42 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

SISTEMA COMPATIBLE INDETERMINADO.

$$14y - 34z = -42$$

$$y = \frac{-42 + 34z}{14}$$

$$y = \frac{\cancel{2}(-21 + 17z)}{\cancel{2} \cdot 7}$$

$$x - 3y + 7z = 10$$

$$x = 10 + 3 \left(\frac{-21 + 17z}{7} \right) - 7z$$

$$x = \frac{70 - 63 + 51z - 49z}{7}$$

$$x = \frac{7 + 2z}{7}$$

Terna Solución:

$$(x, y, z) = \left(\frac{7 + 2\lambda}{7}, \frac{-21 + 17\lambda}{7}, \lambda \right) \text{ con } \lambda \in \mathbb{R}$$

$$\textcircled{5} \quad \left. \begin{array}{l} 2x - y + 2z = 1 \\ x + 2y + z = 2 \\ 3x + y + 3z = 3 \end{array} \right\} \begin{pmatrix} x & y & z & T_i \\ 1 & 2 & 1 & 2 \\ 2 & -1 & 2 & 1 \\ 3 & 1 & 3 & 3 \end{pmatrix} \begin{array}{l} \\ E_2 - 2E_1 \\ E_3 - 3E_1 \end{array} \approx$$

$$\approx \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -5 & 0 & -3 \\ 0 & -5 & 0 & -3 \end{pmatrix} \begin{array}{l} \\ \\ E_3 - E_2 \end{array} \approx \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -5 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$z = \lambda$$

$$-5y = -3$$

$$y = \frac{3}{5}$$

$$x + 2y + z = 2$$

$$x = 2 - 2\left(\frac{3}{5}\right) - \lambda$$

$$x = 2 - \frac{6}{5} - \lambda$$

$$x = \frac{10 - 6 - 5\lambda}{5}$$

$$x = \frac{4 - 5\lambda}{5}$$

SISTEMA COMPATIBLE INDETERMINADO.

$$(x, y, z) = \left(\frac{4-5\lambda}{5}, \frac{3}{5}, \lambda \right) \text{ con } \lambda \in \mathbb{R}$$

$$\textcircled{6} \begin{cases} x + 2y + 2z = -4 \\ 3x - y + z = 0 \\ x - 5y - 3z = 8 \end{cases} \left(\begin{array}{ccc|c} x & y & z & T_i \\ 1 & 2 & 2 & -4 \\ 3 & -1 & 1 & 0 \\ 1 & -5 & -3 & 8 \end{array} \right) \begin{array}{l} \\ E_2 - 3E_1 \\ E_3 - E_1 \end{array} \approx$$

$$\approx \left(\begin{array}{ccc|c} 1 & 2 & 2 & -4 \\ 0 & -7 & -5 & 12 \\ 0 & -7 & -5 & 12 \end{array} \right) \begin{array}{l} \\ \\ E_3 - E_2 \end{array} \approx \left(\begin{array}{ccc|c} 1 & 2 & 2 & -4 \\ 0 & -7 & -5 & 12 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$-7y - 5z = 12$$

$$-7y = 12 + 5z$$

$$y = \frac{12 + 5z}{-7}$$

$$x + 2y + 2z = -4$$

$$x = -4 - 2y - 2z$$

$$x = -4 - 2 \left(\frac{12 + 5z}{-7} \right) - 2z$$

$$x = -4 - \frac{24 + 10z}{-7} - 2z$$

$$x = -4 + \frac{24 + 10z}{7} - 2z$$

$$x = \frac{-28 + 24 + 10z - 14z}{7}$$

$$x = \frac{-4 - 4z}{7}$$

SISTEMA COMPATIBLE INDETERMINADO

$$(x, y, z) = \left(\frac{-4 - 4\lambda}{7}, \frac{12 + 5\lambda}{-7}, \lambda \right) \text{ con } \lambda \in \mathbb{R}$$

$$\textcircled{7} \left. \begin{aligned} x - 3y - 2z &= 7 \\ 2x - y + 16z &= 3 \\ x - 8y - 21z &= 11 \end{aligned} \right\} \begin{pmatrix} x & y & z & T_i \\ 1 & -3 & -2 & 7 \\ 2 & -1 & 16 & 3 \\ 1 & -8 & -21 & 11 \end{pmatrix} \approx \begin{matrix} \\ E_2 - 2E_1 \\ E_3 - E_1 \end{matrix}$$

$$\approx \begin{pmatrix} 1 & -3 & -2 & 7 \\ 0 & 5 & 19 & -11 \\ 0 & -5 & -19 & 4 \end{pmatrix} \xrightarrow{E_3 + E_2} \begin{pmatrix} 1 & -3 & -2 & 7 \\ 0 & 5 & 19 & -11 \\ 0 & 0 & 0 & -7 \end{pmatrix}$$

$$0 = -7 \quad (\text{X})$$

SISTEMA INCOMPATIBLE

$$\textcircled{8} \left. \begin{aligned} 2x - y + 2z &= 1 \\ x + 2y + z &= 2 \\ x - 3y + z &= 5 \end{aligned} \right\} \begin{pmatrix} x & y & z & T_i \\ 1 & 2 & 1 & 2 \\ 2 & -1 & 2 & 1 \\ 1 & -3 & 1 & 5 \end{pmatrix} \approx \begin{matrix} \\ E_2 - 2E_1 \\ E_3 - E_1 \end{matrix}$$

$$\approx \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -5 & 0 & -3 \\ 0 & -5 & 0 & 3 \end{pmatrix} \xrightarrow{E_3 + E_2} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -5 & 0 & -3 \\ 0 & 0 & 0 & 6 \end{pmatrix}$$

$$0 = 6 \quad (\text{X})$$

SISTEMA INCOMPATIBLE

$$\textcircled{9} \quad \begin{cases} x + 2y + 2z = -4 \\ 2x - 3y + z = 1 \\ 3x - y + 3z = 7 \end{cases} \quad \left(\begin{array}{ccc|c} x & y & z & T_1 \\ 1 & 2 & 2 & -4 \\ 2 & -3 & 1 & 1 \\ 3 & -1 & 3 & 7 \end{array} \right) \begin{array}{l} \\ E_2 - 2E_1 \\ E_3 - 3E_1 \end{array} \approx$$

$$\approx \left(\begin{array}{ccc|c} 1 & 2 & 2 & -4 \\ 0 & -7 & -3 & 9 \\ 0 & -7 & -3 & 19 \end{array} \right) \begin{array}{l} \\ \\ E_3 - E_2 \end{array} \approx \left(\begin{array}{ccc|c} 1 & 2 & 2 & -4 \\ 0 & -7 & -3 & 9 \\ 0 & 0 & 0 & 10 \end{array} \right)$$

$$0 = 10 \quad \textcircled{\times}$$

SISTEMA INCOMPATIBLE

$$\textcircled{10} \quad \begin{cases} 3x + 2y + z = 1 \\ 6x + 3y + 3z = 3 \\ x + y + z = 0 \end{cases} \quad \left(\begin{array}{ccc|c} x & y & z & T_1 \\ 1 & 1 & 1 & 0 \\ 3 & 2 & 1 & 1 \\ 6 & 3 & 3 & 3 \end{array} \right) \begin{array}{l} \\ E_2 - 3E_1 \\ E_3 - 6E_1 \end{array} \approx$$

$$\approx \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & -2 & -2 & 3 \end{array} \right) \begin{array}{l} \\ \\ E_3 - 2E_2 \end{array} \approx \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 2 & 1 \end{array} \right)$$

$$2z = 1 \quad -y - 2z = 1$$

$$z = \frac{1}{2}$$

$$-y - 1 = 1$$

$$-y = 1 + 1$$

$$y = -2$$

$$x + y + z = 0$$

$$x = +2 - \frac{1}{2}$$

$$x = \frac{4-1}{2}$$

$$x = \frac{3}{2}$$

SISTEMA COMPATIBLE DETERMINADO.

Terna solución:

$$(x, y, z) = \left(\frac{3}{2}, -2, \frac{1}{2} \right)$$

$$\textcircled{II} \begin{cases} x + 2y + z = 0 \\ x - y - z = -10 \\ 2x - y + z = 5 \end{cases} \left(\begin{array}{cccc} 1 & 2 & 1 & 0 \\ 1 & -1 & -1 & -10 \\ 2 & -1 & 1 & 5 \end{array} \right) \begin{matrix} \\ E_2 - E_1 \\ E_3 - 2E_1 \end{matrix} \approx$$

$$\approx \left(\begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & -3 & -2 & -10 \\ 0 & -5 & -1 & 5 \end{array} \right) \begin{matrix} \\ \\ 3E_3 - 5E_2 \end{matrix} \approx \left(\begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & -3 & -2 & -10 \\ 0 & 0 & 7 & 65 \end{array} \right) \approx$$

$$7z = 65$$

$$z = \frac{65}{7}$$

$$-3y - 2z = -10$$

$$3y + 2z = 10$$

$$3y + \frac{130}{7} = 10$$

$$x + 2y + z = 0$$

$$x = -\frac{40}{7} - \frac{65}{7}$$

$$x = -\frac{25}{7}$$

$$3y = 10 - \frac{130}{7}$$

$$3y = \frac{-60}{7}$$

$$y = -\frac{60}{7} : 3$$

SISTEMA COMPATIBLE DETERMINADO.

$$y = -\frac{60}{21} = -\frac{20}{7}$$

Terna Solución:

$$(x, y, z) = \left(-\frac{25}{7}, -\frac{20}{7}, \frac{65}{7} \right)$$

$$\textcircled{12} \quad \left. \begin{array}{l} -x + 2y - z = 1 \\ 2x - 4y + 2z = 3 \\ x + y + z = 2 \end{array} \right\} \begin{array}{ccc|c} x & y & z & T. \\ -1 & 2 & -1 & 1 \\ 2 & -4 & 2 & 3 \\ 1 & 1 & 1 & 2 \end{array}$$

$$\begin{array}{l} \approx \\ E_2 + 2E_1 \\ E_3 + E_1 \end{array} \begin{pmatrix} -1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 5 \\ 0 & 3 & 0 & 3 \end{pmatrix} \quad \begin{array}{l} 0 = 5 \\ \textcircled{\times} \end{array}$$

SISTEMA INCOMPATIBLE

2º Pon un ejemplo tal que, sin necesidad de resolverlo, se trate de un sistema compatible indeterminado.

$$\left. \begin{array}{l} 3x + 2y + z = 1 \\ x + y - z = 3 \\ 8x + 6y = 8 \end{array} \right\}$$

3º Pon un ejemplo tal que, sin necesidad de resolverlo, se trate de un sistema incompatible.

$$\left. \begin{array}{l} 3x + 2y + z = 1 \\ x + y - z = 3 \\ 8x + 6y = 18 \end{array} \right\}$$

$$\left. \begin{array}{l} x+2y-z=5 \\ x+y+3z=-2 \\ x+4y-7z=1 \end{array} \right\} \xrightarrow{\text{reordenamos}} \left. \begin{array}{l} -x+y+3z=-2 \\ 4x+2y-z=5 \\ 2x+4y-7z=1 \end{array} \right\}$$

$$\left(\begin{array}{ccc|c} x & y & z & TI \\ -1 & 1 & 3 & -2 \\ 4 & 2 & -1 & 5 \\ 2 & 4 & -7 & 1 \end{array} \right) \xrightarrow[E_3+2E_1]{E_2+4E_1} \left(\begin{array}{ccc|c} -1 & 1 & 3 & -2 \\ 0 & 6 & 11 & -3 \\ 0 & 6 & -1 & -3 \end{array} \right) \xrightarrow{E_3-E_2} \left(\begin{array}{ccc|c} -1 & 1 & 3 & -2 \\ 0 & 6 & 11 & -3 \\ 0 & 0 & -12 & 0 \end{array} \right)$$

Queda: $-12z=0 \Rightarrow z=0$

$$6y+11z=3 \Rightarrow 6y+11 \cdot 0=3 \Rightarrow 6y=3 \Rightarrow y=-\frac{1}{2}$$

$$-x+y+3z=-2 \Rightarrow -x-\frac{1}{2}+0=-2 \Rightarrow -x=-\frac{1}{2}-2$$

$$x=\frac{3}{2}$$

SISTEMA COMPATIBLE DETERMINADO, TERNA SOLUCIÓN

$$(x, y, z) = \left(\frac{3}{2}, -\frac{1}{2}, 0 \right)$$

$$\left. \begin{array}{l} x+2y+3z=4 \\ x+2y+z=3 \\ x-2y+2z=-3 \end{array} \right\} \xrightarrow{\text{reordenamos}} \left. \begin{array}{l} x-2y+2z=-3 \\ 5x+2y+3z=4 \\ 2x+2y+z=3 \end{array} \right\}$$

$$\left(\begin{array}{ccc|c} x & y & z & TI \\ 1 & -2 & 2 & -3 \\ 5 & 2 & 3 & 4 \\ 2 & 2 & 1 & 3 \end{array} \right) \xrightarrow[E_3-3E_1]{E_2-5E_1} \left(\begin{array}{ccc|c} 1 & -2 & 2 & -3 \\ 0 & 12 & -7 & 19 \\ 0 & -2 & -3 & 9 \end{array} \right) \xrightarrow{6E_3+E_2} \left(\begin{array}{ccc|c} 1 & -2 & 2 & -3 \\ 0 & 12 & -7 & 19 \\ 0 & 0 & -25 & 73 \end{array} \right)$$

queda: $-25z=73 \Rightarrow z=-\frac{73}{25}$; $12y-7\left(-\frac{73}{25}\right)=19$

$$\Rightarrow y = \quad ; \quad x-2\left(\quad\right)+2\left(-\frac{73}{25}\right)=-3 \quad x =$$

SIST. COMPATIBLE DETERMINADO, TERNA SOLUCIÓN

$$(x, y, z) = \left(\quad \right)$$

$$\begin{cases} x - 3y + 2z = 1 \\ -2x + y - 3z = 0 \\ 3x - 4y + 5z = -1 \end{cases} \rightarrow \left(\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ -2 & 1 & -3 & 0 \\ 3 & -4 & 5 & -1 \end{array} \right) \xrightarrow{\substack{E_2 + 2E_1 \\ E_3 - 3E_1}} \left(\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 0 & -5 & 1 & 2 \\ 0 & 5 & -1 & -2 \end{array} \right) \approx$$

$$\xrightarrow{E_3 + E_2} \left(\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 0 & -5 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow -5y + z = 2 \text{ despejamos } z \text{ en función de "y"; } [z = 5y + 2]$$

La ecuación: $x - 3y + 2z = 1 \Rightarrow x - 3y + 2(5y + 2) = 1$

$$\Rightarrow x - 3y + 10y + 4 = 1 \Rightarrow [x = -7y - 3]$$

SISTEMA COMPATIBLE INDETERMINADO

SOLUCIONES, como $\begin{cases} x = -7y - 3 \\ z = 5y + 2 \end{cases} \rightarrow \begin{cases} x = -7\lambda - 3 \\ y = \lambda \\ z = 5\lambda + 2 \end{cases}$

TERNAS SOLUCIÓN: $(x, y, z) = (-7\lambda - 3, \lambda, 5\lambda + 2) \quad \lambda \in \mathbb{R}$

$$\begin{cases} 3x - y + 2z = -1 \\ -x + 2y - z = 2 \\ x + 3y = 3 \end{cases} \rightarrow \text{reordenamos} \begin{cases} -x + 2y - z = 2 \\ x + 3y = 3 \\ 3x - y + 2z = -1 \end{cases}$$

$$\left(\begin{array}{ccc|c} -1 & 2 & -1 & 2 \\ 1 & 3 & 0 & 3 \\ 3 & -1 & 2 & -1 \end{array} \right) \xrightarrow{\substack{E_2 + E_1 \\ E_3 - 3E_1}} \left(\begin{array}{ccc|c} -1 & 2 & -1 & 2 \\ 0 & 5 & -1 & 5 \\ 0 & 5 & -1 & 5 \end{array} \right) \xrightarrow{E_3 - E_2} \left(\begin{array}{ccc|c} -1 & 2 & -1 & 2 \\ 0 & 5 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

queda $5y - z = 5 \Rightarrow [z = 5y - 5]$, $-x + 2y - z = 2 \Rightarrow$

$$\Rightarrow -x + 2y - (5y - 5) = 2 \Rightarrow -x + 2y - 5y + 5 = 2 \Rightarrow$$

$[x = -3y + 3]$ SISTEMA COMPATIBLE INDETERMINADO

TERNAS SOLUCIÓN $(x, y, z) = (-3\lambda + 3, \lambda, 5\lambda - 5)$

$\lambda \in \mathbb{R}$

$$\left. \begin{aligned} -3x + y - 2z &= 2 \\ 5x + 2y - z &= -1 \\ -2x + 3y + 3z &= -1 \end{aligned} \right\} \text{reordenamos } \left. \begin{aligned} y - 3x - 2z &= 2 \\ 2y + 5x - z &= -1 \\ -3y - 2x + 3z &= -1 \end{aligned} \right\}$$

$$\left(\begin{array}{ccc|c} y & x & z & Ti \\ 1 & -3 & -2 & 2 \\ 2 & 5 & -1 & -1 \\ -3 & -2 & 3 & -1 \end{array} \right) \xrightarrow[\substack{E_2 - 2E_1 \\ E_3 + 3E_1}]{\sim} \left(\begin{array}{ccc|c} 1 & -3 & -2 & 2 \\ 0 & 11 & 3 & -5 \\ 0 & -11 & -3 & 5 \end{array} \right) \xrightarrow{E_3 + E_2} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -3 & -2 & 2 \\ 0 & 11 & 3 & -5 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad \text{queda: } 11y + 3z = -5 \Rightarrow z = \frac{-5 - 11y}{3}$$

$$x - 3y - 2z = 2$$

$$x - 3y - 2\left(\frac{-5 - 11y}{3}\right) = 2$$

$$x - 3y + \frac{10 + 22y}{3} = 2 \Rightarrow x = 3y + \frac{10 + 22y}{3} + 2$$

$$x = \frac{9y + 10 + 22y + 6}{3} = \frac{31y + 16}{3}$$

SIST COMPAT INDET

$$\text{SOLUCIONES } (x, y, z) = \left(\frac{31\lambda + 16}{3}, \lambda, \frac{-5 - 11\lambda}{3} \right)_{\lambda \in \mathbb{R}}$$

$$\left. \begin{aligned} x + y - z &= 1 \\ 3x + 2y - 2z &= -3 \\ 4x + 3y - 3z &= 7 \end{aligned} \right\} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 3 & 2 & -2 & -3 \\ 4 & 3 & -3 & 7 \end{array} \right) \xrightarrow[\substack{E_2 - 3E_1 \\ E_3 - 4E_1}]{\sim} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -1 & 1 & -6 \\ 0 & -1 & 1 & 3 \end{array} \right)$$

$$\xrightarrow{3 - E_2} \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -1 & 1 & -6 \\ 0 & 0 & 0 & 9 \end{array} \right) \rightarrow 0 = 9 \quad (*)$$

SISTEMA INCOMPATIBLE.