

$$9) \sqrt{x-4} - 3 = -\sqrt{x-1}$$

$$\sqrt{x-4} = -\sqrt{x-1} + 3$$

$$(\sqrt{x-4})^2 = (3 - \sqrt{x-1})^2$$

$$\cancel{x-4} = 9 + \cancel{x-1} - 6\sqrt{x-1}$$

$$-4 - 8 = -6\sqrt{x-1}$$

$$\frac{-12}{-6} = \sqrt{x-1}$$

$$2 = \sqrt{x-1}$$

$$2^2 = (\sqrt{x-1})^2$$

$$4 = x-1$$

$$5 = x$$

Verificamos:

$$x=5 \quad \sqrt{5-4} - 3 \stackrel{?}{=} -\sqrt{5-1}$$

$$\sqrt{1} - 3 = -\sqrt{4}$$

$$1 - 3 = -2$$

$$-2 = -2 \quad \text{Sí}$$

$x=5$. SÍ ES SOLUCIÓN.

$$11) 2\sqrt{2x-7} - \sqrt{2x-15} = \sqrt{6x-23}$$

$$(2\sqrt{2x-7} - \sqrt{2x-15})^2 = (\sqrt{6x-23})^2$$

$$4 \cdot (2x-7) + 2x-15 - 4\sqrt{(2x-7)(2x-15)} = 6x-23$$

$$8x-28+2x-15-4\sqrt{4x^2-30x-14x+105} = 6x-23$$

$$10x-43-4\sqrt{4x^2-44x+105} = 6x-23$$

$$-4\sqrt{4x^2-44x+105} = 6x-23-10x+43$$

$$\sqrt{4x^2-44x+105} = \frac{-4x+20}{-4}$$

$$\sqrt{4x^2-44x+105} = \frac{-4(x-5)}{-4}$$

$$(\sqrt{4x^2-44x+105})^2 = (x-5)^2$$

$$4x^2-44x+105 = x^2+25-10x$$

$$3x^2-34x+80=0$$

$$x = \frac{34 \pm \sqrt{34^2 - 4 \cdot 3 \cdot 80}}{6} = \frac{34 \pm \sqrt{196}}{6} =$$

$$= \frac{34 \pm 14}{6} = \begin{cases} x_1 = 8 \\ x_2 = \frac{20}{6} = \frac{10}{3} \end{cases}$$

Verificamos:

$$x=8 \quad 2\sqrt{2 \cdot 8 - 7} - \sqrt{2 \cdot 8 - 15} \stackrel{?}{=} \sqrt{6 \cdot 8 - 23}$$

$$2 \cdot \sqrt{9} - \sqrt{1} = \sqrt{48-23}$$

$$2 \cdot 3 - 1 = \sqrt{25}$$

$$5 = 5 \quad \text{Sí}$$

$x = 8$ SÍ ES SOLUCIÓN

$$x = \frac{10}{3} \quad 2 \sqrt{2 \cdot \frac{10}{3} - 7} - \sqrt{2 \cdot \frac{10}{3} - 15} = \sqrt{6 \cdot \frac{10}{3} - 23}$$

$$2 \sqrt{\frac{20-21}{3}} - \sqrt{\frac{20-45}{3}} = \sqrt{\frac{60-69}{3}}$$

Las raíces son números $x \notin \mathbb{R}$.

Por lo tanto la única solución es $x = 8$

$$12) \sqrt{2x+7} + \sqrt{x+8} - 6 = 0$$

$$\sqrt{2x+7} = 6 - \sqrt{x+8}$$

$$(\sqrt{2x+7})^2 = (6 - \sqrt{x+8})^2$$

$$2x+7 = 36 + x+8 - 12\sqrt{x+8}$$

$$x - 37 = -12\sqrt{x+8}$$

$$\frac{x-37}{-12} = \sqrt{x+8}$$

$$\frac{(x-37)^2}{144} = x+8$$

$$x^2 + 1369 - 74x = 144(x+8)$$

$$x^2 + 1369 - 74x = 144x + 1152$$

$$x^2 - 218x + 217 = 0$$

$$x^2 - 218x + 217 = 0$$

$$x = \frac{218 \pm \sqrt{218^2 - 4 \cdot 217}}{2} = \frac{218 \pm \sqrt{47524 - 868}}{2} =$$

$$= \frac{218 \pm \sqrt{46656}}{2} = \frac{218 \pm 216}{2} = \begin{cases} x_1 = 217 \\ x_2 = 1. \end{cases}$$

Verificamos.

$$x = 217 \quad \sqrt{2 \cdot 217 + 7} + \sqrt{217 + 8} - 6 \stackrel{?}{=} 0$$

$$\sqrt{441} + \sqrt{225} - 6 = 0$$

$$21 + 15 - 6 = 0$$

$$30 \neq 0 \quad \text{NO.}$$

$x = 217$ NO ES SOLUCIÓN.

$$x = 1 \quad \sqrt{2 \cdot 1 + 7} + \sqrt{1 + 8} - 6 \stackrel{?}{=} 0$$

$$\sqrt{9} + \sqrt{9} - 6 = 0$$

$$3 + 3 - 6 = 0$$

$$0 = 0 \quad \text{SÍ}$$

$x = 1$ SÍ ES SOLUCIÓN.

$$13) \sqrt{4x^2+4x-1} = \sqrt{x^2-1} + x$$

$$(\sqrt{4x^2+4x-1})^2 = (\sqrt{x^2-1} + x)^2$$

$$4x^2+4x-1 = x^2-1 + x^2 + 2x\sqrt{x^2-1}$$

$$\cancel{4x^2} + 4x - \cancel{1} = \cancel{2x^2} - \cancel{1} + 2x\sqrt{x^2-1}$$

$$\frac{2x^2+4x}{2x} = \sqrt{x^2-1}$$

$$\frac{(2x^2+4x)^2}{4x^2} = x^2-1$$

$$4x^4 + 16x^2 + 16x^3 = 4x^2(x^2-1)$$

$$\cancel{4x^4} + 16x^2 + 16x^3 = \cancel{4x^4} - 4x^2$$

$$16x^2 + 16x^3 + 4x^2 = 0$$

$$16x^3 + 20x^2 = 0$$

$$4x^2 = 0 \Rightarrow x = 0$$

$$4x^2(4x+5) = 0 \begin{cases} 4x+5=0 \\ 4x=-5 \\ x=-\frac{5}{4} \end{cases}$$

Verificamos.

$$x=0. \quad \sqrt{4 \cdot 0^2 + 4 \cdot 0 - 1} \stackrel{?}{=} \sqrt{0^2 - 1} + 0$$

$$\sqrt{-1} = \sqrt{-1} \quad \text{Sí}$$

$x=0$ SÍ ES SOLUCIÓN.

$$x = -\frac{5}{4} \quad \sqrt{4 \cdot \left(-\frac{5}{4}\right)^2 + 4\left(-\frac{5}{4}\right) - 1} \stackrel{?}{=} \sqrt{\left(-\frac{5}{4}\right)^2 - 1} + \left(-\frac{5}{4}\right)$$

$$\sqrt{4 \cdot \frac{25}{16} - 5 - 1} = \sqrt{\frac{25 - 16}{16}} - \frac{5}{4}$$

$$\sqrt{\frac{25}{4} - 6} = \sqrt{\frac{9}{16}} - \frac{5}{4}$$

$$\sqrt{\frac{25-24}{4}} = \frac{3}{4} - \frac{5}{4}$$

$$\sqrt{\frac{1}{4}} = -\frac{2}{4}$$

$$\frac{1}{2} \neq -\frac{1}{2} \text{ No}$$

$$x = -\frac{5}{4} \text{ NO ES SOLUCIÓN.}$$

$$10) \sqrt{x^2 - 1} + 3 = 2\sqrt{x^2 + 4x + 5} - x$$

$$\sqrt{x^2 - 1} + 3 + x = 2\sqrt{x^2 + 4x + 5}$$

$$(\sqrt{x^2 - 1} + 3 + x)^2 = (2\sqrt{x^2 + 4x + 5})^2$$

$$x^2 - 1 + (3 + x)^2 + 2(3 + x)\sqrt{x^2 - 1} = 4(x^2 + 4x + 5)$$

$$x^2 - 1 + 9 + x^2 + 6x + 2(3 + x)\sqrt{x^2 - 1} = 4x^2 + 16x + 20$$

$$2x^2 + 6x + 8 + 2(3 + x)\sqrt{x^2 - 1} = 4x^2 + 16x + 20$$

$$2(3 + x)\sqrt{x^2 - 1} = 4x^2 + 16x + 20 - 2x^2 - 6x - 8$$

$$2(3 + x)\sqrt{x^2 - 1} = 2x^2 + 10x + 12$$

$$\cancel{2}(3+x) \sqrt{x^2-1} = \cancel{2}(x^2+5x+6)$$

$$(3+x) \sqrt{x^2-1} = (x+3)(x+2)$$

luego: 1º) Si $x+3 \neq 0$ podemos simplificar por $x+3$ los dos miembros

$$(\cancel{x+3}) \sqrt{x^2-1} = (\cancel{x+3})(x+2)$$

$$\sqrt{x^2-1} = x+2$$

$$(\sqrt{x^2-1})^2 = (x+2)^2$$

$$x^2-1 = x^2+4+4x$$

$$4x+5 = 0$$

$$x = -\frac{5}{4}$$

$$2^\circ) \text{ Si } x+3 = 0$$

$$x = -3$$

Verificamos:

$$x = -\frac{5}{4} \quad \sqrt{\left(-\frac{5}{4}\right)^2 - 1} + 3 \stackrel{?}{=} 2 \sqrt{\left(-\frac{5}{4}\right)^2 + 4\left(-\frac{5}{4}\right) + 5} + \frac{5}{4}$$

$$\sqrt{\frac{25-16}{16}} + 3 = 2 \sqrt{\frac{25}{16} - \frac{20}{4} + 5} + \frac{5}{4}$$

$$\sqrt{\frac{9}{16}} + 3 = 2 \sqrt{\frac{25-20+20}{16}} + \frac{5}{4}$$

$$\frac{3}{4} + 3 = 2 \sqrt{\frac{25}{16}} + \frac{5}{4}$$

$$\frac{3}{4} + 3 = 2 \cdot \frac{5}{4} + \frac{5}{4}$$

$$\frac{3+12}{4} = \frac{10+6}{4}$$

$$\frac{15}{4} = \frac{16}{4} \quad \text{SÍ}$$

$$x = -\frac{5}{4} \text{ ES SOLUCIÓN.}$$

$$\begin{aligned} x = -3 \quad \sqrt{(-3)^2 - 1} + 3 &\stackrel{?}{=} 2 \sqrt{(-3)^2 + 4(-3) + 5} + 3 \\ \sqrt{8} + 3 &= 2 \sqrt{9 - 12 + 5} + 3 \\ \sqrt{2^3} + 3 &= 2 \sqrt{2} + 3 \\ 2\sqrt{2} + 3 &= 2\sqrt{2} + 3 \quad \text{SÍ} \end{aligned}$$

$$x = -3 \quad \text{SÍ ES SOLUCIÓN}$$

$$6) \quad x+2 + \sqrt{x^2 - 2x} = 2 \sqrt{x^2 + 2x + 2}$$

$$(x+2 + \sqrt{x^2 - 2x})^2 = (2 \sqrt{x^2 + 2x + 2})^2$$

$$(x+2)^2 + x^2 - 2x + 2(x+2)(\sqrt{x^2 - 2x}) = 4(x^2 + 2x + 2)$$

$$\cancel{x^2} + 4 + \cancel{4x} + \cancel{x^2} - \cancel{2x} + \underbrace{2(x+2)\sqrt{x^2 - 2x}} = 4x^2 + 8x + 8$$

$$2(x+2)\sqrt{x^2 - 2x} = 4x^2 + 8x + 8 - 2x^2 - 2x - 4$$

$$(2(x+2)\sqrt{x^2 - 2x})^2 = (2x^2 + \underbrace{6x + 4})^2$$

$$4(x+2)^2(x^2 - 2x) = 4x^4 + (6x + 4)^2 + 2 \cdot 2x^2 \cdot (6x + 4)$$

$$4(x^2 + 4 + 4x)(x^2 - 2x) = 4x^4 + 36x^2 + 16 + 48x + 4x^2(6x + 4)$$

$$(4x^2 + 16 + 16x)(x^2 - 2x) = 4x^4 + 36x^2 + 16 + 48x + 24x^3 + 16x^2$$

$$4x^4 - 8x^3 + 16x^2 - 32x + 16x^3 - 32x^2 = 4x^4 + 24x^3 + 52x^2 + 48x + 16$$

$$8x^3 - 24x^3 - 16x^2 - 52x^2 - 32x - 48x - 16 = 0$$

$$-16x^3 - 68x^2 - 80x - 16 = 0 \quad \text{Saco factor común } -4$$

$$4x^3 + 17x^2 + 20x + 4 = 0$$

$$\begin{array}{r|rrrr} -2 & 4 & 17 & -20 & 4 \\ & & -8 & -18 & -4 \\ \hline & 4 & 9 & 2 & 0 \end{array}$$

$$T. \text{ indep.} = 4$$

$$\text{Divisores: } \pm 1, \pm 2, \pm 4$$

$$4x^2 + 9x + 2 = 0$$

$$x = \frac{-9 \pm \sqrt{9^2 - 4 \cdot 4 \cdot 2}}{4 \cdot 2} = \frac{-9 \pm \sqrt{81 - 32}}{8} = \frac{-9 \pm \sqrt{49}}{8} =$$

$$= \frac{-9 \pm 7}{8} = \begin{cases} x_1 = -\frac{1}{4} \\ x_2 = -2 \end{cases}$$

Verificamos las soluciones:

$$x = -\frac{1}{4} \quad -\frac{1}{4} + 2 + \sqrt{\left(-\frac{1}{4}\right)^2 - 2 \cdot \left(-\frac{1}{4}\right)} =$$

$$2 \sqrt{\left(-\frac{1}{4}\right)^2 + 2 \cdot \left(-\frac{1}{4}\right)} + 2$$

$$\frac{-1+8}{4} + \sqrt{\frac{1}{16} + \frac{1}{2}} = 2 \sqrt{\frac{1}{16} - \frac{1}{2} + 2}$$

$$\frac{7}{4} + \sqrt{\frac{1+8}{16}} = 2 \sqrt{\frac{1-8+32}{16}}$$

$$\frac{7}{4} + \sqrt{\frac{9}{16}} = 2 \sqrt{\frac{25}{16}}$$

$$\frac{10}{4} = \frac{10}{4} \quad \text{Sí} \quad x = -\frac{1}{4} \quad \text{Sí es solución.}$$

$$x = -2 \quad -2 + 2 + \sqrt{(-2)^2 - 2 \cdot (-2)} \stackrel{?}{=} 2 \sqrt{(-2)^2 + 2 \cdot (-2) + 2}$$

$$\sqrt{4+4} = 2 \sqrt{4-4+2}$$

$$\sqrt{8} = 2 \sqrt{2}$$

$$\sqrt{2^3} = 2 \sqrt{2}$$

$$2\sqrt{2} = 2\sqrt{2} \quad \text{Sí.}$$

$$x = -2 \quad \text{Sí es solución.}$$

Soluciones de la ecuación:

$$x = -\frac{1}{4}$$

$$x = -2$$