

RESOLUCIÓN DE ECUACIONES IRRACIONALES

Resuelve las siguientes ecuaciones irracionales:

$$1) \sqrt{1+\sqrt{x+3}} = 2$$

$$2) \sqrt{x-\sqrt{2x-3}} = 1$$

$$3) \sqrt{2+\sqrt{x-5}} = \sqrt{13-x}$$

$$4) \sqrt{2x-\sqrt{x+5}} = \sqrt{1+x}$$

$$5) \sqrt{12+\sqrt{x+7}} = \sqrt{25-x}$$

$$6) \sqrt{\sqrt{x+16}-\sqrt{x}} = 2$$

$$7) \frac{\sqrt{x+2}}{\sqrt{x-1}} = \frac{4}{\sqrt{x+2}}$$

$$8) \sqrt{x-3} = \frac{\sqrt{x-1}}{\sqrt{x-3}}$$

$$9) \sqrt{x+4}-\sqrt{x-4} = \frac{x+1}{\sqrt{x+4}}$$

$$10) \sqrt{x+3}+\sqrt{x+6} = \frac{3}{\sqrt{x+3}}$$

$$11) \sqrt{2x+1}+2\sqrt{x} = \frac{21}{\sqrt{2x+1}}$$

$$12) \sqrt{2x+10}+\sqrt{3x} = \frac{28}{\sqrt{2x+10}}$$

$$13) \frac{\sqrt{4x+20}}{4+\sqrt{x}} = \frac{4-\sqrt{x}}{\sqrt{x}}$$

$$14) \frac{\sqrt{96+12x}}{4+\sqrt{x}} = \frac{4-\sqrt{x}}{\sqrt{x-3}}$$

Soluciones:

1) $\sqrt{1+\sqrt{x+3}} = 2$

$$1+\sqrt{x+3}=4 \Rightarrow \sqrt{x+3}=3 \Rightarrow x+3=9 \Rightarrow \boxed{x=6}$$

Comprobación:

$$\text{Si } x=6 \Rightarrow \sqrt{1+\sqrt{6+3}}=2 \Rightarrow \sqrt{1+3}=2$$

2) $\sqrt{x-\sqrt{2x-3}}=1$

$$x-\sqrt{2x-3}=1 \Rightarrow x-1=\sqrt{2x-3} \Rightarrow x^2-2x+1=2x-3 \Rightarrow x^2-4x+4=0 \Rightarrow (x-2)^2=0 \Rightarrow \boxed{x=2}$$

Comprobación:

$$\text{Si } x=2 \Rightarrow \sqrt{2-\sqrt{4-3}}=\sqrt{2-1}=1$$

3) $\sqrt{2+\sqrt{x-5}}=\sqrt{13-x}$

$$2+\sqrt{x-5}=13-x \Rightarrow \sqrt{x-5}=11-x \Rightarrow x-5=(11-x)^2 \Rightarrow x-5=121-22x+x^2 \Rightarrow x^2-23x+126=0$$

$$x = \frac{23 \pm \sqrt{23^2 - 4 \cdot 126}}{2} = \frac{23 \pm 5}{2} \Rightarrow \begin{cases} x_1 = \frac{23+5}{2} = 14 \Rightarrow \boxed{x=14} \\ x_2 = \frac{23-5}{2} = 9 \Rightarrow \boxed{x=9} \end{cases}$$

Comprobación:

$$\text{Si } x=14 \Rightarrow \sqrt{2+\sqrt{14-5}}=\sqrt{13-14}!!!$$

$$\text{Si } x=9 \Rightarrow \sqrt{2+\sqrt{9-5}}=\sqrt{13-9} \Rightarrow \sqrt{2+2}=\sqrt{4}$$

4) $\sqrt{2x-\sqrt{x+5}}=\sqrt{1+x}$

$$2x-\sqrt{x+5}=1+x \Rightarrow x-1=\sqrt{x+5} \Rightarrow (x-1)^2=x+5 \Rightarrow x^2-2x+1=x+5 \Rightarrow x^2-3x-4=0$$

$$x = \frac{3 \pm \sqrt{3^2 - 4 \cdot (-4)}}{2} = \frac{3 \pm 5}{2} \Rightarrow \begin{cases} x_1 = \frac{3+5}{2} = 4 \Rightarrow \boxed{x=4} \\ x_2 = \frac{3-5}{2} = -1 \Rightarrow \boxed{x=-1} \end{cases}$$

Comprobación:

$$\text{Si } x=4 \Rightarrow \sqrt{8-\sqrt{4+5}}=\sqrt{1+4} \Rightarrow \sqrt{8-3}=\sqrt{5}$$

$$\text{Si } x=-1 \Rightarrow \sqrt{-2-\sqrt{-1+5}}=\sqrt{1-1} \Rightarrow \sqrt{-2-2} \neq 0$$

5) $\sqrt{12+\sqrt{x+7}}=\sqrt{25-x}$

$$12+\sqrt{x+7}=25-x \Rightarrow \sqrt{x+7}=13-x \Rightarrow x+7=169-26x+x^2 \Rightarrow x^2-27x+162=0$$

$$x = \frac{27 \pm \sqrt{27^2 - 4 \cdot 162}}{2} = \frac{27 \pm 9}{2} \Rightarrow \begin{cases} x_1 = \frac{27+9}{2} = 18 \Rightarrow \boxed{x=18} \\ x_2 = \frac{27-9}{2} = 9 \Rightarrow \boxed{x=9} \end{cases}$$

Comprobación:

$$\text{Si } x=18 \Rightarrow \sqrt{12+\sqrt{18+7}}=\sqrt{25-18} \Rightarrow \sqrt{12+5} \neq \sqrt{7}$$

$$\text{Si } x=9 \Rightarrow \sqrt{12+\sqrt{9+7}}=\sqrt{25-9} \Rightarrow \sqrt{12+4}=\sqrt{16}$$

6) $\sqrt{\sqrt{x+16}-\sqrt{x}}=2$

$$\sqrt{x+16}-\sqrt{x}=4 \Rightarrow \sqrt{x+16}=\sqrt{x}+4 \Rightarrow x+16=x+16+8\sqrt{x} \Rightarrow 8\sqrt{x}=0 \Rightarrow \boxed{x=0}$$

Comprobación:

$$\text{Si } x=0 \Rightarrow \sqrt{\sqrt{16}-\sqrt{0}}=2 \Rightarrow \sqrt{4}=2$$

$$7) \frac{\sqrt{x+2}}{\sqrt{x-1}} = \frac{4}{\sqrt{x+2}}$$

$$x+2 = 4\sqrt{x-1} \Rightarrow x^2 + 4x + 4 = 16x - 16 \Rightarrow x^2 - 12x + 20 = 0$$

$$x = \frac{12 \pm \sqrt{12^2 - 4 \cdot 20}}{2} = \frac{12 \pm 8}{2} \Rightarrow \begin{cases} x_1 = \frac{12+8}{2} = 10 \Rightarrow \boxed{x=10} \\ x_2 = \frac{12-8}{2} = 2 \Rightarrow \boxed{x=2} \end{cases}$$

Comprobación:

$$\text{Si } x = 10 \Rightarrow \frac{\sqrt{12}}{\sqrt{9}} = \frac{4}{\sqrt{12}} \Rightarrow 12 = 3 \cdot 4$$

$$\text{Si } x = 2 \Rightarrow \frac{\sqrt{4}}{\sqrt{1}} = \frac{4}{\sqrt{4}} \Rightarrow 4 = 4$$

$$8) \sqrt{x-3} = \frac{\sqrt{x-1}}{\sqrt{x-3}}$$

$$x-3 = \sqrt{x-1} \Rightarrow x^2 + 9 - 6x = x-1 \Rightarrow x^2 - 7x + 10 = 0$$

$$x = \frac{7 \pm \sqrt{7^2 - 4 \cdot 10}}{2} = \frac{7 \pm 3}{2} \Rightarrow \begin{cases} x_1 = \frac{7+3}{2} = 5 \Rightarrow \boxed{x=5} \\ x_2 = \frac{7-3}{2} = 2 \Rightarrow \boxed{x=2} \end{cases}$$

Comprobación:

$$\text{Si } x = 5 \Rightarrow \sqrt{5-3} = \frac{\sqrt{5-1}}{\sqrt{5-3}} \Rightarrow 2 = \sqrt{4}$$

$$\text{Si } x = 2 \Rightarrow \sqrt{2-3} = \frac{\sqrt{2-1}}{\sqrt{2-3}} !!!$$

$$9) \sqrt{x+4} - \sqrt{x-4} = \frac{x+1}{\sqrt{x+4}}$$

$$x+4 - \sqrt{x^2-16} = x+1 \Rightarrow 3 = \sqrt{x^2-16} \Rightarrow 9 = x^2 - 16 \Rightarrow x^2 = 25 \Rightarrow \boxed{x=5}$$

Comprobación:

$$\text{Si } x = 5 \Rightarrow \sqrt{5+4} - \sqrt{5-4} = \frac{5+1}{\sqrt{5+4}} \Rightarrow 3-1 = \frac{6}{3}$$

$$\text{Si } x = -5 \Rightarrow \sqrt{-5+4} - \sqrt{-5-4} = \frac{-5+1}{\sqrt{-5+4}} !!!$$

$$10) \sqrt{x+3} + \sqrt{x+6} = \frac{3}{\sqrt{x+3}}$$

$$x+3 + \sqrt{x^2+3x+6x+18} = 3 \Rightarrow x^2 + 3x + 6x + 18 = x^2 \Rightarrow 9x = -18 \Rightarrow \boxed{x=-2}$$

Comprobación:

$$\text{Si } x = -2 \Rightarrow \sqrt{-2+3} + \sqrt{-2+6} = \frac{3}{\sqrt{-2+3}} \Rightarrow 1+2 = \frac{3}{1}$$

$$11) \sqrt{2x+1} + 2\sqrt{x} = \frac{21}{\sqrt{2x+1}}$$

$$2x+1 + 2\sqrt{2x^2+x} = 21 \Rightarrow x-10 = -\sqrt{2x^2+x} \Rightarrow x^2 - 20x + 100 = 2x^2 + x \Rightarrow x^2 + 21x - 100 = 0$$

$$x = \frac{-21 \pm \sqrt{(-21)^2 - 4 \cdot (-100)}}{2} = \frac{-21 \pm 29}{2} \Rightarrow \begin{cases} x_1 = \frac{-21+29}{2} = 4 \Rightarrow \boxed{x=4} \\ x_2 = \frac{-21-29}{2} = -25 \Rightarrow \boxed{x=-25} \end{cases}$$

Comprobación:

$$\text{Si } x = 4 \Rightarrow \sqrt{8+1} + 2\sqrt{4} = \frac{21}{\sqrt{8+1}} \Rightarrow 3+4 = \frac{21}{3}$$

$$\text{Si } x = -25 \Rightarrow \sqrt{-50+1} + 2\sqrt{-25} = \frac{21}{\sqrt{-50+1}} !!!$$

$$12) \sqrt{2x+10} + \sqrt{3x} = \frac{28}{\sqrt{2x+10}}$$

$$2x+10 + \sqrt{6x^2+30x} = 28 \Rightarrow 2x-18 = -\sqrt{6x^2+30x} \Rightarrow 4x^2+324-72x = 6x^2+30x$$

$$2x^2+102x-324=0 \Rightarrow x^2+51x-162=0$$

$$x = \frac{-51 \pm \sqrt{(-51)^2 - 4 \cdot (-162)}}{2} = \frac{-51 \pm 57}{2} \Rightarrow \begin{cases} x_1 = \frac{-51+57}{2} = 3 \Rightarrow \boxed{x=3} \\ x_2 = \frac{-51-57}{2} = -54 \Rightarrow \boxed{x=-54} \end{cases}$$

Comprobación:

$$\text{Si } x = 3 \Rightarrow \sqrt{6+10} + \sqrt{9} = \frac{28}{\sqrt{6+10}} \Rightarrow 4+3 = \frac{28}{4}$$

$$13) \frac{\sqrt{4x+20}}{4+\sqrt{x}} = \frac{4-\sqrt{x}}{\sqrt{x}}$$

$$\sqrt{4x^2+20x} = 16-x \Rightarrow 4x^2+20x = 256-32x+x^2 \Rightarrow 3x^2+52x-256=0$$

$$x = \frac{-52 \pm \sqrt{(-52)^2 - 4 \cdot 3 \cdot (-256)}}{2 \cdot 3} = \frac{-52 \pm 76}{6} \Rightarrow \begin{cases} x_1 = \frac{-52+76}{6} = 4 \Rightarrow \boxed{x=4} \\ x_2 = \frac{-52-76}{6} = -64 \Rightarrow \boxed{x=-64} \end{cases}$$

Comprobación:

$$\text{Si } x = 4 \Rightarrow \frac{\sqrt{16+20}}{4+\sqrt{4}} = \frac{4-\sqrt{4}}{\sqrt{4}} \Rightarrow \frac{6}{4+2} = \frac{4-2}{2} \Rightarrow 1 = 1$$

$$14) \frac{\sqrt{96+12x}}{4+\sqrt{x}} = \frac{4-\sqrt{x}}{\sqrt{x}-3}$$

$$\sqrt{96x-288+12x^2-36x} = 16-x \Rightarrow 12x^2+60x-288 = 256-32x+x^2 \Rightarrow 11x^2+92x-544=0$$

$$x = \frac{-92 \pm \sqrt{(-92)^2 - 4 \cdot 11 \cdot (-544)}}{2 \cdot 11} = \frac{-92 \pm 180}{22} \Rightarrow \begin{cases} x_1 = \frac{-92+180}{22} = 4 \Rightarrow \boxed{x=4} \\ x_2 = \frac{-92-180}{22} = -\frac{136}{11} \end{cases}$$

Comprobación:

$$\text{Si } x = 4 \Rightarrow \frac{\sqrt{96+48}}{4+\sqrt{4}} = \frac{4-\sqrt{4}}{\sqrt{4}-3} \Rightarrow \frac{12}{4+2} = \frac{4-2}{1}$$