Radicales

1 Simplifica las siguientes expresiones:

a) $\sqrt[4]{a^3}\cdot\sqrt[5]{a^4}:\sqrt{a}=\sqrt[20]{a^{15}}\cdot\sqrt[20]{a^{16}}:\sqrt{a}=\sqrt[20]{a^{31}}:\sqrt{a}=\sqrt[20]{a^{31}}:\sqrt[20]{a^{10}}=\sqrt[20]{a^{21}}=a\sqrt[20]{a}$

b) $\sqrt[3]{2\sqrt{3}}:\sqrt{\sqrt[3]{4}}=\sqrt[3]{\sqrt{12}}:\sqrt{\sqrt[3]{4}}=\sqrt[6]{12}:\sqrt[6]{4}=\sqrt[6]{3}$

c) $\left(\dfrac{\sqrt[6]{32}}{\sqrt{8}}\right)^8=\left(\dfrac{\sqrt[6]{32}}{\sqrt[6]{8^3}}\right)^8=\left(\sqrt[6]{\dfrac{32}{8^3}}\right)^8=\left(\sqrt[6]{\dfrac{2^5}{2^9}}\right)^8=\left(\sqrt[6]{\dfrac{1}{2^4}}\right)^8=\sqrt[6]{\dfrac{1}{2^{32}}}=\dfrac{1}{2^5}\sqrt[6]{\dfrac{1}{2^2}}=\dfrac{1}{32}\sqrt[3]{\dfrac{1}{2}}$

d) $\dfrac{\sqrt{ab}\cdot\sqrt[3]{ab^2}}{\sqrt[6]{a^2b}}=\dfrac{\sqrt[6]{a^3b^3}\cdot\sqrt[6]{a^2b^4}}{\sqrt[6]{a^2b}}=\dfrac{\sqrt[6]{a^5b^7}}{\sqrt[6]{a^2b}}=\sqrt[6]{\dfrac{a^5b^7}{a^2b}}=\sqrt[6]{a^3b^6}=b\sqrt[6]{a^3}=b\sqrt{a}$

2 Opera y simplifica:

a) $\sqrt[3]{3a^3}-\sqrt[3]{81b^6}=a\sqrt[3]{3}-3b^2\sqrt[3]{3}=\left(a-3b^2\right)\sqrt[3]{3}$

b) $\sqrt{9a+27}-\sqrt{4a+12}=\sqrt{9(a+3)}-\sqrt{4(a+3)}=3\sqrt{a+3}-2\sqrt{a+3}=$

$=(3-2)\sqrt{a+3}=\sqrt{a+3}$

3 Opera y simplifica:

$$\frac{1}{1-\dfrac{\sqrt{3}}{1+\sqrt{3}}}+\frac{1}{1+\dfrac{\sqrt{3}}{1-\sqrt{3}}}=\frac{1}{\dfrac{1+\sqrt{3}-\sqrt{3}}{1+\sqrt{3}}}+\frac{1}{\dfrac{1-\sqrt{3}+\sqrt{3}}{1-\sqrt{3}}}=\frac{1}{\dfrac{1}{1+\sqrt{3}}}+\frac{1}{\dfrac{1}{1-\sqrt{3}}}=$$

$$\left(1+\sqrt{3}\right)+\left(1-\sqrt{3}\right)=2$$

4 Racionaliza y opera:

a) $\dfrac{\sqrt{32}}{5}-\dfrac{3\sqrt{50}}{2}+\dfrac{5}{\sqrt{18}}=\dfrac{\sqrt{2^5}}{5}-\dfrac{3\sqrt{2\cdot5^2}}{2}+\dfrac{5}{\sqrt{2\cdot3^2}}=\dfrac{4\sqrt{2}}{5}-\dfrac{15\sqrt{2}}{2}+\dfrac{5}{3\sqrt{2}}=$

$=\dfrac{4\sqrt{2}}{5}-\dfrac{15\sqrt{2}}{2}+\dfrac{5\cdot\sqrt{2}}{3\sqrt{2}\cdot\sqrt{2}}=\dfrac{4\sqrt{2}}{5}-\dfrac{15\sqrt{2}}{2}+\dfrac{5\sqrt{2}}{6}=\dfrac{24\sqrt{2}-225\sqrt{2}+25\sqrt{2}}{30}$

$=\dfrac{-176\sqrt{2}}{30}=\dfrac{-88\sqrt{2}}{15}$

b) $\dfrac{3\sqrt{8}+\sqrt{18}-2\sqrt{72}}{4\sqrt{8}+\sqrt{2}}=\dfrac{3\sqrt{2^3}+\sqrt{2\cdot3^2}-2\sqrt{2^3\cdot3^2}}{4\sqrt{2^3}+\sqrt{2}}=\dfrac{6\sqrt{2}+3\sqrt{2}-12\sqrt{2}}{8\sqrt{2}+\sqrt{2}}=$

$\dfrac{-3\sqrt{2}}{9\sqrt{2}}=\dfrac{-3}{9}=-\dfrac{1}{3}$

c) $\dfrac{-\sqrt{27}+\sqrt{48}+5\sqrt{75}}{2\sqrt{75}-\sqrt{3}}=\dfrac{-\sqrt{3^3}+\sqrt{2^4\cdot3}+5\sqrt{3\cdot5^2}}{2\sqrt{3\cdot5^2}-\sqrt{3}}=\dfrac{-3\sqrt{3}+4\sqrt{3}+25\sqrt{3}}{10\sqrt{3}-\sqrt{3}}=\dfrac{26\sqrt{3}}{9\sqrt{3}}=\dfrac{26}{9}$