EJERCICIOS RESUELTOS POTENCIAS Y RAICES

1) Cálculo y reducción

Introduce los factores dentro del radical:

a) $2\cdot\sqrt[4]{3}$ $\qquad 2\cdot\sqrt[4]{3} = \sqrt[4]{2^4\cdot 3} = \sqrt[4]{16\cdot 3} = \sqrt[4]{48}$

b) $x^2\sqrt[7]{x^3}$ $\qquad x^2\sqrt[7]{x^3} = \sqrt[7]{(x^2)^7\cdot x^3} = \sqrt[7]{x^{14}\cdot x^3} = \sqrt[7]{x^{17}}$

Extrae los factores del radical:

a) $\sqrt[4]{128}$ $\qquad \sqrt[4]{128} = \sqrt[4]{2^7} = 2\sqrt[4]{2^3} = 2\sqrt[4]{8}$

b) $\sqrt[7]{x^{30}}$ $\qquad \sqrt[7]{x^{30}} = \sqrt[7]{x^{28+2}} = \sqrt[7]{x^{28}\cdot x^2} = x^4\sqrt[7]{x^2}$

Calcular las siguientes raíces:

a) $\sqrt[5]{1024}$ $\qquad \sqrt[5]{1024} = \sqrt[5]{2^{10}} = 2^2 = 4$

b) $\sqrt[7]{x^{84}}$ $\qquad \sqrt[7]{x^{84}} = \sqrt[7]{x^{12\cdot 7}} = \sqrt[7]{(x^{12})^7} = x^7$

Reduce a índice común

a) $\sqrt{3};\sqrt[3]{5}$ $\qquad \sqrt{2} = \sqrt[6]{2^3} = \sqrt[6]{8}$; $\sqrt[3]{5} = \sqrt[6]{5^2} = \sqrt[6]{25}$

b) $\sqrt[4]{x^3};\sqrt[6]{x^5}$ $\qquad \sqrt[4]{x^3} = \sqrt[12]{x^9}$; $\sqrt[6]{x^5} = \sqrt[12]{x^{10}}$

2) Raíz de un cociente, raíz de un producto, raíz de una potencia, raíz de una raíz, simplificación y racionalización.

Escribe con una sóla raíz:

a) $\sqrt[5]{\sqrt{3}}$ $\qquad \sqrt[5]{\sqrt{3}} = \sqrt[10]{3}$

b) $\sqrt[7]{x^4\sqrt{x}}$ $\qquad \sqrt[7]{x^4\sqrt{x}} = \sqrt[7]{\sqrt{x^8 \cdot x}} = \sqrt[14]{x^9}$

Escribe con una sóla raíz:

a) $\sqrt[4]{3}\cdot\sqrt[4]{27}$ $\qquad \sqrt[4]{3}\cdot\sqrt[4]{27} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$

b) $\sqrt[5]{x}\cdot\sqrt[5]{x^2}$ $\qquad \sqrt[5]{x}\cdot\sqrt[5]{x^2} = \sqrt[5]{x^3}$

Escribe con una sóla raíz:

a) $\dfrac{\sqrt[3]{16}}{\sqrt[3]{2}}$ $\qquad \dfrac{\sqrt[3]{16}}{\sqrt[3]{2}} = \sqrt[3]{\dfrac{16}{2}} = \sqrt[3]{8} = 2$

b) $\dfrac{\sqrt[5]{x^4}}{\sqrt[5]{x^3}}$ $\qquad \dfrac{\sqrt[5]{x^4}}{\sqrt[5]{x^3}} = \sqrt[5]{\dfrac{x^4}{x^3}} = \sqrt[5]{x}$

Racionaliza.

a) $\dfrac{1}{\sqrt[5]{9}}$ $\qquad \dfrac{1}{\sqrt[5]{9}} = \dfrac{1}{\sqrt[5]{3^2}} = \dfrac{1\cdot\sqrt[5]{3^2}}{\sqrt[5]{3^2}\cdot\sqrt[5]{3^3}} = \dfrac{\sqrt[5]{3^2}}{\sqrt[5]{3^5}} = \dfrac{\sqrt[5]{9}}{3}$

b) $\dfrac{2}{5\cdot\sqrt[3]{4}}$ $\qquad \dfrac{2}{5\cdot\sqrt[3]{4}} = \dfrac{2}{5\cdot\sqrt[3]{2^2}} = \dfrac{2\cdot\sqrt[3]{2}}{5\cdot\sqrt[3]{2^2}\cdot\sqrt[3]{2}} = \dfrac{2\cdot\sqrt[3]{2}}{5\cdot\sqrt[3]{2^3}} = \dfrac{2\cdot\sqrt[3]{2}}{5\cdot 2} = \dfrac{\sqrt[3]{2}}{5}$

Racionaliza:

a) $\dfrac{1}{\sqrt[7]{x^4}}$ $\qquad \dfrac{1}{\sqrt[7]{x^4}} = \dfrac{1\cdot\sqrt[7]{x^3}}{\sqrt[7]{x^4}\cdot\sqrt[7]{x^3}} = \dfrac{\sqrt[7]{x^3}}{\sqrt[7]{x^7}} = \dfrac{\sqrt[7]{x^3}}{x}$

b) $\dfrac{1}{x^2\sqrt[7]{x^3}}$ $\qquad \dfrac{1}{x^2\sqrt[7]{x^3}} = \dfrac{1\cdot\sqrt[7]{x^4}}{x^2\sqrt[7]{x^3}\cdot\sqrt[7]{x^4}} = \dfrac{\sqrt[7]{x^4}}{x^2\sqrt[7]{x^7}} = \dfrac{\sqrt[7]{x^4}}{x^2\cdot x} = \dfrac{\sqrt[7]{x^4}}{x^3}$

Racionaliza:

a) $\dfrac{1}{\sqrt{3}-\sqrt{2}}$ $\qquad \dfrac{1}{\sqrt{3}-\sqrt{2}} = \dfrac{1\cdot(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})\cdot(\sqrt{3}+\sqrt{2})} = \dfrac{(\sqrt{3}+\sqrt{2})}{3-2} = (\sqrt{3}+\sqrt{2})$

b) $\dfrac{\sqrt{2}}{\sqrt{5}+2}$ $\qquad \dfrac{\sqrt{2}}{\sqrt{5}+2} = \dfrac{\sqrt{2}\cdot(\sqrt{5}-2)}{(\sqrt{5}+2)\cdot(\sqrt{5}-2)} = \dfrac{\sqrt{10}-2\sqrt{2}}{5-4} = \sqrt{10}-2\sqrt{2}$

c) $\dfrac{1}{3-\sqrt{x}}$ $\qquad \dfrac{1}{3-\sqrt{x}} = \dfrac{1\cdot(3+\sqrt{x})}{(3-\sqrt{x})\cdot(3+\sqrt{x})} = \dfrac{3+\sqrt{x}}{9-x}$

3) Operaciones

Calcular la suma:

a) $\sqrt{40}+\sqrt{90}$ $\qquad \sqrt{40}+\sqrt{90}=\sqrt{4\cdot 10}+\sqrt{9\cdot 10}=2\sqrt{10}+3\sqrt{10}=5\sqrt{10}$

b) $2\sqrt{32}-\sqrt{8}$ $\qquad 2\sqrt{32}-\sqrt{8}=2\sqrt{2^5}-\sqrt{2^3}=2\cdot 2^2\sqrt{2}-2\sqrt{2}=8\sqrt{2}-2\sqrt{2}=6\sqrt{2}$

c) $\sqrt[3]{4}+\sqrt[6]{16}$ $\qquad \sqrt[3]{4}+\sqrt[6]{16}=\sqrt[3]{4}+\sqrt[6]{4^2}=\sqrt[3]{4}+\sqrt[3]{4}=2\sqrt[3]{4}$

d) $2\sqrt{\frac{1}{2}}+5\sqrt{8}$ $\qquad 2\sqrt{\frac{1}{2}}+5\sqrt{8}=\sqrt{\frac{4\cdot 1}{2}}+5\sqrt{2^3}=\sqrt{2}+10\sqrt{2}=12\sqrt{2}$

Calcular y simplificar:

a) $\sqrt[4]{3}\cdot\sqrt[4]{27}$ $\qquad \sqrt[4]{3}\cdot\sqrt[4]{27}=\sqrt[4]{81}=\sqrt[4]{3^4}=3$

b) $\sqrt[3]{x}\cdot\sqrt[9]{x^2}$ $\qquad \sqrt[5]{x}\cdot\sqrt[5]{x^2}=\sqrt[5]{x^3}$

c) $\sqrt[5]{x^3\sqrt{x}}\cdot\sqrt{x}$ $\qquad \sqrt[5]{x^3\sqrt{x}}\cdot\sqrt{x}=\sqrt[5]{\sqrt{x\cdot x^3}}\cdot\sqrt{x}=\sqrt[10]{x^4}\cdot\sqrt{x}=\sqrt[10]{x^4}\cdot\sqrt[10]{x^5}=\sqrt[10]{x^9}$

d) $\sqrt[3]{2}\cdot\sqrt{2}\cdot\sqrt[4]{8}$ $\qquad \sqrt[3]{2}\cdot\sqrt{2}\cdot\sqrt[4]{8}=\sqrt[3]{2}\cdot\sqrt{2}\cdot\sqrt[4]{2^3}=\sqrt[12]{2^4}\cdot\sqrt[12]{2^6}\cdot\sqrt[12]{2^9}=\sqrt[12]{2^{19}}=2\sqrt[12]{2^7}$

Calcular y simplificar:

a) $\frac{\sqrt[3]{16}}{\sqrt[5]{2}}$ $\qquad \frac{\sqrt[3]{16}}{\sqrt[5]{2}}=\frac{\sqrt[3]{2^4}}{\sqrt[5]{2}}=\frac{\sqrt[15]{2^{20}}}{\sqrt[15]{2^3}}=\sqrt[15]{2^{17}}=2\sqrt[15]{2^2}=2\sqrt[15]{4}$

b) $\frac{\sqrt[7]{x^4}}{\sqrt[14]{x^3}}$ $\qquad \frac{\sqrt[7]{x^4}}{\sqrt[14]{x^3}}=\frac{\sqrt[14]{x^8}}{\sqrt[14]{x^3}}=\sqrt[14]{x^5}$

a) $\frac{\sqrt[6]{8^4}}{\sqrt[8]{4^3}}$ $\qquad \frac{\sqrt[6]{8^4}}{\sqrt[8]{4^3}}=\frac{\sqrt[6]{\left(2^3\right)^4}}{\sqrt[8]{\left(2^2\right)^3}}=\frac{\sqrt[6]{2^{12}}}{\sqrt[8]{2^6}}=\frac{\sqrt[24]{\left(2^{12}\right)^4}}{\sqrt[24]{\left(2^6\right)^3}}=\frac{\sqrt[24]{2^{48}}}{\sqrt[24]{2^{18}}}=\sqrt[24]{2^{30}}=\sqrt[4]{2^5}=2\sqrt[4]{2}$

b) $\frac{\sqrt[3]{x^4\sqrt{x}}}{\sqrt[4]{x}}$ $\qquad \frac{\sqrt[3]{x^4\sqrt{x}}}{\sqrt[4]{x}}=\frac{\sqrt[3]{\sqrt{x\cdot x^8}}}{\sqrt[4]{x}}=\frac{\sqrt[6]{x^9}}{\sqrt[4]{x}}=\frac{\sqrt[12]{x^{18}}}{\sqrt[12]{x^3}}=\sqrt[12]{x^{15}}=x\sqrt[12]{x^3}$

Calcular y simplificar

a) $\frac{\sqrt{2}\cdot\sqrt[3]{4}}{\sqrt[4]{8}}$ $\qquad \frac{\sqrt{2}\cdot\sqrt[3]{4}}{\sqrt[4]{8}}=\frac{\sqrt{2}\cdot\sqrt[3]{2^2}}{\sqrt[4]{2^3}}=\frac{\sqrt[12]{2^6}\cdot\sqrt[12]{2^8}}{\sqrt[12]{2^9}}=\frac{\sqrt[12]{2^{24}}}{\sqrt[12]{2^9}}=\sqrt[12]{2^{15}}=\sqrt[4]{2^5}=2\sqrt[4]{2}$

b) $\frac{\sqrt[5]{2\sqrt{2}}\cdot\sqrt[3]{4}}{\sqrt{8}}$

$$\frac{\sqrt[5]{2\sqrt{2}}\cdot\sqrt[3]{4}}{\sqrt{8}}=\frac{\sqrt[5]{\sqrt{2\cdot 2^2}}\cdot\sqrt[3]{2^2}}{\sqrt{2^3}}=\frac{\sqrt[10]{2^3}\cdot\sqrt[3]{2^2}}{\sqrt{2^3}}=\frac{\sqrt[30]{2^9}\cdot\sqrt[30]{2^{20}}}{\sqrt[30]{2^{45}}}=\frac{\sqrt[30]{2^{29}}}{\sqrt[30]{2^{45}}}=$$

$$=\frac{1}{\sqrt[30]{2^{16}}}=\frac{\sqrt[30]{2^{14}}}{\sqrt[30]{2^{16}}\cdot\sqrt[30]{2^{14}}}=\frac{\sqrt[30]{2^{14}}}{\sqrt[30]{2^{30}}}=\frac{\sqrt[30]{2^{14}}}{2}=\frac{\sqrt[15]{2^7}}{2}$$

4) Otros

1 $\sqrt{\sqrt[3]{\sqrt[4]{2}}} =$

$$\sqrt{\sqrt[3]{\sqrt[4]{2}}} = \sqrt[24]{2}$$

2 $\sqrt{2\sqrt[3]{2\sqrt[4]{2}}} =$

$$\sqrt{2\sqrt[3]{2\sqrt[4]{2}}} = \sqrt{\sqrt[3]{2^3 \cdot 2\sqrt[4]{2}}} = \sqrt{\sqrt[3]{2^4\sqrt[4]{2}}} =$$

$$= \sqrt{\sqrt[3]{\sqrt[4]{(2^4)^4 \cdot 2}}} = \sqrt{\sqrt[3]{\sqrt[4]{2^{16} \cdot 2}}} = \sqrt[24]{2^{17}}$$

3 $\sqrt{\sqrt[3]{\sqrt{\sqrt[3]{2\sqrt{2}}}}} =$

$$\sqrt{\sqrt[3]{\sqrt{\sqrt[3]{2\sqrt{2}}}}} = \sqrt{\sqrt[3]{\sqrt{\sqrt[3]{\sqrt{2 \cdot 2^2}}}}} = \sqrt{\sqrt[3]{\sqrt{\sqrt[3]{\sqrt{2^3}}}}} = \sqrt[72]{2^3} = \sqrt[24]{2}$$

4. $\left(\sqrt{7} - \sqrt{2}\right)^2 =$

$$= \left(\sqrt{7}\right)^2 - 2 \cdot \sqrt{7} \cdot \sqrt{2} + \left(\sqrt{2}\right)^2 =$$

$$= 7 - 2\sqrt{14} + 2 = 9 - 2\sqrt{14}$$

5. $\left(\sqrt{5}+2\right)\cdot\left(\sqrt{5}-2\right)=$

$$=\left(\sqrt{5}\right)^2-2^2=5-4=1$$

6. $\left(2\sqrt{5}+3\sqrt{2}\right)\cdot\left(2\sqrt{5}-3\sqrt{2}\right)=$

$$=\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2=$$

$$=2^2\cdot\left(\sqrt{5}\right)^2-3^2\left(\sqrt{2}\right)^2=$$

$$=4\cdot5-9\cdot2=20-18=2$$

7. $\dfrac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}=$

$$=\frac{\sqrt{2}\cdot\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}=$$

$$=\frac{\sqrt{6}+\sqrt{2^2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{2+\sqrt{6}}{3-2}=2+\sqrt{6}$$

8. $\dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}=$

$$=\frac{\left(3\sqrt{2}-2\sqrt{3}\right)\cdot\left(3\sqrt{2}-2\sqrt{3}\right)}{\left(3\sqrt{2}+2\sqrt{3}\right)\cdot\left(3\sqrt{2}-2\sqrt{3}\right)}=$$

$$= \frac{\left(3\sqrt{2} - 2\sqrt{3}\right)^2}{\left(3\sqrt{2}\right)^2 - \left(2\sqrt{3}\right)^2} =$$

$$= \frac{\left(3\sqrt{2}\right)^2 - 2 \cdot 3 \cdot \sqrt{2} \cdot 2\sqrt{3} + \left(2\sqrt{3}\right)^2}{\left(3\sqrt{2}\right)^2 - \left(2\sqrt{3}\right)^2} =$$

$$= \frac{9 \cdot 2 - 12\sqrt{6} + 4 \cdot 3}{9 \cdot 2 - 4 \cdot 3} =$$

$$= \frac{18 - 12\sqrt{6} + 12}{18 - 12} =$$

$$= \frac{30 - 12\sqrt{6}}{6} = 5 - 2\sqrt{6}$$