POTENCIAS Y RADICALES

Notas teóricas

- Operaciones con potencias:

I. $a^m : a^n = \dfrac{a^m}{a^n} = a^{m-n}$

II. $\left(a^m\right)^n = a^{m\cdot n}$

III. $a^p \cdot b^p = (a\cdot b)^p$

IV. $\left(a^p \cdot b^q\right)^m = a^{p\cdot m}\cdot b^{q\cdot m}$

V. $a^0 = 1$

VI. $a^1 = a$

VII. $a^{-1} = \dfrac{1}{a}$

VIII. $a^{-b} = \dfrac{1}{a^b}$

IX. $\left(\dfrac{a}{b}\right)^{-1} = \dfrac{1}{\frac{a}{b}} = \dfrac{b}{a}$

X. $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{1}{\left(\frac{a}{b}\right)^n} = \left(\dfrac{b}{a}\right)^n$

- Operaciones con radicales:

XI. $\sqrt{a} = a^{\frac{1}{2}}$

XII. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

XIII. $\sqrt[n]{\sqrt[m]{a^p}} = \left(\left(a^p\right)^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{p}{mn}}$

XIV. $\sqrt[n]{a^m}\cdot\sqrt[p]{a^q} = a^{\frac{m}{n}}\cdot a^{\frac{p}{q}} =$
$= \sqrt[nq]{a^{mq+np}} = \sqrt[nq]{a^{mq}\cdot a^{np}}$

- Racionalizar:

Racionalizar es quitar del denominador las raíces. Se pueden presentar dos casos:

a) En el denominador hay sólo una raíz. en este caso, la raíz se elimina multiplicando el numerador y el denominador el mismo número de veces que el radical de la raíz.

b) En el denominador hay una raíz y otro término que la suma o la resta. En este caso, las raíz o raíces se eliminan multiplicando el numerador y el denominador por el conjugado del denominador.

- La jerarquía que hay que seguir a la hora de operar con radicales :

Simplificar
⇓
Operar dentro del paréntesis
⇓
Cálculo de potencias y raíces
⇓
Multiplicaciones y divisiones de izquierda a derecha
⇓
Sumas y restas en el orden que aparecen

<u>Ejercicios resueltos</u>

Opera con las siguientes potencias y raíces

1. $16^{-2}\cdot 4^{3}=\left(2^{4}\right)^{-2}\cdot\left(2^{2}\right)^{3}=2^{-8}\cdot 2^{6}=2^{-8+6}=2^{-2}=\dfrac{1}{4}$

2. $\left(7^{2}\right)^{-3}\cdot 7^{3}=7^{2\cdot(-3)}\cdot 7^{3}=7^{-6}\cdot 7^{3}=7^{-6+3}=7^{-3}=\dfrac{1}{7^{3}}$

3. $\left(3^{-2}:3^{3}\right)\cdot 3^{-2}=3^{-2-3}\cdot 3^{-2}=3^{-5}\cdot 3^{-2}=3^{-5+(-2)}=3^{-5-2}=3^{-7}=\dfrac{1}{3^{7}}$

4. $\dfrac{4^{2}\cdot 12^{3}\cdot 15^{2}}{9^{3}\cdot 8^{2}\cdot 3^{3}}=\dfrac{\left(2^{2}\right)^{2}\cdot\left(2^{2}\cdot 3\right)^{3}\cdot(3\cdot 5)^{2}}{\left(3^{2}\right)^{3}\cdot\left(2^{3}\right)^{2}\cdot 3^{3}}=\dfrac{2^{4}\cdot 2^{6}\cdot 3^{3}\cdot 3^{2}\cdot 5^{2}}{3^{6}\cdot 2^{6}\cdot 3^{3}}=\dfrac{2^{10}\cdot 3^{5}\cdot 5^{2}}{2^{6}\cdot 3^{9}}=$

$=2^{4}\cdot 3^{-4}\cdot 5^{2}$

5. $\dfrac{8^{4}\cdot 15^{3}\cdot 18^{2}\cdot 12^{-3}}{20^{3}\cdot 27^{2}\cdot 3^{-3}}=\dfrac{\left(2^{3}\right)^{4}\cdot(3\cdot 5)^{3}\cdot\left(2\cdot 3^{2}\right)^{2}\cdot\left(2^{2}\cdot 3\right)^{-3}}{\left(2^{2}\cdot 5\right)^{3}\cdot\left(3^{3}\right)^{2}\cdot 3^{-3}}=$

$$=\frac{2^{12}\cdot 3^3\cdot 5^3\cdot 2^2\cdot 3^4\cdot 2^{-6}\cdot 3^{-3}}{2^6\cdot 5^3\cdot 3^6\cdot 3^{-3}}=\frac{2^8\cdot 3^4\cdot 5^3}{2^6\cdot 3^3\cdot 5^3}=2^2\cdot 3=12$$

6. $$\frac{27^{-1}\cdot 81\cdot 3^4\cdot\left(\frac{2^3}{3}\right)^{-1}\cdot 2^3}{36\cdot\left(\frac{1}{3}\right)^{-2}\cdot\frac{4}{3}\cdot\frac{27}{16}\cdot\left(2^0\right)^{-2}}=\frac{\left(3^3\right)^{-1}\cdot 3^4\cdot 3^4\cdot\frac{3}{2^3}\cdot 2^3}{3^2\cdot 2^2\cdot 3^2\cdot\frac{2^2}{3}\cdot\frac{3^3}{2^4}\cdot 1}=\frac{3^6}{3^6}=1$$

7. $$\frac{(-27)^3\cdot 32^{-5}\cdot(-8)^5\cdot\left(25^2\right)^{-6}}{(-72)^4\cdot\left(-50^3\right)^4}=\frac{\left(3^3\right)^3\cdot\left(2^5\right)^{-5}\cdot\left(2^3\right)^5\cdot\left(5^4\right)^{-6}}{\left(3^2\cdot 2^3\right)^4\cdot\left[\left(5^2\cdot 2\right)^3\right]^4}=$$

$$=\frac{3^9\cdot 2^{-25}\cdot 2^{15}\cdot 5^{-24}}{3^8\cdot 2^{12}\cdot 5^{24}\cdot 2^{12}}=\frac{3}{2^{34}\cdot 5^{48}}$$

8. $$2^{\frac{3}{2}}\cdot 2^{\frac{1}{5}}=2^{\frac{3}{2}+\frac{1}{5}}=2^{\frac{3\cdot 5}{10}+\frac{1\cdot 2}{10}}=2^{\frac{15}{10}+\frac{2}{10}}=2^{\frac{15+2}{10}}=2^{\frac{17}{10}}=\sqrt[10]{2^{17}}$$

9. $$\sqrt[3]{19^5}:\sqrt[4]{19^3}=19^{\frac{5}{3}}:19^{\frac{3}{4}}=19^{\frac{5}{3}-\frac{3}{4}}=19^{\frac{5\cdot 4}{12}-\frac{3\cdot 3}{12}}=19^{\frac{20}{12}-\frac{9}{12}}=19^{\frac{20-9}{12}}=$$

$$=19^{\frac{11}{12}}=\sqrt[12]{19^{11}}$$

10. $$\frac{5^5\cdot 5^{\frac{1}{2}}}{\sqrt{5}\cdot 5^{-3}}=\frac{5^5\cdot\cancel{\sqrt{5}}}{\cancel{\sqrt{5}}\cdot 5^{-3}}=5^{5-(-3)}=5^{5+3}=5^8$$

11. $$\frac{2^{\frac{1}{5}}\cdot 2^3\cdot 2^{-\frac{1}{2}}}{2^3\cdot 2^{\frac{25}{125}}}=\frac{\cancel{2^{\frac{1}{5}}}\cdot\cancel{2^3}\cdot 2^{-\frac{1}{2}}}{\cancel{2^3}\cdot\cancel{2^{\frac{1}{5}}}}=2^{-\frac{1}{2}}=\frac{1}{2^{\frac{1}{2}}}=\frac{1}{\sqrt{2}}$$

12. $$\frac{2^{\frac{1}{2}}\cdot 2^{-\frac{1}{3}}\cdot 2^2}{2^2\cdot 2^{\frac{1}{2}}}=2^{-\frac{1}{3}}=\frac{1}{2^{\frac{1}{3}}}=\frac{1}{\sqrt[3]{2}}$$

13. $$\frac{\sqrt[4]{27}}{\sqrt[3]{18}}=\frac{\sqrt[4]{3^3}}{\sqrt[3]{2\cdot 3^2}}=\sqrt[12]{\frac{\left(3^3\right)^3}{\left(2\cdot 3^2\right)^4}}=\sqrt[12]{\frac{3^9}{2^4\cdot 3^8}}=\sqrt[12]{\frac{3}{2^4}}=\sqrt[12]{\frac{3}{16}}$$

14. $$\sqrt[4]{-80}:\sqrt[3]{18}=\frac{-\sqrt[4]{2^4\cdot 5}}{\sqrt[3]{2\cdot 3^2}}=-\frac{2\sqrt[4]{5}}{\sqrt[3]{2\cdot 3^2}}=\frac{2\sqrt[4]{5^3}}{\sqrt[4]{\left(2\cdot 3^2\right)^4}}=2\cdot\sqrt[4]{\frac{5^3}{2^4\cdot 3^8}}=$$

$$= \frac{\cancel{2}}{\cancel{2}\cdot 3^2}\cdot\sqrt[4]{5^3} = \frac{\sqrt[4]{75}}{9}$$

15. $\left(\sqrt[15]{-\frac{1}{243}}\right)^3 = \left(-\sqrt[15]{\frac{1}{3^5}}\right)^3 = -\sqrt[15]{\left(\frac{1}{3^5}\right)^3} = -\sqrt[5]{\frac{1}{3^{15}}} = -\frac{1}{3^3} = -\frac{1}{27}$

16. $\sqrt[3]{\sqrt{2}}\cdot\sqrt[3]{16} = \sqrt[6]{2}\cdot\sqrt[3]{16} = \sqrt[6]{2\cdot 16^2} = \sqrt[6]{2\cdot\left(2^4\right)^2} = \sqrt[6]{2^9} = \sqrt[6]{2^6\cdot 2^3} = 2\cdot\sqrt[6]{2^3} = 2\cdot\sqrt{2}$

17. $\sqrt[3]{\sqrt{2}}\cdot\sqrt[3]{16} = \sqrt[6]{2}\cdot\sqrt[3]{16} = \sqrt[6]{2\cdot 16^2} = \sqrt[6]{2\cdot\left(2^4\right)^2} = \sqrt[6]{2^9} = \sqrt[6]{2^6\cdot 2^3} = 2\cdot\sqrt[6]{2^3} = 2\cdot\sqrt{2}$

18. $\sqrt{\sqrt[3]{\sqrt[4]{64^4}}} = \sqrt{\sqrt[3]{\sqrt[4]{\left(2^6\right)^4}}} = \sqrt[2\cdot 3\cdot 4]{2^{24}} = \sqrt[24]{2^{24}} = 2$

19. $\sqrt{\frac{3\sqrt{2}}{8}} = \sqrt{\frac{\sqrt{3^2\cdot 2}}{2\cdot 2^2}} = \frac{1}{2}\sqrt{\sqrt{\frac{3^2\cdot 2}{2^2}}} = \frac{1}{2}\sqrt[4]{\frac{3^2\cdot 2}{2^2}} = \frac{1}{2}\sqrt[4]{\frac{9}{2}}$

20. $\frac{\left(\sqrt[4]{3^2}\right)^2\cdot\left(\sqrt[3]{3}\right)^6}{\left(\sqrt[12]{3^4}\right)^6} = \frac{\sqrt[4]{3^4}\cdot\sqrt[3]{3^6}}{\sqrt[12]{3^{24}}} = \frac{3\cdot 3^2}{3^2} = 3$

21. $\frac{\left(\sqrt[5]{3}\right)^4\cdot\left(\sqrt[3]{3}\right)^2}{\left(\sqrt{3^4}\right)^3} = \frac{\sqrt[5]{3^4}\cdot\sqrt[3]{3^2}}{\sqrt{3^{12}}} = \frac{\sqrt[15]{\left(3^4\right)^3\cdot\left(3^2\right)^5}}{3^2} = \frac{\sqrt[15]{3^{12}\cdot 3^{10}}}{3^2} = \frac{\sqrt[15]{3^{22}}}{3^2} = \sqrt[15]{\frac{3^{22}}{3^{30}}} =$

$$= \sqrt[15]{\frac{1}{3^8}}$$

22. $\frac{\left(\sqrt[4]{3^4}\right)^2\cdot\sqrt[4]{\sqrt[5]{3^{25}}}}{\left[\sqrt[9]{\sqrt[5]{3}}\right]^{15}\cdot 3} = \frac{\left[\left(3^4\right)^{\frac{1}{4}}\right]^4\cdot\left[\left(3^{25}\right)^{\frac{1}{5}}\right]^{\frac{1}{4}}}{\left[\left(3^{\frac{1}{5}}\right)^{\frac{1}{9}}\right]^{15}\cdot 3} = \frac{3^{4\cdot\frac{1}{4}\cdot 4}\cdot 3^{25\cdot\frac{1}{5}\cdot\frac{1}{4}}}{3^{\frac{1}{5}\cdot\frac{1}{9}\cdot 15}\cdot 3} = \frac{3^4\cdot\cancel{3^{\frac{5}{4}}}}{\cancel{3^{\frac{5}{4}}}\cdot 3} = 3^5$

23. $\frac{\left(\sqrt[9]{2^3}\right)^2\cdot 2}{\sqrt{\left(\sqrt[4]{2}\right)^4}} = \frac{\left(2^3\right)^{\frac{2}{9}}\cdot 2}{\left(\left(2^{\frac{1}{4}}\right)^4\right)^{\frac{1}{2}}} = \frac{2^{\frac{6}{9}}\cdot 2}{2^{\frac{1}{2}}} = \frac{2^{\frac{2}{3}+1}}{2^{\frac{1}{2}}} = \frac{2^{\frac{5}{3}}}{2^{\frac{1}{2}}} = 2^{\frac{5}{3}-\frac{1}{2}} = 2^{\frac{10-3}{6}} = 2^{\frac{7}{6}} = \sqrt[6]{2^7} = 2\sqrt[6]{2}$

24. $\frac{\left(\sqrt[4]{5^2}\right)^4 \cdot \sqrt[4]{\sqrt[5]{5^{20}}}}{\left[\sqrt[3]{\sqrt[5]{5}}\right]^{15}\cdot 25} = \frac{\left(\left(5^2\right)^{\frac{1}{4}}\right)^4\cdot\left(\left(5^{20}\right)^{\frac{1}{5}}\right)^{\frac{1}{4}}}{\left[\left(5^{\frac{1}{5}}\right)^{\frac{1}{3}}\right]^{15}\cdot 5^2} = \frac{5^2\cdot 5}{5\cdot 5^2} = 1$

25. $\frac{\sqrt{\frac{a}{b}\sqrt[3]{2a^{-2}\sqrt{\frac{b^3}{a}}}}}{2\sqrt{ab^2}} = \frac{\sqrt{\sqrt[3]{2a^{-2}\left(\frac{a}{b}\right)^3\sqrt{\frac{b^3}{a}}}}}{\sqrt{4ab^2}} = \frac{\sqrt{\sqrt[3]{\sqrt{\left[2a^{-2}\left(\frac{a}{b}\right)^3\right]^2\cdot\frac{b^3}{a}}}}}{\sqrt{4ab^2}} =$

$= \frac{\sqrt[12]{\left[2a^{-2}\left(\frac{a}{b}\right)^3\right]^2\cdot\frac{b^3}{a}}}{\sqrt{4ab^2}} = \frac{\sqrt[12]{\frac{4a}{b^3}}}{2b\sqrt{a}} = \frac{1}{2b}\sqrt[12]{\frac{4a}{a^6b^3}} = \frac{1}{2b}\sqrt[12]{\frac{4}{a^5b^3}}$

26. $\sqrt{8}-\sqrt{50}-\frac{1}{2}\sqrt{98} = \sqrt{2^2\cdot 2}-\sqrt{2\cdot 5^2}-\frac{1}{2}\sqrt{7^2\cdot 2} = 2\sqrt{2}-5\sqrt{2}-7\sqrt{2} = -10\sqrt{2}$

27. $\frac{1}{2}\sqrt{3}-\sqrt{12}-\frac{3}{4}\sqrt{75} = \frac{1}{2}\sqrt{3}-\sqrt{2^2\cdot 3}-\frac{3}{4}\sqrt{5^2\cdot 3} = \frac{1}{2}\sqrt{3}-2\sqrt{3}-\frac{3\cdot 5}{4}\sqrt{3} =$

$= \frac{1}{2}\sqrt{3}-2\sqrt{3}-\frac{15}{4}\sqrt{3} = -\frac{21}{4}\sqrt{3}$

28. $\sqrt{9xy}+\frac{xy}{\sqrt{4xy}}+\frac{\sqrt[6]{(xy)^{21}}}{x^3y^3} = 3\sqrt{xy}-\frac{xy}{2\sqrt{xy}}-\sqrt[6]{\frac{(xy)^{21}}{\left(x^3y^3\right)^6}} = 3\sqrt{xy}-\frac{1}{2}\sqrt{\frac{(xy)^2}{xy}}-\sqrt[6]{(xy)^3} =$

$= 3\sqrt{xy}-\frac{1}{2}\sqrt{xy}-\sqrt{xy} = \frac{3}{2}\sqrt{xy}$

29. $\sqrt{256x^2y}+\frac{1}{3}\sqrt[4]{\frac{81y^2}{x^{-4}}}-x\sqrt{225y} = x\cdot\sqrt{2^8y}+\frac{3}{3}x\cdot\sqrt[4]{y^2}-x\cdot\sqrt{3^2\cdot 5^2y} =$

$= 16x\cdot\sqrt{y}+x\cdot\sqrt{y}-15x\cdot\sqrt{y} = 2x\cdot\sqrt{y}$

Racionaliza

30. $\frac{1}{2\cdot\sqrt[3]{5}} = \frac{1}{2\cdot\sqrt[3]{5}}\frac{\sqrt[3]{5}}{\sqrt[3]{5}}\frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \frac{\sqrt[3]{25}}{2\cdot 5} = \frac{\sqrt[3]{25}}{10}$

31. $\dfrac{1}{\sqrt[5]{x^4}}=\dfrac{1}{\sqrt[5]{x^4}}\left(\dfrac{\sqrt[5]{x^4}}{\sqrt[5]{x^4}}\right)^4=\dfrac{\left(\sqrt[5]{x^4}\right)^4}{\left(\sqrt[5]{x^4}\right)^5}=\dfrac{\left(\left(x^4\right)^{\frac{1}{5}}\right)^4}{x^4}=\dfrac{x^{\frac{16}{5}}}{x^4}=\dfrac{\sqrt[5]{x^{16}}}{x^4}=\dfrac{\sqrt[5]{x^{15}\cdot x}}{x^4}=\dfrac{x^3\cdot\sqrt[5]{x}}{x^4}=\dfrac{\sqrt[5]{x}}{x}$

32. $\dfrac{\sqrt[3]{x}}{\sqrt[6]{x^5}}=\dfrac{\sqrt[3]{x}}{\sqrt[6]{x^5}}\left(\dfrac{\sqrt[6]{x^5}}{\sqrt[6]{x^5}}\right)^5=\dfrac{\sqrt[3]{x}\left(\sqrt[6]{x^5}\right)^5}{\left(\sqrt[6]{x^5}\right)^6}=\dfrac{x^{\frac{1}{3}}\left(x^5\right)^{\frac{1}{6}\times 5}}{x^5}=\dfrac{x^{\frac{1}{3}}\cdot x^{\frac{25}{6}}}{x^5}=\dfrac{x^{\frac{2+25}{6}}}{x^5}=\dfrac{x^{\frac{27}{6}}}{x^5}=\dfrac{\sqrt[6]{x^{27}}}{x^5}=\dfrac{\sqrt[6]{x^3}}{x}$

33. $\dfrac{\sqrt{2}}{\sqrt{3}+1}=\dfrac{\sqrt{2}\cdot\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)\cdot\left(\sqrt{3}-1\right)}=\dfrac{\sqrt{2}\cdot\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2-1^2}=\dfrac{\sqrt{2}\cdot\left(\sqrt{3}-1\right)}{3-1}=\dfrac{\sqrt{2}\cdot\left(\sqrt{3}-1\right)}{2}$

34. $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}=$

$\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}\cdot\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}=\dfrac{\left(\sqrt{2}+\sqrt{3}\right)^2}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}=\dfrac{\left(\sqrt{2}+\sqrt{3}\right)^2}{2-3}=-\left(\sqrt{2}+\sqrt{3}\right)^2$

35. $\dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}=\dfrac{\left(2\sqrt{3}+\sqrt{2}\right)\cdot\left(2\sqrt{3}+\sqrt{2}\right)}{\left(2\sqrt{3}-\sqrt{2}\right)\cdot\left(2\sqrt{3}+\sqrt{2}\right)}=\dfrac{\left(2\sqrt{3}\right)^2+2\cdot 2\sqrt{3}+\left(\sqrt{2}\right)^2}{\left(2\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\dfrac{4\cdot 3+4\sqrt{3}+2}{4\cdot 3-2}=$

$=\dfrac{7+2\sqrt{6}}{5}$