

1. Realiza la operación:

$$\begin{array}{llllll}
 1. \frac{(2x^3)^3 x}{-2x \cdot 2} & 2. \frac{-3(2x^2)^3}{3(-3x^2)} & 3. \frac{(-x)^2(-2)}{(-x^2)^3(-1)} & 4. \frac{(3x^2)^2 2}{-2x^3(-3x)} & 5. \frac{(2x^2)^2(-2)}{-3x \cdot x} & 6. \frac{(x^3)^2 3x^2}{(-2x)^3 3x^3} \\
 7. \frac{2(-2)}{(-2x^3)^2 2x} & 8. \frac{(3x^3)^2 3x^2}{-3(-3)} & 9. \frac{(x^3)^2(-x^2)}{(-2x^2)^3(-3)} & 10. \frac{(-x^2)^3(3x)^3}{(3x)^3(-2x^3)} & 11. \frac{-2x^3(3x^3)^2}{-3x^2 \cdot 2} & 12. \frac{(2x)^2(3x^3)^2}{(-2x^3)^2(-1)} \\
 13. \frac{-2x^2(-2x^3)^3}{2(-2)} & 14. \frac{2(x^2)^2}{(3x^2)^3(-2x)^2} & 15. \frac{(3x^2)^3(2x^3)^2}{(2x^2)^3(3x^2)^2} & 16. \frac{(-x^2)^2(-3x^3)^3}{-3(-2x^2)} & 17. \frac{-2x}{(-2x^3)^2(-3x)^3} & 18. \frac{(-3x^3)^2(-2x^2)}{3x(-2)} \\
 19. \frac{(3x^2)^2(3x^2)^3}{(-x^3)^3(x^2)^3} & 20. \frac{(-2x)^2(-2x^3)^2}{(-3x)^2(-x^2)^3} & 21. \frac{-3x^2(-1)3x}{(x^2)^3 3(3x^2)^2} & 22. \frac{-2(-x^3)^3(x^2)^2}{-2x(-3)} & 23. \frac{(2x^2)^3(-3x^3)^2}{(3x)^3(3x)^2} & 24. \frac{(-3x^3)^3(-2x^3)^2}{(2x)^3(-x^2)^3}
 \end{array}$$

2. Realiza la operación:

$$\begin{array}{llllll}
 1. \frac{-2(y^3)^2}{(-3y)^3 x} & 2. \frac{3(-3xy)^2}{(2y^3)^2 3y^3} & 3. \frac{(3x)^2(-2x)}{-x(-3)} & 4. \frac{-2y \cdot 3}{-3y^2(-xy^2)} & 5. \frac{xy \cdot xy}{-x(-3xy^3)^3} & 6. \frac{-xy^2(3y^3)^2}{(y^3)^2(-1)} \\
 7. \frac{-xy^2(3y^3)^3}{y(-2y^3)} & 8. \frac{-3y}{2y^3(-3xy^3)} & 9. \frac{(-xy)^3 3}{(-3y^3)^3(-3y)} & 10. \frac{(2y^2)^2(-xy^2)}{-3(-2xy^2)} & 11. \frac{(-3xy)^3(y^2)^3}{-y^3(3x)^2} & 12. \frac{(-xy^2)^3(2x)^2}{(-x)^2(2y)^2} \\
 13. \frac{-2y^3 y}{(-2y^3)^2(-y^2)^2} & 14. \frac{y^3(2y^3)^3}{(3x)^2(2xy^3)^2} & 15. \frac{2(2y^3)^3}{(-xy^3)^3(-2x)^2} & 16. \frac{3(-2y)^2}{(-2xy^3)^2(-3y)} & 17. \frac{2y^3(-y^2)}{(-xy^2)^2(xy^2)^3} & 18. \frac{(2xy)^2 3x}{(-2x)^2(2xy^2)^3}
 \end{array}$$

3. Halla el valor numérico de la siguiente expresión:

$$\begin{array}{llll}
 1. -2x^4 - 3x^2 - 2; x = 2 & 2. -3x^3 - 2x - 2; x = -1 & 3. 2x^3 - 2x^2 - 1; x = -2 & 4. -3x^2 + x^2 + 2x; x = 2 \\
 5. x^3 + 3x^2 - x + 3; x = -2 & 6. x^4 + 3x^3 + 3x - 3; x = -1 & 7. 3x^3 + 2x^2 + 3x - 2; x = 2 & 8. -x^4 - 3x^3 + x^2 + 2; x = -3 \\
 9. 2x^4 + 2x^3 - 2x^2 - 3; x = 1 & 10. x^4 - 3x^3 + 2x^2 + 2x; x = 2 & 11. -x(2x+2) - 3x(x+1); x = 1 & 12. -3(x+2) + x(3x-2); x = -3 \\
 13. -3(2x-2) - 3(3x-1); x = -1 & 14. -2x(x-3) + 3(2x+1); x = -2 & 15. -3x(2x+3) - x(x-2); x = -3 & 16. 2x(2x+2) + 2(2x+1); x = -2 \\
 17. -3(2x+3) - 3x(3x+1); x = -1 & 18. -3x(3x-2) + x(3x+2); x = -1 & 19. -2x - 2(x-3) + x(2x-3); x = 2 & 20. -x - x(3x+1) - x(2x-3); x = -2
 \end{array}$$

4. Realiza la operación:

$$\begin{array}{llll}
 1. -x \cdot (2x^2 + x + 2) & 2. 2x \cdot (-4x^2 + x + 2) & 3. -(3x+2) + (-4x+2) & 4. (-x^2 - x) - (2x^2 - 2) \\
 5. -(2x^2 + 4) - (-3x + 1) & 6. (-4x^2 - 4) + (-4x + 3) & 7. -3x + (-4x^2 + 4x - 2) & 8. 3x^2 - (-4x^2 - 3x + 3) \\
 9. (3x^2 + x + 3) - (-3x - 1) & 10. -(2x - 3) - (-4x^2 + 2x) & 11. -(3x + 1) + (-x^2 - 2x - 2) & 12. (-3x - 2) + (2x^2 + 3x + 4) \\
 13. -(-4x - 4) - (4x^2 - 3x + 3) & 14. -(2x^2 + 4x + 2) + (-4x - 1) & 15. (x^2 + 3x - 3) - (-4x^2 + x + 2) & 16. (2x^2 - 4x + 4) + (2x^2 + x + 1) \\
 17. (-4x^2 - 3x - 4) - (4x^2 + x - 4) & 18. -(4x^2 - 4x) + (-2x^2 - 3x - 4) & 19. -(4x^2 - 4x + 1) - (-x^2 + 2x - 1) & 20. (2x^2 - 3x + 4) + (-3x^2 + 3x - 3)
 \end{array}$$

5. Realiza la operación:

$$\begin{array}{lllll}
 1. 4x(-2x^2 - 2) & 2. 4x^2(x^2 - 3x) & 3. -3x(2x^3 + 3) & 4. (-4x^3 + x^2)(-x + 2) & 5. (-3x + 4)(-3x^2 - 2x) \\
 6. (4x^3 + 4x^2)(3x - 4x^2) & 7. (2x^3 - 2x)(4x^2 + 4) & 8. (-3x^3 + x^2)(-2x^2 - x) & 9. (3x + 1)(3x - 1) & 10. (4x - 3)(4x + 3) \\
 11. (2x + 5)(2x - 5) & 12. (x^2 + 1)(x^2 - 1) & 13. (3x^2 + 3)(3x^2 - 3) & 14. (2x^2 - 4x)(2x^2 + 4x) & 15. (3x + 1)^2 \\
 16. (4x - 3)^2 & 17. (-2x - 3)^2 & 18. (x^2 - 2)^2 & 19. (2x^2 + 3)^2 & 20. (x^3 + 2x^2)^2
 \end{array}$$

6. Realiza la operación:

$$\begin{array}{lll}
 1. x(2x+1) - 2(x-1) & 2. x(x+1) + 2(3x-2) & 3. 3(2x-3) + 3(2x-1) \\
 4. 2x(3x-2) - 2(x-2) & 5. 2(x+2) - (x-3)^2 & 6. -2(3x^2-x) - (x-1)^2 \\
 7. -(x+1)(x-1) - (x-3)^2 & 8. 3(x^2-2) - (x^2+3x)^2 & 9. -(2x-2)^2 + 3(2x+3) \\
 10. -(x+2)(x-2) + (x^2-3)^2 & 11. -(x-2)(x+2) + (2x-3)^2 & 12. -(2x+1)^2 - 3x^2(2x-2) \\
 13. -(x+2)(x-2) - 3x^2(x-3) & 14. -2x - x^2(3x-3) + (x-3)^2 & 15. x^2 - 2(x+3) + (x+3)(x-3)
 \end{array}$$

16.  $3x-2x(2x+2)-(3x-3)^2$       17.  $-(x-2)^2+3x(2x^2+2x-2)$       18.  $-2x^2+x^2(3x+1)-(2x+2)^2$   
 19.  $-(2x+1)(2x-1)-(2x^2+2x)^2$       20.  $3x^2-(2x+1)(2x-1)-(3x-1)^2$       21.  $-(3x^2+1)(3x^2-1)+x^2(3x-2)$   
 22.  $-x^2-(2x-1)(2x+1)+3(3x+2)$       23.  $-3x-(3x+2)(3x-2)-(2x+1)^2$       24.  $-x+(3x^2+2)^2-x(2x^2-3x+3)$   
 25.  $-(3x^2-3x)(3x^2+3x)+3(x-2)$       26.  $2x-(3x-3)(3x+3)+(2x^2-x)^2$       27.  $3x+(2x-3)(2x+3)-(2x^2-3x)^2$   
 28.  $3x^2+(3x-1)(3x+1)-2x^2(2x+1)$       29.  $-2x(2x^2+2x-1)+(2x+2)(2x-2)$       30.  $3x^2-(3x^2-3x)^2+(3x-3)(3x+3)$

7. Realiza la siguiente división:

- |   |  |   |  |
|---|--|---|--|
| 1. $\begin{cases} D: 9-8x^2 \\ d: -4x-4 \end{cases}$            | 2. $\begin{cases} D: 8x^2+10x-1 \\ d: -4x-3 \end{cases}$         | 3. $\begin{cases} D: 9-6x-9x^2 \\ d: 2-3x \end{cases}$                | 4. $\begin{cases} D: -4x^3+2x^2-6 \\ d: -2x-2 \end{cases}$         |
| 5. $\begin{cases} D: 20x-8-12x^3 \\ d: 2-3x-3x^2 \end{cases}$   | 6. $\begin{cases} D: 9x^3+3x^2-5x+2 \\ d: 2-2x-3x^2 \end{cases}$ | 7. $\begin{cases} D: x^2-6x^3+20x-16 \\ d: 3-2x \end{cases}$          | 8. $\begin{cases} D: -x^4+11x^2+6x-4 \\ d: x^2-3x-2 \end{cases}$   |
| 9. $\begin{cases} D: 3x^3-9x^2+5x-1 \\ d: x^2-2x-1 \end{cases}$ | 10. $\begin{cases} D: 9x+x^2-3x^3-7 \\ d: 3-x^2 \end{cases}$     | 11. $\begin{cases} D: -16x^4-4x^3+10x^2+1 \\ d: 4x^2-x-2 \end{cases}$ | 12. $\begin{cases} D: -3x^3+21x-9-9x^2 \\ d: x^2+4x-3 \end{cases}$ |

8. Realiza, usando la regla de Ruffini, la división:

- |  |   |   |   |
|--|---|---|---|
| 1. $\begin{cases} D: -1-3x^2 \\ d: 1+x \end{cases}$      | 2. $\begin{cases} D: x^2-3x-6 \\ d: x-4 \end{cases}$        | 3. $\begin{cases} D: 4x^2-16x+15 \\ d: x-3 \end{cases}$         | 4. $\begin{cases} D: 2+x-x^3 \\ d: x+1 \end{cases}$               |
| 5. $\begin{cases} D: -2x^3+7x-2 \\ d: x+2 \end{cases}$   | 6. $\begin{cases} D: 2x^3-7x^2-16 \\ d: x-4 \end{cases}$    | 7. $\begin{cases} D: 2x^2-4x^3-2x \\ d: x-1 \end{cases}$        | 8. $\begin{cases} D: 2x-7x^2-4x^3+2 \\ d: x+2 \end{cases}$        |
| 9. $\begin{cases} D: x^4-2x^3-x+6 \\ d: x-2 \end{cases}$ | 10. $\begin{cases} D: 5-4x+5x^2-2x^3 \\ d: x-2 \end{cases}$ | 11. $\begin{cases} D: -3x^4+10x^3-x^2-6x \\ d: x-3 \end{cases}$ | 12. $\begin{cases} D: 3x^2-11+9x^3+3x^4+5x \\ d: 3+x \end{cases}$ |

9. Calcula m, sabiendo que la división del polinomio  $x^2+x-2m$  por  $x-2$  es exacta.
10. Dado el polinomio  $-3mx^2+8x-1$ , halla m sabiendo que el resto de su división por  $x+3$  es 2.
11. Dado el polinomio  $3x^2-2mx-2$ , calcula el valor de m, sabiendo que es divisible por  $x+2$ .
12. Determina el valor de m, sabiendo que la división del polinomio  $3x^2-x+3m$  por  $x-1$  es exacta.
13. Dado el polinomio  $3x^2-x-3m$ , determina m, sabiendo que es divisible por  $x-1$ .
14. Dado el polinomio  $x^2+6x+3m$ , determina m sabiendo que el resto de su división por  $x+3$  es 1.
15. Halla m para que el polinomio  $-mx^2-x-2$  sea divisible por  $x-2$ .
16. Dado el polinomio  $x^2+3x-3m$ , halla m sabiendo que el resto de su división por  $x+2$  es 4.
17. Dado el polinomio  $x^2+2mx+6$ , determina m para que su división por  $x+3$  sea exacta.
18. Calcula m para que el polinomio  $x^2-x+3m$  sea divisible por  $x-2$ .
19. Descompón, al máximo, en producto de factores:
- |                  |                  |                    |                   |                   |                   |
|------------------|------------------|--------------------|-------------------|-------------------|-------------------|
| 1. $3x+6$        | 2. $x^2+2x$      | 3. $3x^2-9x$       | 4. $x^2-4$        | 5. $x^4-9$        | 6. $9x^2-4$       |
| 7. $x^2+6x+9$    | 8. $x^2-4x+4$    | 9. $x^4-4x^2+4$    | 10. $4x^4-x^2$    | 11. $3x^3-3x$     | 12. $27x^2-3$     |
| 13. $-12x^2+3$   | 14. $9x^2-x^6$   | 15. $3-27x^4$      | 16. $18x^3-2x$    | 17. $27x^4-12$    | 18. $12x^5-3x$    |
| 19. $8x-2x^5$    | 20. $12x^2-27$   | 21. $2x^4-2x^2$    | 22. $18x-8x^3$    | 23. $8x-18x^3$    | 24. $3x^2-3x^4$   |
| 25. $8x^4-18x^2$ | 26. $27x^3-12x$  | 27. $3x-27x^3$     | 28. $2x^2-4x+2$   | 29. $3x^2+6x+3$   | 30. $-2x^2-4x-2$  |
| 31. $-3x^2+6x-3$ | 32. $-2x^2+8x-8$ | 33. $x^4-2x^3+x^2$ | 34. $x^3-4x^2+4x$ | 35. $8x^4-8x^2+2$ | 36. $9x^5+6x^3+x$ |

37.  $4x^3+4x^2+x$     38.  $-x^4+2x^3-x^2$     39.  $18x^2-12x+2$     40.  $2x^2+12x+18$     41.  $18x^3-12x^2+2x$     42.  $-27x^2+36x-12$   
 43.  $12x^2-36x+27$     44.  $18x^5+12x^3+2x$     45.  $18x^3-24x^2+8x$     46.  $27x^4+36x^2+12$     47.  $-12x^3-12x^2-3x$     48.  $-3x^6+18x^4-27x^2$   
 49.  $27x^5+36x^3+12x$     50.  $-12x^4-36x^2-27$     51.  $18x^4+24x^3+8x^2$     52.  $12x^3+36x^2+27x$     53.  $12x^5+36x^3+27x$     54.  $27x^3-36x^2+12x$

20. Descompón, al máximo, en producto de factores:

1.  $y^3-xy^2$     2.  $3xy+9x$     3.  $xy^2-2xy$     4.  $2xy-4x^2$     5.  $3xy^2-3xy$   
 6.  $y^2+2xy+x^2$     7.  $y^2-6xy+9x^2$     8.  $y^4-4xy^2+4x^2$     9.  $9x^2-12xy+4y^2$     10.  $y^4-6xy^2+9x^2$   
 11.  $x^2y^2-y^6$     12.  $9xy^3-x^3y$     13.  $12xy^5-3xy$     14.  $3y^4-3x^2y^2$     15.  $8xy^2-2x^3y^6$   
 16.  $8x^2y^2+8xy+2$     17.  $-y^3+2xy^2-x^2y$     18.  $xy^4-4x^2y^2+4x^3$     19.  $2x^2y^4+4xy^3+2y^2$     20.  $-2y^3+12xy^2-18x^2y$

21. Halla el M.c.d. y m.c.m. de los siguientes polinomios:

1.  $\begin{cases} 9x-3 \\ 9x+3 \\ 9x^2-1 \end{cases}$     2.  $\begin{cases} 4x^2-2x \\ 4x^2+2x \\ 8x^3-2x \end{cases}$     3.  $\begin{cases} 2x^2+2x \\ 2x^3-2x \\ x^2-2x+1 \end{cases}$     4.  $\begin{cases} 3-9x \\ 3-27x^2 \\ 9x^2-6x+1 \end{cases}$     5.  $\begin{cases} x^4-9 \\ x^3+3x \\ x^4+6x^2+9 \end{cases}$     6.  $\begin{cases} 3-3x \\ 3-3x^2 \\ 3x^2+6x+3 \end{cases}$   
 7.  $\begin{cases} 2-2x^2 \\ x^2+2x+1 \\ 2x^2+4x+2 \end{cases}$     8.  $\begin{cases} 4x^2-1 \\ 2-8x^2 \\ 8x^2+8x+2 \end{cases}$     9.  $\begin{cases} 2-2x \\ 2-2x^2 \\ 2x^2-4x+2 \end{cases}$     10.  $\begin{cases} x^2+2x \\ x^2-4x+4 \\ x^3+4x^2+4x \end{cases}$     11.  $\begin{cases} 4x^2+4x+1 \\ 4x^2-4x+1 \\ 4x^3+4x^2+x \end{cases}$     12.  $\begin{cases} 6x+3 \\ 4x^2+4x+1 \\ 12x^2-12x+3 \end{cases}$   
 13.  $\begin{cases} 9x^4-4 \\ 4x-9x^5 \\ 9x^4-12x^2+4 \end{cases}$     14.  $\begin{cases} 6-9x^2 \\ 12-27x^4 \\ 9x^4+12x^2+4 \end{cases}$     15.  $\begin{cases} x^2-4 \\ x^3+2x^2 \\ x^4+4x^3+4x^2 \end{cases}$     16.  $\begin{cases} x^2-1 \\ 3x^3-3x \\ 3x^3-6x^2+3x \end{cases}$     17.  $\begin{cases} x^2-1 \\ 2x^3-2x \\ 2x^3+4x^2+2x \end{cases}$     18.  $\begin{cases} 8x^2-2 \\ 4x^2-4x+1 \\ -8x^2+8x-2 \end{cases}$   
 19.  $\begin{cases} 2x+6 \\ x^2+6x+9 \\ 2x^2+12x+18 \end{cases}$     20.  $\begin{cases} x^2-4 \\ x^2+4x+4 \\ 2x^3+8x^2+8x \end{cases}$     21.  $\begin{cases} 6x+4 \\ 8-18x^2 \\ 18x^2+24x+8 \end{cases}$     22.  $\begin{cases} -4x-6 \\ 4x^2-12x+9 \\ -8x^2-24x-18 \end{cases}$     23.  $\begin{cases} 2x^3-2x \\ 2x^3+4x^2+2x \\ 2x^4+4x^3+2x^2 \end{cases}$     24.  $\begin{cases} x^2-2x \\ x^2+2x+1 \\ 2x^4-4x^3+2x^2 \end{cases}$   
 25.  $\begin{cases} 6x^2-4 \\ 9x^4+12x^2+4 \\ 18x^4+24x^2+8 \end{cases}$     26.  $\begin{cases} 2x-3x^2 \\ 9x^2-12x+4 \\ -9x^3-12x^2-4x \end{cases}$     27.  $\begin{cases} 6x^3-4x \\ 6x^4-4x^2 \\ 18x^5+24x^3+8x \end{cases}$     28.  $\begin{cases} 3x^3-9x^2 \\ 3x^4-27x^2 \\ 3x^3+18x^2+27x \end{cases}$     29.  $\begin{cases} 3x^2+6 \\ x^4-4x^2+4 \\ 3x^4-12x^2+12 \end{cases}$     30.  $\begin{cases} 3x^3+6x \\ x^4-4x^2+4 \\ 3x^5+12x^3+12x \end{cases}$

22. Halla el M.c.d. y m.c.m. de los siguientes polinomios:

1.  $\begin{cases} y-x \\ xy^2-x^2y \\ xy^2+x^2y \end{cases}$     2.  $\begin{cases} 2y \\ y^2-x^2 \\ y^2-2xy+y^2 \end{cases}$     3.  $\begin{cases} y+2x \\ xy^2+2x^2y \\ y^2+4xy+4x^2 \end{cases}$     4.  $\begin{cases} y^4-4x^2 \\ 3y^3-6xy \\ y^4-4xy^2+4x^2 \end{cases}$     5.  $\begin{cases} y^2-x^2 \\ xy^2+x^2y \\ xy^3-x^3y \end{cases}$     6.  $\begin{cases} x^2y^3-y \\ x^2y^4-y^2 \\ x^2y^3-2xy^2+y \end{cases}$   
 7.  $\begin{cases} 8y^2-2x^2 \\ 4y+2x \\ 8y^2+8xy+2x^2 \end{cases}$     8.  $\begin{cases} 3xy+3x \\ 3xy^2-6xy+3x \\ 3xy^2+6xy+3x \end{cases}$     9.  $\begin{cases} xy+x^2 \\ xy^2-2x^2y+x^3 \\ xy^2+2x^2y+x^3 \end{cases}$     10.  $\begin{cases} xy-x^2 \\ y^2-2xy+x^2 \\ xy^2-2x^2y+x^3 \end{cases}$     11.  $\begin{cases} x^2y^2-9 \\ x^3y^2-9x \\ x^2y^2+6xy+9 \end{cases}$     12.  $\begin{cases} x^2y^2-4 \\ x^2y^2-4xy+4 \\ 2x^2y^2+8xy+8 \end{cases}$

23. Simplifica, al máximo, la fracción:

1.  $\frac{3x+3}{3x-3}$     2.  $\frac{x^2-2x+1}{x^2-1}$     3.  $\frac{3-3x}{3-3x^2}$     4.  $\frac{6x+3}{12x^2-3}$     5.  $\frac{x^3+2x^2}{x^4-4x^2}$     6.  $\frac{x^2-6x+9}{x^2-9}$   
 7.  $\frac{2x^3+2x^2}{2x^3-2x^2}$     8.  $\frac{9x^2+6x+1}{9x^2-1}$     9.  $\frac{4x^2-1}{3-12x^2}$     10.  $\frac{-2x^2-6x}{x^2-9}$     11.  $\frac{-2x^2+3x}{-2x^2-3x}$     12.  $\frac{2x^2-4x}{x^2-4}$   
 13.  $\frac{4x^2-1}{4x^2+4x+1}$     14.  $\frac{-x^2-2x}{x^2-4}$     15.  $\frac{2-32x^4}{2-8x^2}$     16.  $\frac{2x+2}{x^2+2x+1}$     17.  $\frac{3x^2-9x}{x^2-9}$     18.  $\frac{4x+4}{4x^2+8x+4}$   
 19.  $\frac{3x^2-9x}{x^2-6x+9}$     20.  $\frac{2x^2-2x}{x^2-1}$     21.  $\frac{2x-4}{x^2-4x+4}$     22.  $\frac{4x-4}{4x^2-4}$     23.  $\frac{2x^2-2x}{1-x^2}$     24.  $\frac{-2x^3-6x}{x^4+6x^2+9}$   
 25.  $\frac{-8x^2+8x-2}{4x^2-4x+1}$     26.  $\frac{-3x^2+6x-3}{-3x^2+3}$     27.  $\frac{x^2-3x}{x^3-6x^2+9x}$     28.  $\frac{-9x^5+6x^3-x}{-3x^3+x}$     29.  $\frac{9x^3+12x^2+4x}{9x^3-4x}$     30.  $\frac{2x^4+4x^3+2x^2}{2x^4-2x^2}$

$$31. \frac{18x^4+12x^3+2x^2}{18x^4-2x^2} \quad 32. \frac{3x^2-2x}{9x^3-12x^2+4x} \quad 33. \frac{18x^3-24x^2+8x}{18x^3-8x} \quad 34. \frac{-27x^3+18x^2-3x}{9x^2-6x+1} \quad 35. \frac{4x^2+4x+1}{-12x^4-12x^3-3x^2} \quad 36. \frac{27x^3+36x^2+12x}{9x^2+6x}$$

24. Halla el valor numérico de la siguiente fracción:

$$1. \frac{2x-2}{x^2-1}; x=1 \quad 2. \frac{3x-3}{x^2-1}; x=-1 \quad 3. \frac{2x}{2x^2+2x}; x=0 \quad 4. \frac{2x-2}{x^2-2x+1}; x=1 \quad 5. \frac{3x-6}{x^2-4}; x=2$$

$$6. \frac{3x+9}{x^2-9}; x=-3 \quad 7. \frac{2x^2-6x+4}{x^2-4x+4}; x=2 \quad 8. \frac{18x^3-18x^2}{9x^4-9x^2}; x=1 \quad 9. \frac{4x^3-12x^2}{4x^4-36x^2}; x=-3 \quad 10. \frac{18x^4+36x^3}{9x^4-36x^2}; x=-2$$

$$11. \frac{3x^2-6x+3}{x^2-1}; x=1 \quad 12. \frac{6x^2+18x}{3x^2+18x+27}; x=-1 \quad 13. \frac{3x^2+3x}{x^2-1}; x=\frac{1}{2} \quad 14. \frac{2x^2+6x}{x^2-9}; x=\frac{2}{3} \quad 15. \frac{6x^2+15x+9}{3x^2+6x+3}; x=\frac{-3}{2}$$

25. Realiza la operación:

$$1. \frac{1}{2} - \frac{x-4}{6x} - \frac{x+1}{3x} \quad 2. \frac{2}{3} - \frac{x-6}{6x} - \frac{x+1}{2x} \quad 3. \frac{1}{2x} + \frac{x+1}{3x^2} - \frac{5}{6x} \quad 4. \frac{4}{3x} - \frac{x+1}{3x^2} - \frac{x-1}{x^2}$$

$$5. \frac{x-1}{x} - 2 + \frac{x^2+x+1}{x^2} \quad 6. \frac{x+1}{2x^2} + \frac{2x+1}{6x^2} - \frac{5}{6x} \quad 7. \frac{x+1}{x} + \frac{x+1}{3x} - \frac{5x+8}{6x} \quad 8. \frac{x}{x-1} - \frac{x-4}{2x-2} + 1$$

$$9. \frac{3}{x+1} - \frac{1}{x} - \frac{x-1}{x^2+x} \quad 10. \frac{x+1}{2x-6} + \frac{x-2}{x-3} - \frac{3}{2} \quad 11. \frac{5}{4} - \frac{x+2}{x+1} - \frac{x-7}{4x+4} \quad 12. \frac{x+4}{6x-3} - \frac{2}{3} + \frac{x+1}{2x-1}$$

$$13. \frac{x^2+x+3}{x^2+x} - 1 + \frac{x-3}{x} \quad 14. \frac{x+1}{2x-1} + \frac{2x+1}{8x-4} - \frac{3}{4} \quad 15. \frac{1}{x} + \frac{x-2}{x^2+2x} + \frac{1}{x+2} \quad 16. \frac{2x+5}{4x+2} + \frac{2x+3}{8x+4} - \frac{1}{4}$$

$$17. 4 - \frac{x^2+x-10}{x^2-4} - \frac{x-1}{x-2} \quad 18. \frac{x-1}{2x-3} + \frac{2x-1}{8x-12} - \frac{3}{4} \quad 19. 4 - \frac{x^2+x-42}{x^2-9} - \frac{x+2}{x-3} \quad 20. \frac{5}{6} + \frac{2x+7}{12x+6} + \frac{x+1}{2x+1}$$

$$21. \frac{3x+2}{27x-9} - \frac{4}{9} + \frac{3x+1}{9x-3} \quad 22. \frac{4}{9} - \frac{3x+1}{27x-18} + \frac{x+1}{3x-2} \quad 23. \frac{19}{9x-9} - \frac{1}{x+1} - \frac{x+37}{9x^2-9} \quad 24. \frac{3x^2+3x+1}{3x^2+3x} - \frac{7}{9} + \frac{x-3}{9x}$$

$$25. \frac{4}{3x-3} - \frac{1}{3x} - \frac{x+1}{3x^2-3x} \quad 26. \frac{x^2+x+1}{3x^2-x} - \frac{3x+1}{3x-1} + \frac{x+1}{x} \quad 27. \frac{9}{4} - \frac{4x^2+4x+5}{16x^2-4} - \frac{2x-3}{4x-2} \quad 28. \frac{4x+3}{4x} - \frac{3}{2} + \frac{4x^2+4x-9}{8x^2+12x}$$

$$29. \frac{3x^3+3x+4}{3x^2-3} + \frac{x-16}{9x-9} - \frac{4}{9} \quad 30. \frac{5}{4x-6} - \frac{2x+27}{8x^2-18} - \frac{1}{2x+3} \quad 31. \frac{x^2+x+1}{3x^2-2x} + \frac{2x+1}{2x} - \frac{6x+3}{6x-4} \quad 32. \frac{5x+18}{x+2} - \frac{x^2+x-34}{x^2-4} - \frac{x+1}{x-2}$$

26. Realiza la operación:

$$1. \frac{2x+2}{6x} : \frac{x+1}{3x} \quad 2. \frac{3x+9}{9x^2} : \frac{x+3}{3x} \quad 3. \frac{x^2+3x}{3x-6} : \frac{x}{x-2} \quad 4. \frac{2x-6}{3x^2-6x} : \frac{x-3}{3x}$$

$$5. \frac{x^2-4}{3x^2+3x} : \frac{3x}{x+2} \quad 6. \frac{3x^2+9x}{x^2+x} : \frac{x+3}{x+1} \quad 7. \frac{x^2-2x+1}{6x} : \frac{x-1}{3} \quad 8. \frac{x^2+2x}{3x^2+9x} : \frac{x}{x+3}$$

$$9. \frac{9}{x^2-6x+9} : \frac{x-3}{3} \quad 10. \frac{x^2-1}{x^2-4x+4} : \frac{x+1}{x-2} \quad 11. \frac{2x^2-2x}{x^2-4x+4} : \frac{x-1}{x-2} \quad 12. \frac{x^2+6x+9}{2x^2} : \frac{2x}{x+3}$$

$$13. \frac{x^2+4x+4}{3x^2} : \frac{x}{x+2} \quad 14. \frac{6x^2+18x}{6x^3-6x^2} : \frac{x+3}{x^2-x} \quad 15. \frac{x^4-4x^2}{x^3+2x^2} : \frac{x^2-2x}{x+2} \quad 16. \left(\frac{6x^3-6x}{9x^2-9}\right)^2 : \frac{4x^2}{9}$$

$$17. \left(\frac{9x-18}{9x^2-18x}\right)^2 : \frac{3x^2}{2} \quad 18. \left(\frac{6x^2+6x}{9x^2+9x}\right)^2 : \frac{9}{4x-8} \quad 19. \frac{6x+6}{4x^2-36} : \frac{2x^2-18}{3x+3} \quad 20. \frac{3x^2-9x}{9x^2-36} : \frac{3x^2-12}{x-3}$$

$$21. \frac{9x^2-36x+36}{9x+9} : \frac{x+1}{3x-6} \quad 22. \frac{3x^2+18x+27}{x^3-x^2} : \frac{3x+9}{x^2-x} \quad 23. \left(\frac{6x^3+12x^2+6x}{9x^2+18x+9}\right)^2 : \frac{9}{4x} \quad 24. \left(\frac{9x^2-9x}{6x-6}\right)^2 : \frac{4}{9x^3-27x^2}$$

$$25. \left(\frac{2x^2-8x+8}{4x^2-16x+16}\right)^2 : \frac{x}{2x-6} \quad 26. \frac{4x^2-24x+36}{9x^4-54x^3+81x^2} : \frac{9x^2}{4} \quad 27. \left(\frac{6x^2-36x+54}{9x^2-54x+18}\right)^2 : \frac{2x-6}{9} \quad 28. \frac{x^2-9}{3x^3+6x^2+3x} : \frac{x+3}{3x^2+3x}$$

$$29. \frac{3x^2-9x}{3x^3+12x^2+12x} : \frac{x^2+2x}{x-3} \quad 30. \frac{2x^2+4x}{4x^2-24x+36} : \frac{x^2-6x+9}{x+2} \quad 31. \left(\frac{4x+8}{6x^2+12x}\right)^2 : \frac{4x-4}{9x^3-27x^2} \quad 32. \left(\frac{2x^3+12x^2+18x}{x^4+6x^3+9x^2}\right)^2 : \frac{x^2+2x}{4}$$

27. Realiza la operación:

1.  $\left(\frac{1}{x} + \frac{x+6}{2x^2} - \frac{3}{2x}\right) : \frac{3}{2x}$
2.  $\left(\frac{x-1}{x^2} - \frac{2}{x} + \frac{x^2+x+3}{x^3}\right) : \left(\frac{x+1}{x} - 1\right)$
3.  $\left(\frac{2x-1}{2x-4} + \frac{2x+1}{2x+4} - 2\right) : \frac{3}{x-2}$
4.  $\left(4 - \frac{x+2}{x+1} - \frac{x-2}{x-1}\right) : \left(1 + \frac{x+1}{x-1}\right)$
5.  $\left(\frac{x+2}{x+1} - 2 + \frac{x^2+x-3}{x^2+x}\right) : \frac{3x}{x-3}$
6.  $\left(\frac{x^2+x+3}{x^2-x} - 2 + \frac{x-2}{x-1}\right) : \frac{x}{x+3}$
7.  $\left(\frac{x+2}{x+1} - 2 + \frac{x^2+x+6}{x^2-1}\right) : \frac{x-1}{x+3}$
8.  $\left(4 - \frac{x+4}{x+3} - \frac{x-8}{x-3}\right) : \left(2 - \frac{x+4}{x+3}\right)$
9.  $\left(\frac{x^2+x-8}{x^2-4} + \frac{x}{x-2} - 2\right) : \frac{1}{x-2}$
10.  $\left(\frac{x^2+x+3}{x^2-3x} + \frac{x+1}{x-3} + \frac{x+1}{x}\right) : 3$
11.  $\left(\frac{x^2+x-18}{x^2-9} + \frac{x-1}{x-3} - 2\right) : \frac{x+3}{x-1}$
12.  $\left(\frac{x+5}{x+3} + \frac{x^2+x+3}{x^2-9} - 2\right) : \frac{x+2}{x+3}$
13.  $\left(\frac{x-18}{x^2-9} + \frac{7}{x-3} + \frac{1}{x+3}\right) : \frac{x+3}{3}$
14.  $\left(\frac{2x-1}{2x-4} + \frac{2x+7}{2x+4} - 2\right) : \left(\frac{x-1}{x-2} - 1\right)$
15.  $\left(\frac{2x+1}{2x-2} + \frac{2x+3}{2x+2} - 1\right) : \left(2 - \frac{x-4}{x-1}\right)$
16.  $\left(\frac{x+3}{x+1} + \frac{x^2+x-8}{x^2-1} - 2\right) : \left(2 - \frac{x-5}{x-3}\right)$
17.  $\left(\frac{x+8}{x+3} + \frac{x^2+x+9}{x^2+3x} - 2\right) : \left(2 - \frac{x+6}{x+3}\right)$
18.  $\left(\frac{x-1}{x^2+2x} - \frac{1}{x+2} + \frac{1}{x}\right) : \left(2 - \frac{x-1}{x}\right)$
19.  $\left(2 - \frac{x+1}{x+3} - \frac{x^2+x-13}{x^2-9}\right) : \left(2 - \frac{x-7}{x-2}\right)$
20.  $\left(\frac{x+1}{x+2} + \frac{x^2+x+10}{x^2-4} + 2\right) : \left(1 - \frac{x+2}{2x}\right)$
21.  $\left(\frac{x^2+x+12}{x^2-9} + \frac{x-1}{x-3} + 1\right) : \left(2 - \frac{x+5}{x+3}\right)$
22.  $\left(\frac{1}{x+2} + \frac{x}{x^2-4} + \frac{1}{x-2}\right) : \left(2 - \frac{x+4}{x+2}\right)$
23.  $\left(4 - \frac{x-4}{x+3} - \frac{x^2+x-48}{x^2-9}\right) : \left(2 - \frac{x-9}{x-3}\right)$
24.  $\left(\frac{1}{x} - \frac{1}{3x} - \frac{x+1}{3x^2}\right)^2 : \frac{18x^4}{x^2-2x+1}$
25.  $\left(\frac{2}{3} + \frac{x-3}{3x} + \frac{x+1}{x}\right)^2 : \left(\frac{x+5}{x-1} + 5\right)$
26.  $\left(\frac{x+3}{3x} - \frac{1}{3} + \frac{x+1}{x}\right)^2 : \left(\frac{x+7}{x} + \frac{x^2+x+8}{x^2}\right)$
27.  $\left(\frac{x-4}{6x^2} - \frac{7}{6x} + \frac{x+1}{x^2}\right)^2 : (9x^4+9x^3)$
28.  $\left(\frac{x+4}{x+3} + \frac{x^2+x}{x^2-9} - 2\right)^2 : \frac{x^2+6x+9}{12}$
29.  $\left(4 - \frac{x-4}{x} - \frac{x^2+x-4}{x^2-x}\right)^2 : \frac{x^2-2x+1}{8x^2}$
30.  $\left(\frac{3}{2} + \frac{x-11}{2x+6} + \frac{x+1}{x+3}\right)^2 : \frac{9x^3}{x^2+6x+9}$

28. Realiza la operación:

1.  $1 + \frac{y+6x-2}{3xy-6x} - \frac{1}{3x}$
2.  $\frac{y+2x^2-1}{x^2y-x^2} + 1 - \frac{1}{x^2}$
3.  $\frac{y+11x-3}{3y-3x} + \frac{2}{3} + \frac{y+1}{y-x}$
4.  $\frac{1}{3} - \frac{x+3}{3x} + \frac{3y+2x+3}{3xy+3x}$
5.  $\frac{xy+3x^2-3}{x^3y-3x^2} + 1 - \frac{x^2+1}{x^2}$
6.  $\frac{y+5x-3}{xy-3x} + \frac{y+1}{y-3} + \frac{2x+1}{x}$
7.  $\frac{xy+4x^3-1}{4x^3y-4x^2} - \frac{x^2+1}{4x^2} + \frac{1}{4}$
8.  $\frac{3x-4}{3x} + \frac{y-9x^2+2x}{3xy+6x^2} + \frac{1}{x}$
9.  $\frac{3y+18x^2-1}{18xy-6x} - \frac{3x+1}{6x} + \frac{1}{2}$
10.  $\frac{2xy+3x-2}{2x^2y^2-2xy} + \frac{x-1}{xy} + \frac{1}{2y}$
11.  $\left(\frac{y^2+y+3x}{y^2-x^2} + \frac{y-x-3}{y-x} - 1\right) : \frac{y-2}{y+x}$
12.  $\left(\frac{1}{x} - \frac{1}{3x}\right) 3x - \frac{1}{y} : \left(\frac{y+3x+1}{y+3x} - 1\right)$
13.  $\left(\frac{y+1}{y^2-4} + \frac{4x-1}{4y+8} - \frac{3}{4y-8}\right)^2 : \frac{y^2+4y+4}{x^3}$
14.  $\left(\frac{2y-6x+9}{2y-6x} - 2 + \frac{2y+6x+9}{2y+6x}\right) : \left(2 + \frac{y-6x}{y+3x}\right)$
15.  $\left(1 - \frac{y-3x^2+x}{2y}\right) : \frac{y-x}{2} + \frac{3x}{y} : \left(1 + \frac{1}{3x-1}\right)$

— Soluciones —

- 1.1.  $-2x^9$  1.2.  $\frac{8x^4}{3}$  1.3.  $\frac{-2}{x^4}$  1.4. 3 1.5.  $\frac{8x^2}{3}$  1.6.  $\frac{-x^2}{8}$  1.7.  $\frac{-1}{2x^7}$  1.8.  $3x^8$  1.9.  $\frac{-x^2}{24}$  1.10.  $\frac{x^3}{2}$  1.11.  $3x^7$  1.12.  $-9x^2$  1.13.  $-4x^{11}$  1.14.  $\frac{1}{54x^4}$  1.15.  $\frac{3x^2}{2}$  1.16.  $\frac{-9x^{11}}{2}$  1.17.  $\frac{1}{54x^8}$  1.18.  $3x^7$  1.19.  $\frac{-243}{x^5}$  1.20.  $\frac{-16}{9}$  1.21.  $\frac{1}{3x^7}$  1.22.  $\frac{x^{12}}{3}$  1.23.  $\frac{8x^7}{27}$  1.24.  $\frac{27x^6}{2}$  2.1.  $\frac{2y^3}{27x}$  2.2.  $\frac{9x^2}{4y^7}$  2.3.  $-6x^2$  2.4.  $\frac{-2}{xy^3}$  2.5.  $\frac{1}{27x^2y^7}$  2.6.  $9xy^2$  2.7.  $\frac{27xy^7}{2}$  2.8.  $\frac{1}{2xy^5}$  2.9.  $\frac{-x^3}{27y^7}$  2.10.  $\frac{-2y^4}{3}$  2.11.  $3xy^6$  2.12.  $-x^3y^4$  2.13.  $\frac{-1}{2y^6}$  2.14.  $\frac{2y^6}{9x^4}$  2.15.  $\frac{-4}{x^5}$  2.16.  $\frac{-1}{x^2y^5}$  2.17.  $\frac{-2}{x^5y^5}$  2.18.  $\frac{3}{8x^2y^4}$  3.1. -46 3.2. 3 3.3. -25 3.4. -16 3.5. 9 3.6. -8 3.7. 36 3.8. 11 3.9. -1 3.10. 4 3.11. -10 3.12. 36 3.13. 24 3.14. -29 3.15. -42 3.16. 2 3.17. -9 3.18. -14 3.19. 0 3.20. -22 4.1.  $-2x^2-2x-2$  4.2.  $4x^2+x-2$  4.3.  $-7x$  4.4.  $-3x^2-x+2$  4.5.  $-2x^2+3x-5$  4.6.  $-4x^2-4x-1$  4.7.  $-4x^2+x-2$  4.8.  $7x^2+3x-3$  4.9.  $3x^2+4x+4$  4.10.  $4x^2-4x+3$  4.11.  $-x^2+x-3$  4.12.  $2x^2+2$  4.13.  $-4x^2+7x+1$  4.14.  $-2x^2-8x-3$  4.15.  $5x^2+2x-5$  4.16.  $4x^2-3x+5$  4.17.  $-8x^2-4x$  4.18.  $-6x^2+x-4$  4.19.  $-3x^2+2x$  4.20.  $-x^2+1$  5.1.  $-8x^3-8x$  5.2.  $4x^4-12x^3$  5.3.  $-6x^4-9x$  5.4.  $4x^4-9x^3+2x^2$  5.5.  $9x^3-6x^2-8x$  5.6.  $-16x^5-4x^4+12x^3$  5.7.  $8x^5-8x$  5.8.  $6x^5+x^4-x^3$  5.9.  $9x^2-1$  5.10.  $16x^2-9$  5.11.  $4x^2-25$  5.12.  $x^4-1$  5.13.  $9x^4-9$  5.14.  $4x^4-16x^2$  5.15.  $9x^2+6x+1$  5.16.  $16x^2-24x+9$  5.17.  $4x^2+12x+9$  5.18.  $x^4-4x^2+4$  5.19.  $4x^4+12x^2+9$  5.20.  $x^6+4x^5+4x^4$  6.1.  $2x^2-x+2$  6.2.  $x^2+7x-4$  6.3.  $12x-12$  6.4.  $6x^2-6x+4$  6.5.  $-x^4-2x^3-x^2+2x+4$  6.6.  $-7x^2+4x-1$  6.7.  $-2x^2+6x-8$

- 6.8.  $-x^4-6x^3-6x^2-6$  6.9.  $-4x^2+14x+5$  6.10.  $x^4-7x^2+13$  6.11.  $3x^2-12x+13$  6.12.  $-6x^3+5x^2-4x-1$  6.13.  $-3x^3+8x^2+4$  6.14.  $-3x^3+4x^2-8x+9$  6.15.  $2x^2-2x-15$   
6.16.  $-13x^2+17x-9$  6.17.  $6x^3+5x^2-2x-4$  6.18.  $3x^3-5x^2-8x-4$  6.19.  $-4x^4-8x^3-8x^2+1$  6.20.  $-10x^2+6x$  6.21.  $-9x^4+3x^3-2x^2+1$  6.22.  $-5x^2+9x+7$  6.23.  $-13x^2-7x+3$   
6.24.  $9x^4-2x^3+15x^2-4x+4$  6.25.  $-9x^4+9x^2+3x-6$  6.26.  $4x^4-4x^3-8x^2+2x+9$  6.27.  $-4x^4+12x^3-5x^2+3x-9$  6.28.  $-4x^3+10x^2-1$  6.29.  $-4x^3+2x-4$  6.30.  
 $-9x^4+18x^3+3x^2-9$  7.1.  $\begin{cases} C: 2x-2 \\ r: 1 \end{cases}$  7.2.  $\begin{cases} C: -2x-1 \\ r: -4 \end{cases}$  7.3.  $\begin{cases} C: 3x+4 \\ r: 1 \end{cases}$  7.4.  $\begin{cases} C: 2x^2-3x+3 \\ r: 0 \end{cases}$  7.5.  $\begin{cases} C: 4x-4 \\ r: 0 \end{cases}$  7.6.  $\begin{cases} C: -3x+1 \\ r: 3x \end{cases}$  7.7.  $\begin{cases} C: 3x^2+4x-4 \\ r: -4 \end{cases}$  7.8.  $\begin{cases} C: -x^2-3x \\ r: -4 \end{cases}$  7.9.  
 $\begin{cases} C: 3x-3 \\ r: 2x-4 \end{cases}$  7.10.  $\begin{cases} C: 3x-1 \\ r: -4 \end{cases}$  7.11.  $\begin{cases} C: -4x^2-2x \\ r: -4x+1 \end{cases}$  7.12.  $\begin{cases} C: -3x+3 \\ r: 0 \end{cases}$  8.1.  $\begin{cases} C: -3x+3 \\ r: -4 \end{cases}$  8.2.  $\begin{cases} C: x+1 \\ r: -2 \end{cases}$  8.3.  $\begin{cases} C: 4x-4 \\ r: 3 \end{cases}$  8.4.  $\begin{cases} C: -x^2+x \\ r: 2 \end{cases}$  8.5.  $\begin{cases} C: -2x^2+4x-1 \\ r: 0 \end{cases}$  8.6.  
 $\begin{cases} C: 2x^2+x+4 \\ r: 0 \end{cases}$  8.7.  $\begin{cases} C: -4x^2-2x-4 \\ r: -4 \end{cases}$  8.8.  $\begin{cases} C: -4x^2+x \\ r: 2 \end{cases}$  8.9.  $\begin{cases} C: x^3-1 \\ r: 4 \end{cases}$  8.10.  $\begin{cases} C: -2x^2+x-2 \\ r: 1 \end{cases}$  8.11.  $\begin{cases} C: -3x^3+x^2+2x \\ r: 0 \end{cases}$  8.12.  $\begin{cases} C: 3x^3+3x-4 \\ r: 1 \end{cases}$  9. 3 10. -1 11.  $\frac{-5}{2}$  12.  
 $\frac{-2}{3}$  13.  $\frac{2}{3}$  14.  $\frac{10}{3}$  15. -1 16. -2 17.  $\frac{5}{2}$  18.  $\frac{-2}{3}$  19.1.  $3(x+2)$  19.2.  $x(x+2)$  19.3.  $3x(x-3)$  19.4.  $(x+2)(x-2)$  19.5.  $(x^2-3)(x^2+3)$  19.6.  $(3x+2)(3x-2)$  19.7.  
 $(x+3)^2$  19.8.  $(x-2)^2$  19.9.  $(x^2-2)^2$  19.10.  $x^2(2x+1)(2x-1)$  19.11.  $3x(x+1)(x-1)$  19.12.  $3(3x+1)(3x-1)$  19.13.  $-3(2x+1)(2x-1)$  19.14.  $-x^2(x^2+3)(x^2-3)$  19.15.  
 $-3(3x^2+1)(3x^2-1)$  19.16.  $2x(3x+1)(3x-1)$  19.17.  $3(3x^2+2)(3x^2-2)$  19.18.  $3x(2x^2+1)(2x^2-1)$  19.19.  $-2x(x^2+2)(x^2-2)$  19.20.  $3(2x+3)(2x-3)$  19.21.  
 $2x^2(x+1)(x-1)$  19.22.  $-2x(2x+3)(2x-3)$  19.23.  $-2x(3x+2)(3x-2)$  19.24.  $-3x^2(x+1)(x-1)$  19.25.  $2x^2(2x+3)(2x-3)$  19.26.  $3x(3x+2)(3x-2)$  19.27.  $-3x(3x+1)(3x-1)$   
19.28.  $2(x-1)^2$  19.29.  $3(x+1)^2$  19.30.  $-2(x+1)^2$  19.31.  $-3(x-1)^2$  19.32.  $-2(x-2)^2$  19.33.  $x^2(x-1)^2$  19.34.  $x(x-2)^2$  19.35.  $2(2x^2-1)^2$  19.36.  $x(3x^2+1)^2$   
19.37.  $x(2x+1)^2$  19.38.  $-x^2(x-1)^2$  19.39.  $2(3x-1)^2$  19.40.  $2(x+3)^2$  19.41.  $2x(3x-1)^2$  19.42.  $-3(3x-2)^2$  19.43.  $3(2x-3)^2$  19.44.  $2x(3x^2+1)^2$  19.45.  $2x(3x-2)^2$   
19.46.  $3(3x^2+2)^2$  19.47.  $-3x(2x+1)^2$  19.48.  $-3x^2(x^2-3)^2$  19.49.  $3x(3x^2+2)^2$  19.50.  $-3(2x^2+3)^2$  19.51.  $2x^2(3x+2)^2$  19.52.  $3x(2x+3)^2$  19.53.  $3x(2x^2+3)^2$   
19.54.  $3x(3x-2)^2$  20.1.  $y^2(y-x)$  20.2.  $3x(y+3)$  20.3.  $xy(y-2)$  20.4.  $2x(y-2x)$  20.5.  $3xy(y-1)$  20.6.  $(y+x)^2$  20.7.  $(y-3x)^2$  20.8.  $(y^2-2x)^2$  20.9.  $(3x-2y)^2$   
20.10.  $(y^2-3x)^2$  20.11.  $y^2(x+y^2)(x-y^2)$  20.12.  $xy(3y+x)(3y-x)$  20.13.  $3xy(2y^2+1)(2y^2-1)$  20.14.  $3y^2(y+x)(y-x)$  20.15.  $2xy^2(2+xy^2)(2-xy^2)$  20.16.  
 $2(2xy+1)^2$  20.17.  $-y(y-x)^2$  20.18.  $x(y^2-2x)^2$  20.19.  $2y^2(xy+1)^2$  20.20.  $-2y(y-3x)^2$  21.1. 1;  $3(3x-1)(3x+1)$  21.2.  $2x$ ;  $2x(2x+1)(2x-1)$  21.3. 1;  $2x(x+1)(x-1)^2$   
21.4.  $3x-1$ ;  $3(3x+1)(3x-1)^2$  21.5.  $x^2+3$ ;  $x(x^2-3)(x^2+3)^2$  21.6. 3;  $3(x-1)(x+1)^2$  21.7.  $x+1$ ;  $2(x-1)(x+1)^2$  21.8.  $2x+1$ ;  $2(2x-1)(2x+1)^2$  21.9.  $2(x-1)$ ;  $2(x+1)(x-1)^2$   
21.10. 1;  $x(x+2)^2(x-2)^2$  21.11. 1;  $x(2x+1)^2(2x-1)^2$  21.12. 1;  $3(2x+1)^2(2x-1)^2$  21.13.  $3x^2-2$ ;  $x(3x^2+2)(3x^2-2)^2$  21.14. 1;  $3(3x^2-2)(3x^2+2)^2$  21.15.  $x+2$ ;  
 $x^2(x-2)(x+2)^2$  21.16.  $x-1$ ;  $3x(x+1)(x-1)^2$  21.17.  $x+1$ ;  $2x(x-1)(x+1)^2$  21.18.  $2x-1$ ;  $2(2x+1)(2x-1)^2$  21.19.  $x+3$ ;  $2(x+3)^2$  21.20.  $x+2$ ;  $2x(x-2)(x+2)^2$  21.21.  $2(3x+2)$ ;  
 $2(3x-2)(3x+2)^2$  21.22. 1;  $2(2x-3)^2(2x+3)^2$  21.23.  $2x(x+1)$ ;  $2x^2(x-1)(x+1)^2$  21.24. 1;  $2x^2(x-1)^2(x+1)^2$  21.25. 1;  $2(3x^2-2)(3x^2+2)^2$  21.26. 1;  $x(3x-2)^2(3x+2)^2$   
21.27.  $2x$ ;  $2x^2(3x^2-2)(3x^2+2)^2$  21.28.  $3x$ ;  $3x^2(x-3)(x+3)^2$  21.29. 1;  $2(x^2+2)(x^2-2)^2$  21.30. 1;  $3x(x^2+2)^2(x^2-2)^2$  22.1. 1;  $xy(y-x)(y+x)$  22.2. 1;  $2y(y+x)y-x)^2$   
22.3.  $y+2x$ ;  $xy(y+2x)^2$  22.4.  $y^2-2x$ ;  $3y(y^2+2x)(y^2-2x)^2$  22.5.  $y+x$ ;  $xy(y+x)(y-x)$  22.6.  $y(xy-1)$ ;  $y^2(xy+1)(xy-1)^2$  22.7.  $2(2y+x)$ ;  $2(2y-x)(2y+x)^2$  22.8.  $3x$ ;  
 $3x(y-1)^2(y+1)^2$  22.9.  $x$ ;  $x(y+x)^2(y-x)^2$  22.10.  $y-x$ ;  $x(y-x)^2$  22.11.  $xy+3$ ;  $x(xy-3)(xy+3)^2$  22.12. 1;  $2(xy+2)^2(xy-2)^2$  23.1.  $\frac{x+1}{x-1}$  23.2.  $\frac{x-1}{x+1}$  23.3.  $\frac{1}{x+1}$  23.4.  $\frac{1}{2x-1}$   
23.5.  $\frac{1}{x-2}$  23.6.  $\frac{x-3}{x+3}$  23.7.  $\frac{x+1}{x-1}$  23.8.  $\frac{3x+1}{3x-1}$  23.9.  $\frac{-1}{3}$  23.10.  $\frac{-2x}{x-3}$  23.11.  $\frac{2x-3}{2x+3}$  23.12.  $\frac{2x}{x+2}$  23.13.  $\frac{2x-1}{2x+1}$  23.14.  $\frac{-x}{x-2}$  23.15.  $4x^2+1$  23.16.  $\frac{2}{x+1}$  23.17.  
 $\frac{3x}{x+3}$  23.18.  $\frac{1}{x+1}$  23.19.  $\frac{3x}{x-3}$  23.20.  $\frac{2x}{x+1}$  23.21.  $\frac{2}{x-2}$  23.22.  $\frac{1}{x+1}$  23.23.  $\frac{-2x}{x+1}$  23.24.  $\frac{-2x}{x^2+3}$  23.25. -2 23.26.  $\frac{x-1}{x+1}$  23.27.  $\frac{1}{x-3}$  23.28.  $3x^2-1$  23.29.  
 $\frac{3x+2}{3x-2}$  23.30.  $\frac{x+1}{x-1}$  23.31.  $\frac{3x+1}{3x-1}$  23.32.  $\frac{1}{3x-2}$  23.33.  $\frac{3x-2}{3x+2}$  23.34.  $-3x$  23.35.  $\frac{-1}{3x^2}$  23.36.  $3x+2$  24.1. 1 24.2. no tiene 24.3. 1 24.4. no tiene 24.5.  $\frac{3}{4}$   
24.6.  $\frac{-1}{2}$  24.7. no tiene 24.8. 1 24.9. no tiene 24.10. 1 24.11. 0 24.12. -1 24.13. -3 24.14.  $\frac{-4}{7}$  24.15. 0 25.1.  $\frac{1}{3x}$  25.2.  $\frac{1}{2x}$  25.3.  $\frac{1}{3x^2}$  25.4.  $\frac{2}{3x^2}$   
25.5.  $\frac{1}{x^2}$  25.6.  $\frac{2}{3x^2}$  25.7.  $\frac{1}{2}$  25.8.  $\frac{3x+2}{2x-2}$  25.9.  $\frac{1}{x+1}$  25.10.  $\frac{3}{x-3}$  25.11.  $\frac{1}{x+1}$  25.12.  $\frac{3}{2x-1}$  25.13.  $\frac{x-2}{x+1}$  25.14.  $\frac{2}{2x-1}$  25.15.  $\frac{3}{x+2}$  25.16.  $\frac{x+3}{2x+1}$  25.17.  
 $\frac{2x+2}{x+2}$  25.18.  $\frac{1}{2x-3}$  25.19.  $\frac{2x}{x+3}$  25.20.  $\frac{3x+3}{2x+1}$  25.21.  $\frac{1}{3x-1}$  25.22.  $\frac{2x}{3x-2}$  25.23.  $\frac{1}{x+1}$  25.24.  $\frac{x}{3x+3}$  25.25.  $\frac{2}{3x-3}$  25.26.  $\frac{x+2}{3x-1}$  25.27.  $\frac{3x+2}{2x+1}$  25.28.  $\frac{1}{2x+3}$   
25.29.  $\frac{2x}{3x+3}$  25.30.  $\frac{1}{2x+3}$  25.31.  $\frac{x-1}{3x-2}$  25.32.  $\frac{3x-2}{x-2}$  26.1. 1 26.2.  $\frac{1}{x}$  26.3.  $\frac{x+3}{3}$  26.4.  $\frac{2}{x-2}$  26.5.  $\frac{x-2}{x+1}$  26.6. 3 26.7.  $\frac{x-1}{2x}$  26.8.  $\frac{x+2}{3x}$  26.9.  $\frac{3}{x-3}$   
26.10.  $\frac{x-1}{x-2}$  26.11.  $\frac{2x}{x-2}$  26.12.  $\frac{x+3}{x}$  26.13.  $\frac{x+2}{3x}$  26.14. 1 26.15.  $\frac{x+2}{x}$  26.16. 1 26.17.  $\frac{3}{2}$  26.18.  $\frac{1}{x-2}$  26.19. 1 26.20.  $x$  26.21.  $\frac{x-2}{3}$  26.22.  $\frac{x+3}{x}$   
26.23.  $x$  26.24.  $\frac{1}{x-3}$  26.25.  $\frac{x-3}{2x}$  26.26. 1 26.27.  $\frac{2}{x-3}$  26.28.  $\frac{x-3}{x+1}$  26.29.  $\frac{x}{x+2}$  26.30.  $\frac{x}{2}$  26.31.  $\frac{x-3}{x-1}$  26.32.  $\frac{x+2}{x}$  27.1.  $\frac{2}{x}$  27.2.  $\frac{3}{x^2}$  27.3.  $\frac{2}{x+2}$  27.4.  
 $\frac{x}{x+1}$  27.5.  $\frac{3}{x+1}$  27.6.  $\frac{1}{x-1}$  27.7.  $\frac{2}{x+1}$  27.8.  $\frac{2x}{x-3}$  27.9.  $\frac{3x}{x+2}$  27.10.  $\frac{x}{x-3}$  27.11.  $\frac{3}{x-3}$  27.12.  $\frac{3}{x-3}$  27.13.  $\frac{3x}{x-3}$  27.14.  $\frac{3x}{x+2}$  27.15.  $\frac{x}{x+1}$  27.16.  $\frac{3}{x+1}$  27.17.  
 $\frac{3}{x+3}$  27.18.  $\frac{1}{x+2}$  27.19.  $\frac{1}{x-3}$  27.20.  $\frac{2x}{x+2}$  27.21.  $\frac{3x}{x-3}$  27.22.  $\frac{3}{x-2}$  27.23.  $\frac{2x}{x+3}$  27.24. 2 27.25.  $\frac{2x-2}{3x}$  27.26.  $\frac{1}{2}$  27.27.  $\frac{x+1}{x}$  27.28.  $\frac{1}{3}$  27.29.  $\frac{1}{2}$   
27.30.  $x$  28.1.  $\frac{y}{y-2}$  28.2.  $\frac{y+1}{y-1}$  28.3.  $\frac{2y+3x}{y-x}$  28.4.  $\frac{2}{3y+3}$  28.5.  $\frac{3}{xy-3}$  28.6.  $\frac{3y}{y-3}$  28.7.  $\frac{x}{xy-1}$  28.8.  $\frac{y-x}{y+2x}$  28.9.  $\frac{3x}{3y-1}$  28.10.  $\frac{3x}{2xy-2}$  28.11.  $\frac{y}{y-x}$  28.12.  
 $\frac{y-3x}{y}$  28.13.  $\frac{1}{x}$  28.14.  $\frac{3}{y-3x}$  28.15.  $\frac{3x}{y-x}$