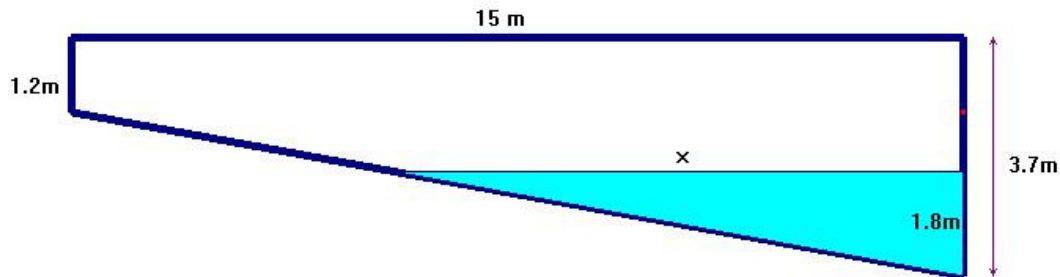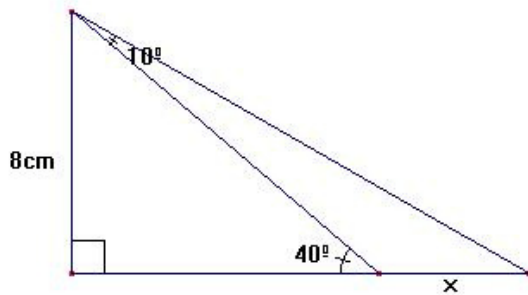EXAM 3_1   (Geometry-Trigonometry)

1. The diagram shows the side view of a swimming pool being filled with water. Calculate the length x.   (1.5 points)
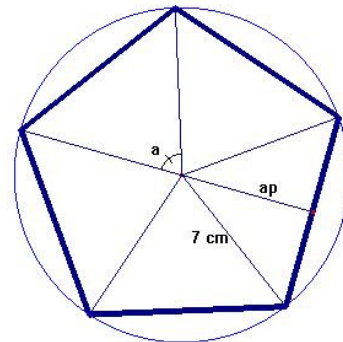

15 m
1.2m
x
3.7m
1.8m

2. In the following diagram, find the length x.   (2 points)

18º
8cm
40º
x

3. A regular pentagon is inscribed in a circle of radius 7 cm.   (2.5 points)

a) Find the angle a.

b) Find the length of the side of the pentagon.

c) Calculate the area of the pentagon.

a
ap
7 cm

4. Find the height of a rectangular box of length 8 cm, width 6 cm and where the length of the diagonal is 11 cm.   (1.5 points)
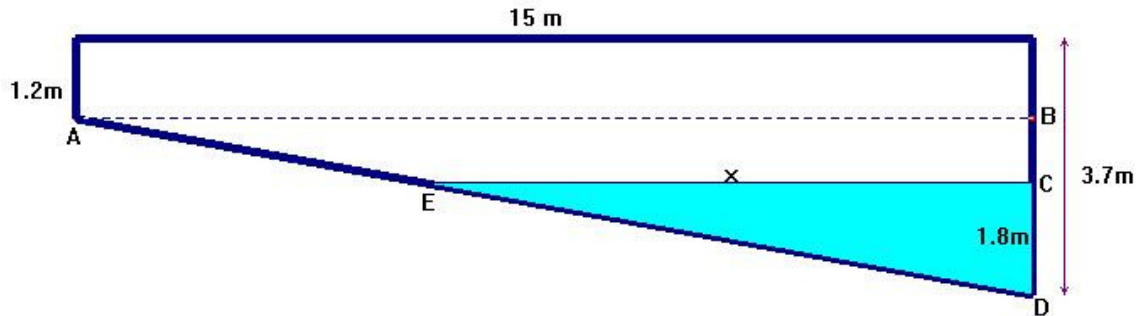
5. Solve: (2.5 points)

a) $\dfrac{(x+1)^2}{16} - \dfrac{1+x}{2} = \dfrac{(x-1)^2}{16} - \dfrac{2+x}{4}$ b) $\sqrt{2x} + \sqrt{5x-6} = 4$
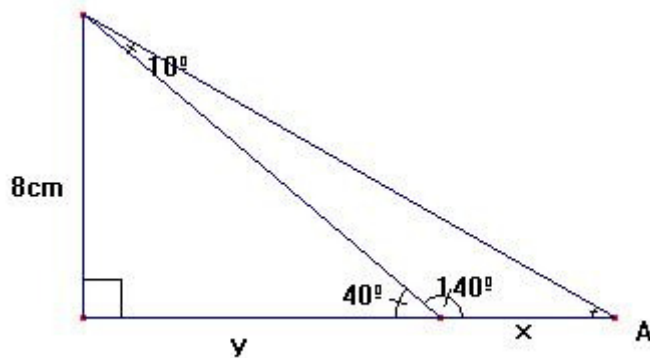
SOLUTION

1. The diagram shows the side view of a swimming pool being filled with water. Calculate the length x.

Triangles ABD and CDE are similar, they have congruent angles.

$$\frac{BD}{CD} = \frac{15}{CE} \rightarrow \frac{2.5}{1.8} = \frac{15}{x} \rightarrow 2.5x = 15 \cdot 1.8 \rightarrow x = 10.8 \text{ m}$$

2. In the following diagram, find the length x.

Angle A=180-150=30°

$$\tan 40 = \frac{8}{y} \rightarrow y = \frac{8}{\tan 40} = 9.53$$

$$\tan 30 = \frac{8}{x+y} \rightarrow x+9.53 = \frac{8}{\tan 30} \rightarrow x+9.53 = 13.86 \rightarrow x = 4.33 \text{ cm}$$

The length of x is 4.33 cm

3. A regular pentagon is inscribed in a circle of radius 7 cm.

a) Find the angle a.

Angle a: 360:5 =72º

b) Find the length of the side x.
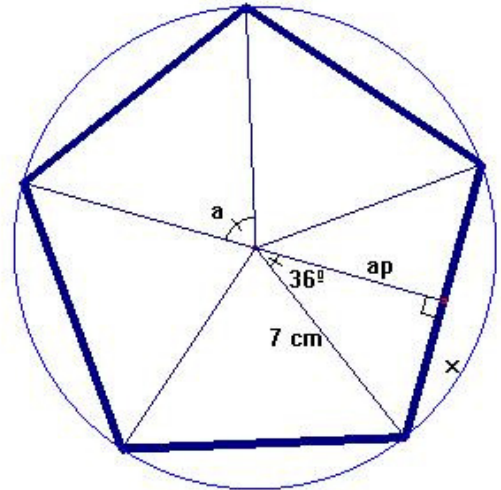
$\sin 36 = \dfrac{x}{7} \rightarrow x = 7 \cdot \sin 36 = 4.11$
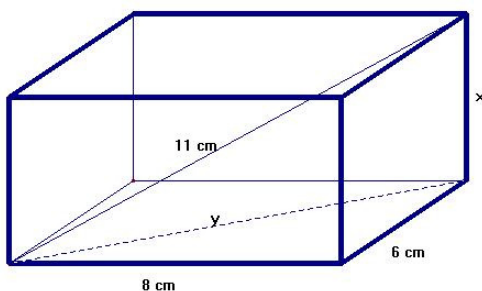
Side of pentagon: $2 \cdot 4.11 = 8.22$ cm

c) Calculate the area of the pentagon.

$A = \dfrac{P \cdot ap}{2}$; Perimeter $P = 5 \cdot 8.22 = 41.11$ cm

$ap \rightarrow \cos 36 = \dfrac{ap}{7} \rightarrow ap = 7 \cdot \cos 36 = 5.66$ cm; $A = \dfrac{P \cdot ap}{2} = \dfrac{41.11 \cdot 5.66}{2} = 116.34$ cm^2

4. Find the height of a rectangular box of length 8 cm, width 6 cm and where the length of the diagonal is 11 cm.

Pythagorean Theorem:

$y^2 = 8^2 + 6^2 = 100 \rightarrow y = 10$ cm

$x^2 = 11^2 - y^2 = 121 - 100 = 21 \rightarrow x = 4.58$ cm

The height of the box is 4.58 cm

5. Solve:

a) $\dfrac{(x+1)^2}{16} - \dfrac{1+x}{2} = \dfrac{(x-1)^2}{16} - \dfrac{2+x}{4} \rightarrow x^2 + 2x + 1 - 8 - 8x = x^2 - 2x + 1 - 8 - 4x$

$2x + 1 - 8 - 8x = -2x + 1 - 8 - 4x \rightarrow 2x - 8x + 2x + 4x = 1 - 8 - 1 + 8 \rightarrow 0x = 0$

Solution: all real numbers

b) $\sqrt{2x} + \sqrt{5x-6} = 4 \to \sqrt{5x-6} = 4 - \sqrt{2x} \to \left(\sqrt{5x-6}\right)^2 = \left(4 - \sqrt{2x}\right)^2$

$5x - 6 = 16 - 8\sqrt{2x} + 2x \to 5x - 6 - 16 - 2x = -8\sqrt{2x} \to 3x - 22 = -8\sqrt{2x}$

$(3x-22)^2 = \left(-8\sqrt{2x}\right)^2 \to 9x^2 - 132x + 484 = 64 \cdot 2x \to 9x^2 - 260x + 484 = 0$

$x = \dfrac{260 \pm \sqrt{260^2 - 4 \cdot 9 \cdot 484}}{18} = \dfrac{260 \pm 224}{18} = \begin{cases} \dfrac{242}{9} \\ 2 \end{cases}$

Checking:

$\sqrt{2x} + \sqrt{5x-6} = 4 \to \sqrt{2 \cdot \dfrac{242}{9}} + \sqrt{5 \cdot \dfrac{242}{9} - 6} = 4 \to \dfrac{22}{3} + \dfrac{34}{3} = 4 \to \dfrac{56}{3} = 4$ NO!

$\sqrt{2x} + \sqrt{5x-6} = 4 \to \sqrt{2 \cdot 2} + \sqrt{5 \cdot 2 - 6} = 4 \to 2 + 2 = 4 \to 4 = 4$ YES!

Solution: x = 2