IES MURILLO

Maths 4th ESO

EXAM 2_2 (Functions 2)

Name:.....

1. Solve algebraically and graphically the simultaneous equations (to graph the functions write their characteristics and **don't** use a table data):

$$\left.\begin{array}{l} y = \sqrt{2x+1} \\ y = \dfrac{x}{2}+1 \end{array}\right\}$$ (2 points)

2. The labels stuck on tins of tomatoes are rectangular which an area of 18 $cm^2$. What are the possible lengths of the base and height? Make a table with some of the possibilities and graph the function. Find the rule. What type of function is it? (2 points)

3. Sketch the graph of the compound function (to graph the function write its characteristics and don't use a table data):

$$f(x) = \begin{cases} 2-2x & x < 0 \\ 2-x^2 & 0 \le x < 3 \\ -3 & x \ge 3 \end{cases}$$ (2 points)

   a) Domain and range
   b) Increasing and decreasing intervals
   c) Continuity

4. In the following equations, find x: (2 points)

   a) $\log_3 9^x = 2$ b) $\log_5 (x+2) = 3$

   c) $3^{x^2-6} = \dfrac{1}{27}$ d) $\log_x 16 = -2$

5. Sketch the graph of the function $y = \left(\dfrac{1}{2}\right)^x$ What type of function is it? Write its characteristics.

Using the basic $y = \left(\dfrac{1}{2}\right)^x$, sketch the graph of $y = \left(\dfrac{1}{2}\right)^x - 2$ and $y = \left(\dfrac{1}{2}\right)^{x+3}$. Find their intersections with x-axis and y-axis, if possible, their domains, ranges and asymptotes. (2 points)

SOLUTION

1. $$\left.\begin{array}{l} y=\sqrt{2x+1} \\ y=\dfrac{x}{2}+1 \end{array}\right\} \Rightarrow \sqrt{2x+1}=\frac{x}{2}+1 \Rightarrow 2\sqrt{2x+1}=x+2 \Rightarrow \left(2\sqrt{2x+1}\right)^2=(x+2)^2$$

$$4(2x+1)=x^2+4x+4 \Rightarrow 8x+4=x^2+4x+4 \Rightarrow x^2-4x=0 \Rightarrow x=0,\ x=4$$

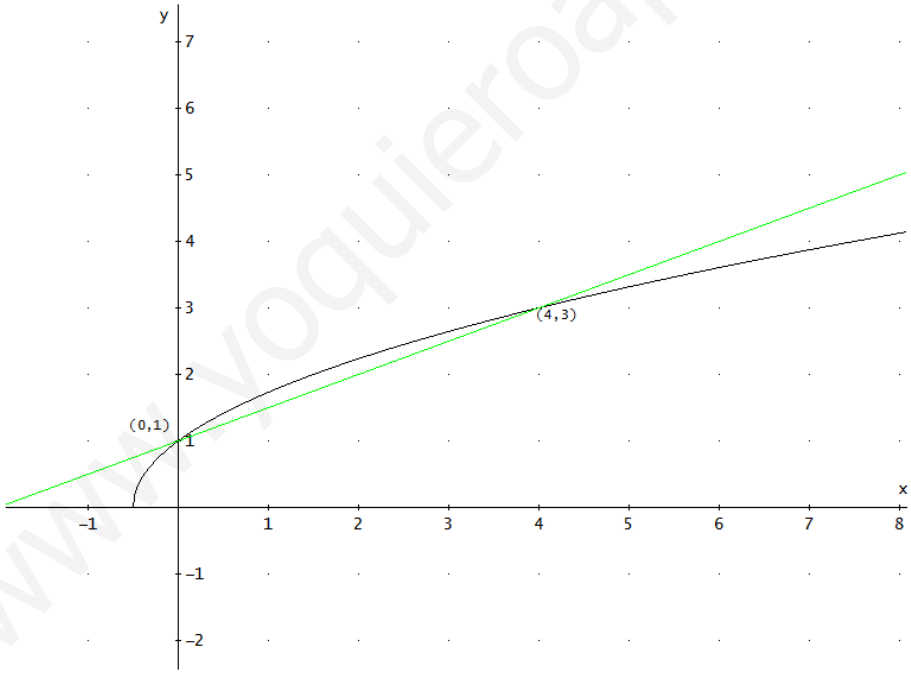
Checking: $\sqrt{2x+1}=\frac{x}{2}+1 \Rightarrow \begin{cases} \sqrt{2\cdot 0+1}=0+1 \Rightarrow 1=1 \vee \\ \sqrt{2\cdot 4+1}=2+1 \Rightarrow 3=3 \vee \end{cases}$

Solution: they intercepts in $(0,1)$ and $(6,4)$

Graphically: $y=\sqrt{2x+1}$ it is a radical function (semi-parabola) with domain:

$$2x+1\geq 0 \Rightarrow 2x\geq 1 \Rightarrow x\geq \frac{1}{2} \rightarrow \text{Dom}=\left[\frac{1}{2},+\infty\right)$$

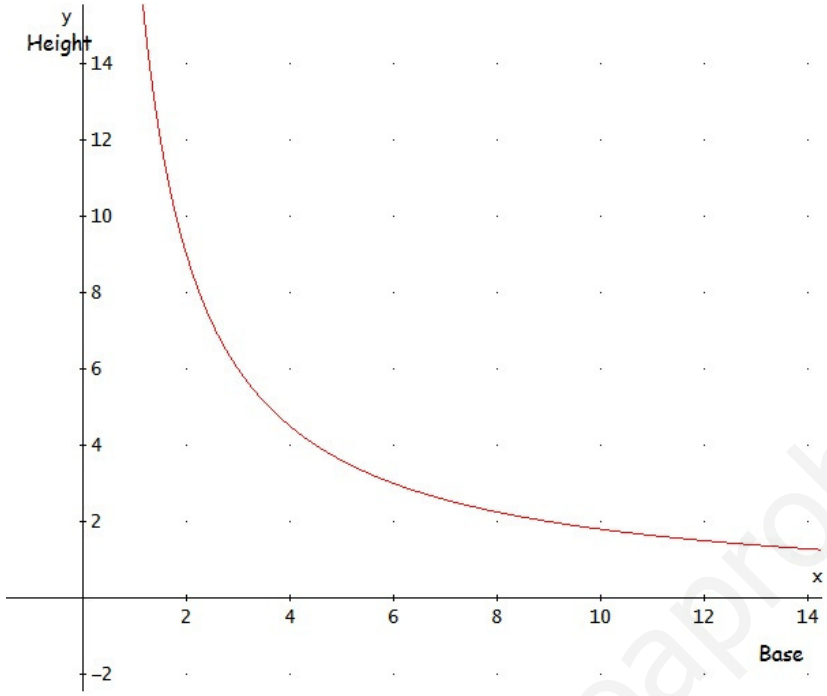
$y=\frac{x}{2}+1$ is a straight line, with slope 1/2 (increasing) and y-intercept (0, 1)




2. The labels stuck on tins of tomatoes are rectangular which an area of 18 $cm^2$. What are the possible lengths of the base and height? Make a table with some of the possibilities and graph the function. Find the rule. What type of function is it?

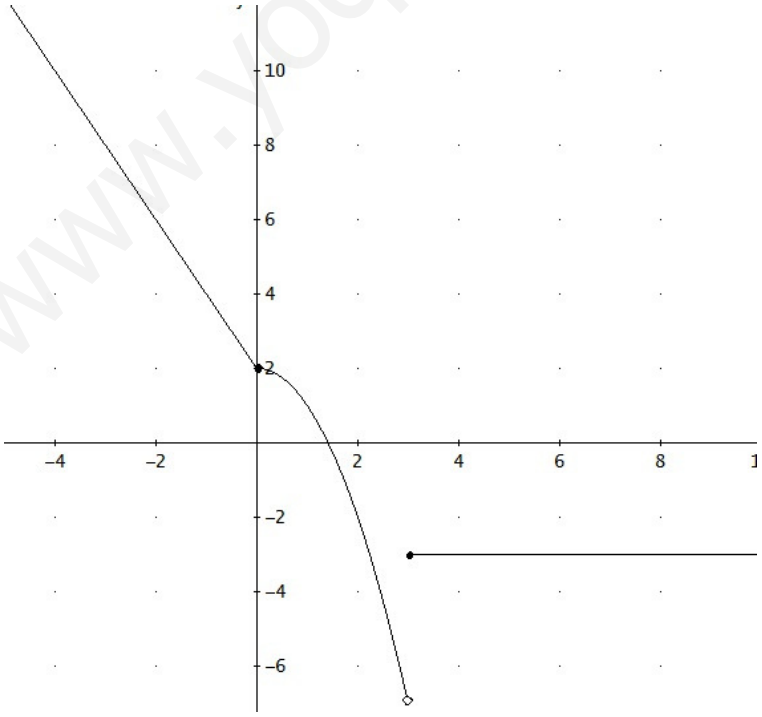
| Base | 1 | 2 | 3 | 6 | 9 | 18 |
|---|---|---|---|---|---|---|
| Height | 18 | 9 | 6 | 3 | 2 | 1 |

Rule: $y = \frac{18}{x}$ Inversely proportional function (hyperbola)




3. Sketch the graph of the compound function (to graph the function write its characteristics and don't use a table data):

$$f(x) = \begin{cases} 2-2x & x < 0 \rightarrow \text{straigh line decreasing} \\ 2-x^2 & 0 \le x < 3 \rightarrow \text{parabola, } \cup, \text{vertex}(0,2), \text{intercepts } x \rightarrow \pm\sqrt{2} \\ -3 & x \ge 3 \rightarrow \text{horizontal line} \end{cases}$$



a) Domain and range Dom = R , R= $(-7,+\infty)$

b) Increasing and decreasing intervals: decreasing in $(-\infty,3)$ ,constant in $(3,+\infty)$

c) Continuity: It is continuous in $R-\{3\}$, It has a jump discontinuity in x = 3

4. In the following equations, find x:

a) $\log_3 9^x = 2 \rightarrow 3^2 = 9^x \rightarrow 3^2 = 3^{2x} \Rightarrow x = 1$

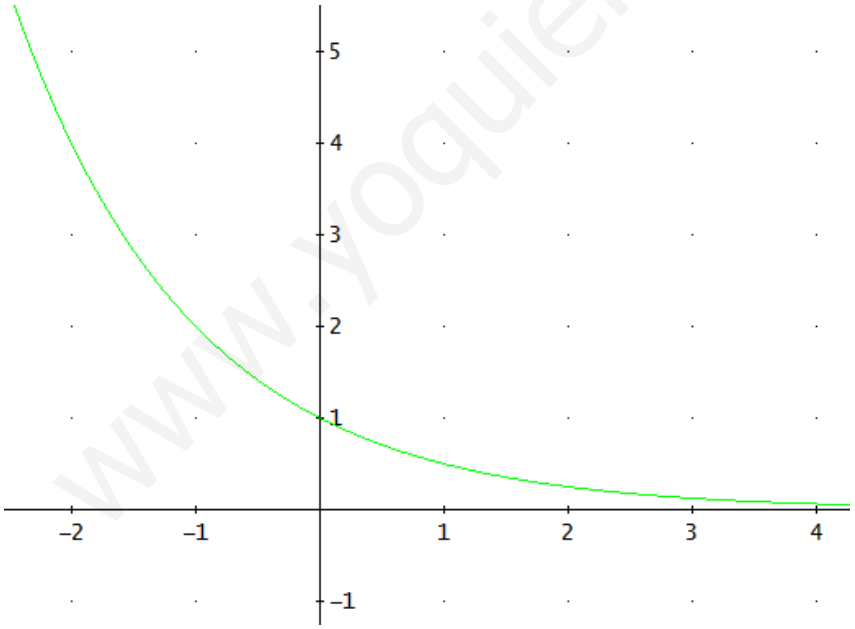
b) $\log_5 (x+2) = 3 \rightarrow 5^3 = x+2 \rightarrow 125 = x+2 \Rightarrow x = 123$

c) $3^{x^2-6} = \frac{1}{27} \rightarrow 3^{x^2-6} = 3^{-3} \rightarrow x^2 - 6 = -3 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

d) $\log_x 16 = -2 \rightarrow x^{-2} = 16 \rightarrow x^2 = \frac{1}{16} \rightarrow x = \frac{1}{4}$ (just positive)

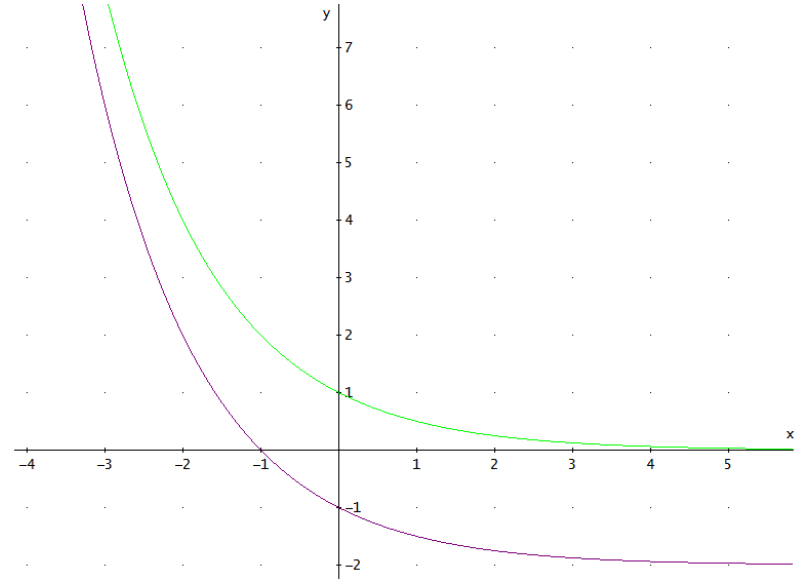
5. Sketch the graph of the function $y = \left(\frac{1}{2}\right)^x$ What type of function is it? Write its characteristics.

It is an exponential function, base < 1, Domain R, Range $(0,+\infty)$ decreasing, continuous in R, horizontal asymptote x-axis (y=0)




Using the basic $y = \left(\frac{1}{2}\right)^x$, sketch the graph of $y = \left(\frac{1}{2}\right)^x - 2$ and $y = \left(\frac{1}{2}\right)^{x+3}$. Find their intersections with x-axis and y-axis, if possible, their domains, ranges and asymptotes.

$y = \left(\frac{1}{2}\right)^x - 2$ the same graph, two units down



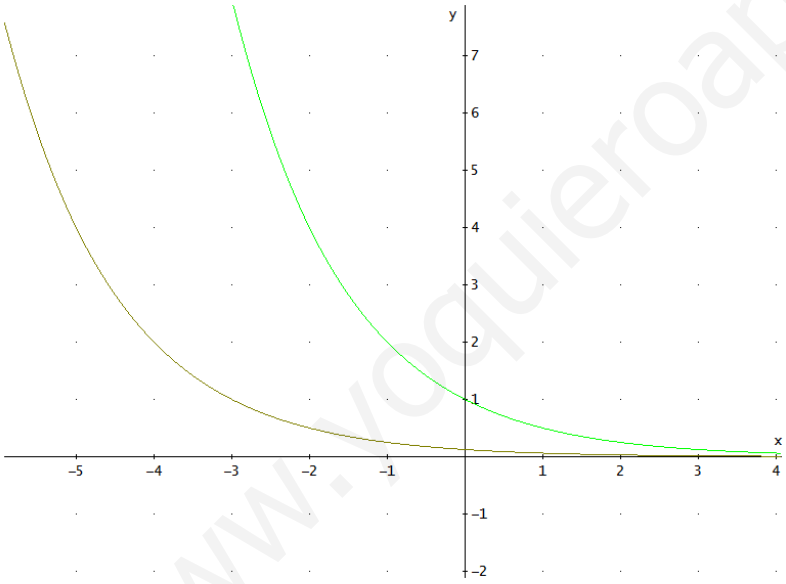

Dom=R

Range = $(-2,+\infty)$

Horizontal

Asymptote x = -2

$y=\left(\frac{1}{2}\right)^{x+3}$ the same graph, 3 units left




Dom=R

Range = $(0,+\infty)$

Horizontal

Asymptote x = 0