EJERCICIOS RESUELTOS DE FRACCIONES, POTENCIAS Y LOGARITMOS

1. Operaciones con fracciones:

a) $$\frac{-3-\left[-\frac{8}{2}-50\cdot\left(1-\frac{24}{25}\right)\right]}{-4-\left(\frac{1}{2}-1\right)}=\frac{-3-\left[-4-50\cdot\frac{1}{25}\right]}{-4-\left(-\frac{1}{2}\right)}=\frac{-3-[-6]}{-4+\frac{1}{2}}=\frac{3}{-\frac{7}{2}}=3:\left(-\frac{7}{2}\right)=-\frac{6}{7}$$

b) $$\frac{3-\left[-\frac{1}{2}-5\cdot\left(1-\frac{1}{2}\right)\right]}{-4+\frac{1}{2}}=\frac{3-\left[-\frac{1}{2}-\frac{5}{2}\right]}{-\frac{7}{2}}=\frac{3-\left[-\frac{6}{2}\right]}{-\frac{7}{2}}=\frac{3+3}{-\frac{7}{2}}=\frac{6}{-\frac{7}{2}}=-\frac{12}{7}$$

c) $$1-\frac{1-\frac{1}{10}}{1-\frac{1}{1-\frac{1}{10}}}=1-\frac{\frac{10-1}{10}}{1-\frac{1}{\frac{10-1}{10}}}=1-\frac{\frac{9}{10}}{1-\frac{1}{\frac{9}{10}}}=1-\frac{\frac{9}{10}}{1-\frac{10}{9}}=1-\frac{\frac{9}{10}}{\frac{9-10}{9}}=$$

$$=1-\frac{\frac{9}{10}}{\frac{-1}{9}}=1+\frac{9}{10}:\frac{1}{9}=1+\frac{81}{10}=\frac{10+81}{10}=\frac{91}{10}$$

d) $$\frac{\frac{\frac{2}{3}+\frac{1}{4}}{\frac{3}{4}-\frac{1}{2}}\cdot\frac{3-\frac{3}{2}}{2+\frac{1}{5}}}{5\cdot\frac{1}{2}}=\frac{\frac{\frac{11}{12}}{\frac{1}{4}}\cdot\frac{\frac{3}{2}}{\frac{11}{5}}}{\frac{5}{2}}=\frac{\frac{11\cdot4}{12}\cdot\frac{3\cdot5}{2\cdot11}}{\frac{5}{2}}=\frac{\cancel{11}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{5}\cdot\cancel{2}}{\cancel{12}\cdot\cancel{2}\cdot\cancel{11}\cdot\cancel{5}}=1$$

2. Operaciones con potencias:

a) $$16^{-2}\cdot4^{3}=\left(2^{4}\right)^{-2}\cdot\left(2^{2}\right)^{3}=2^{-8}\cdot2^{6}=2^{-8+6}=2^{-2}=\frac{1}{4}$$

b) $$\frac{4^{2}\cdot12^{3}\cdot15^{2}}{9^{3}\cdot8^{2}\cdot3^{3}}=\frac{\left(2^{2}\right)^{2}\cdot\left(2^{2}\cdot3\right)^{3}\cdot\left(3\cdot5\right)^{2}}{\left(3^{2}\right)^{3}\cdot\left(2^{3}\right)^{2}\cdot3^{3}}=\frac{2^{4}\cdot2^{6}\cdot3^{3}\cdot3^{2}\cdot5^{2}}{3^{6}\cdot2^{6}\cdot3^{3}}=\frac{2^{10}\cdot3^{5}\cdot5^{2}}{2^{6}\cdot3^{9}}=2^{4}\cdot3^{-4}\cdot5^{2}$$

c) $$\frac{8^{4}\cdot15^{3}\cdot18^{2}\cdot12^{-3}}{20^{3}\cdot27^{2}\cdot3^{-3}}=\frac{\left(2^{3}\right)^{4}\cdot\left(3\cdot5\right)^{3}\cdot\left(2\cdot3^{2}\right)^{2}\cdot\left(2^{2}\cdot3\right)^{-3}}{\left(2^{2}\cdot5\right)^{3}\cdot\left(3^{3}\right)^{2}\cdot3^{-3}}=$$

$$= \frac{2^{12}\cdot 3^3\cdot 5^3\cdot 2^2\cdot 3^4\cdot 2^{-6}\cdot 3^{-3}}{2^6\cdot 5^3\cdot 3^6\cdot 3^{-3}} = \frac{2^8\cdot 3^4\cdot 5^3}{2^6\cdot 3^3\cdot 5^3} = 2^2\cdot 3 = 12$$

d) $$\frac{27^{-1}\cdot 81\cdot 3^4\cdot\left(\frac{2^3}{3}\right)^{-1}\cdot 2^3}{36\cdot\left(\frac{1}{3}\right)^{-2}\cdot\frac{4}{3}\cdot\frac{27}{16}\cdot\left(2^0\right)^{-2}} = \frac{\left(3^3\right)^{-1}\cdot 3^4\cdot 3^4\cdot\frac{3}{2^3}\cdot 2^3}{3^2\cdot 2^2\cdot 3^2\cdot\frac{2^2}{3}\cdot\frac{3^3}{2^4}\cdot 1} = \frac{3^6}{3^6} = 1$$

3. Haz las siguientes operaciones con radicales:

a) $$\frac{\sqrt[4]{27}}{\sqrt[3]{18}} = \frac{\sqrt[4]{3^3}}{\sqrt[3]{2\cdot 3^2}} = \sqrt[12]{\frac{\left(3^3\right)^3}{\left(2\cdot 3^2\right)^4}} = \sqrt[12]{\frac{3^9}{2^4\cdot 3^8}} = \sqrt[12]{\frac{3}{2^4}} = \sqrt[12]{\frac{3}{16}}$$

b) $$\sqrt[4]{-80} : \sqrt[3]{18} = \frac{-\sqrt[4]{2^4\cdot 5}}{\sqrt[3]{2\cdot 3^2}} = -\frac{2\sqrt[4]{5}}{\sqrt[3]{2\cdot 3^2}} = \frac{2\sqrt[4]{5^3}}{\sqrt[4]{\left(2\cdot 3^2\right)^4}} = 2\cdot\sqrt[4]{\frac{5^3}{2^4\cdot 3^8}} =$$

$$= \frac{\cancel{2}}{\cancel{2}\cdot 3^2}\cdot\sqrt[4]{5^3} = \frac{\sqrt[4]{75}}{9}$$

c) $$\left(\sqrt[15]{-\frac{1}{243}}\right)^3 = \left(-\sqrt[15]{\frac{1}{3^5}}\right)^3 = -\sqrt[15]{\left(\frac{1}{3^5}\right)^3} = -\sqrt[5]{\frac{1}{3^{15}}} = -\frac{1}{3^3} = -\frac{1}{27}$$

d) $$\sqrt[3]{\sqrt{2}}\cdot\sqrt[3]{16} = \sqrt[6]{2}\cdot\sqrt[3]{16} = \sqrt[6]{2\cdot 16^2} = \sqrt[6]{2\cdot\left(2^4\right)^2} = \sqrt[6]{2^9} = \sqrt[6]{2^6\cdot 2^3} = 2\cdot\sqrt[6]{2^3} = 2\cdot\sqrt{2}$$

e) $$\sqrt[3]{\sqrt{2}}\cdot\sqrt[3]{16} = \sqrt[6]{2}\cdot\sqrt[3]{16} = \sqrt[6]{2\cdot 16^2} = \sqrt[6]{2\cdot\left(2^4\right)^2} = \sqrt[6]{2^9} = \sqrt[6]{2^6\cdot 2^3} = 2\cdot\sqrt[6]{2^3} = 2\cdot\sqrt{2}$$

f) $$\sqrt{\frac{3\sqrt{2}}{8}} = \sqrt{\frac{\sqrt{3^2\cdot 2}}{2\cdot 2^2}} = \frac{1}{2}\sqrt{\sqrt{\frac{3^2\cdot 2}{2^2}}} = \frac{1}{2}\sqrt[4]{\frac{3^2\cdot 2}{2^2}} = \frac{1}{2}\sqrt[4]{\frac{9}{2}}$$

g) $$\frac{\left(\sqrt[4]{3^2}\right)^2\cdot\left(\sqrt[3]{3}\right)^6}{\left(\sqrt[12]{3^4}\right)^6} = \frac{\sqrt[4]{3^4}\cdot\sqrt[3]{3^6}}{\sqrt[12]{3^{24}}} = \frac{3\cdot 3^2}{3^2} = 3$$

h) $$\frac{\left(\sqrt[5]{3}\right)^4\cdot\left(\sqrt[3]{3}\right)^2}{\left(\sqrt{3^4}\right)^3} = \frac{\sqrt[5]{3^4}\cdot\sqrt[3]{3^2}}{\sqrt{3^{12}}} = \frac{\sqrt[15]{\left(3^4\right)^3\cdot\left(3^2\right)^5}}{3^2} = \frac{\sqrt[15]{3^{12}\cdot 3^{10}}}{3^2} = \frac{\sqrt[15]{3^{22}}}{3^2} = \sqrt[15]{\frac{3^{22}}{3^{30}}} = \sqrt[15]{\frac{1}{3^8}}$$

i) $$\sqrt{\sqrt[3]{\sqrt[4]{64^4}}} = \sqrt{\sqrt[3]{\sqrt[4]{\left(2^6\right)^4}}} = \sqrt[2\cdot 3\cdot 4]{2^{24}} = \sqrt[24]{2^{24}} = 2$$

j) $$\sqrt{8} - \sqrt{50} - \frac{1}{2}\sqrt{98} = \sqrt{2^2\cdot 2} - \sqrt{2\cdot 5^2} - \frac{1}{2}\sqrt{7^2\cdot 2} = 2\sqrt{2} - 5\sqrt{2} - 7\sqrt{2} = -10\sqrt{2}$$

k) $\frac{1}{2}\sqrt{3}-\sqrt{12}-\frac{3}{4}\sqrt{75}=\frac{1}{2}\sqrt{3}-\sqrt{2^2\cdot 3}-\frac{3}{4}\sqrt{5^2\cdot 3}=\frac{1}{2}\sqrt{3}-2\sqrt{3}-\frac{3\cdot 5}{4}\sqrt{3}=$

$=\frac{1}{2}\sqrt{3}-2\sqrt{3}-\frac{15}{4}\sqrt{3}=-\frac{21}{4}\sqrt{3}$

l) $\sqrt{9xy}+\frac{xy}{\sqrt{4xy}}+\frac{\sqrt[6]{(xy)^{21}}}{x^3y^3}=3\sqrt{xy}-\frac{xy}{2\sqrt{xy}}-\sqrt[6]{\frac{(xy)^{21}}{(x^3y^3)^6}}=3\sqrt{xy}-\frac{1}{2}\sqrt{\frac{(xy)^2}{xy}}-\sqrt[6]{(xy)^3}=$

$=3\sqrt{xy}-\frac{1}{2}\sqrt{xy}-\sqrt{xy}=\frac{3}{2}\sqrt{xy}$

4. Racionaliza:

a) $\frac{1}{2\cdot\sqrt[3]{5}}=\frac{1}{2\cdot\sqrt[3]{5}}\frac{\sqrt[3]{5}}{\sqrt[3]{5}}\frac{\sqrt[3]{5}}{\sqrt[3]{5}}=\frac{\sqrt[3]{25}}{2\cdot 5}=\frac{\sqrt[3]{25}}{10}$

b) $\frac{1}{\sqrt[5]{x^4}}=\frac{1}{\sqrt[5]{x^4}}\left(\frac{\sqrt[5]{x^4}}{\sqrt[5]{x^4}}\right)^4=\frac{\left(\sqrt[5]{x^4}\right)^4}{\left(\sqrt[5]{x^4}\right)^5}=\frac{\left(\left(x^4\right)^{\frac{1}{5}}\right)^4}{x^4}=\frac{x^{\frac{16}{5}}}{x^4}=\frac{\sqrt[5]{x^{16}}}{x^4}=\frac{\sqrt[5]{x^{15}\cdot x}}{x^4}=\frac{x^3\cdot\sqrt[5]{x}}{x^4}=\frac{\sqrt[5]{x}}{x}$

c) $\frac{\sqrt[3]{x}}{\sqrt[6]{x^5}}=\frac{\sqrt[3]{x}}{\sqrt[6]{x^5}}\left(\frac{\sqrt[6]{x^5}}{\sqrt[6]{x^5}}\right)^5=\frac{\sqrt[3]{x}\left(\sqrt[6]{x^5}\right)^5}{\left(\sqrt[6]{x^5}\right)^6}=\frac{x^{\frac{1}{3}}\left(x^5\right)^{\frac{1}{6}\times 5}}{x^5}=\frac{x^{\frac{1}{3}}\cdot x^{\frac{25}{6}}}{x^5}=\frac{x^{\frac{2+25}{6}}}{x^5}=\frac{x^{\frac{27}{6}}}{x^5}=\frac{\sqrt[6]{x^{27}}}{x^5}=\frac{\sqrt[6]{x^3}}{x}$

d) $\frac{\sqrt{2}}{\sqrt{3}+1}=\frac{\sqrt{2}\cdot\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)\cdot\left(\sqrt{3}-1\right)}=\frac{\sqrt{2}\cdot\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2-1^2}=\frac{\sqrt{2}\cdot\left(\sqrt{3}-1\right)}{3-1}=\frac{\sqrt{2}\cdot\left(\sqrt{3}-1\right)}{2}$

e) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}=\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}\cdot\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}=\frac{\left(\sqrt{2}+\sqrt{3}\right)^2}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}=\frac{\left(\sqrt{2}+\sqrt{3}\right)^2}{2-3}=-\left(\sqrt{2}+\sqrt{3}\right)^2$

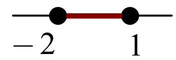
f) $\frac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}=\frac{\left(2\sqrt{3}+\sqrt{2}\right)\cdot\left(2\sqrt{3}+\sqrt{2}\right)}{\left(2\sqrt{3}-\sqrt{2}\right)\cdot\left(2\sqrt{3}+\sqrt{2}\right)}=\frac{\left(2\sqrt{3}\right)^2+2\cdot 2\sqrt{3}+\left(\sqrt{2}\right)^2}{\left(2\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{4\cdot 3+4\sqrt{3}+2}{4\cdot 3-2}=$

$=\frac{7+2\sqrt{6}}{5}$

5. Expresa mediante intervalos los siguientes conjuntos y represéntalos en la recta real.

a) $A = \{x / x \in \mathbb{R}, -2 \le x \le 1\}$

Solución:

Intevalo	Recta real
$[-2,1]$	

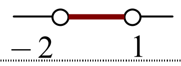
b) $B = \{x / x \in \mathbb{R}, -\infty < x < 1\}$

Solución:

Intevalo	Recta real
$(-\infty,1)$	

c) $C = \{x / x \in \mathbb{R}, -2 < x < 1\}$

Solución:

Intevalo	Recta real
$(-2,1)$	

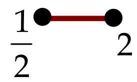
d) $D = \{x / x \in \mathbb{R}, -2 \le x < \infty\}$

Solución:

Intevalo	Recta real
$[-2,\infty)$	

e) $E = \left\{x / x \in \mathbb{R}, \frac{1}{2} \le x \le 2\right\}$

Solución:

Intevalo	Recta real
$\left[\frac{1}{2},2\right]$	

f) $F=\{x / x\in\mathbb{R}, 2\leq x<12\}$

Solución:

Intevalo	Recta real
$[2,12)$	

6. Teniendo en cuenta los conjuntos anteriores, escribe como un solo intervalo:

a) $A\cup B$ y $A\cap B$

Solución:

La unión de A y B son el conjunto de valores que pertenecen a A, a B o a ambos, mientras que la intersección son el conjunto de valores comunes a A y B. Entonces:

- $A\cup B=(-\infty,1]$
- $A\cap B=[-2,1)$

b) $C\cup D$ y $C\cap D$

Solución:

- $C\cup D=[-2,\infty)$
- $C\cap D=(-2,1)$

c) $E\cup F$ y $E\cap F$

Solución:

- $E\cup F=\left[\frac{1}{2},12\right)$
- $C\cap D=2$

7. Halla el valor de los siguientes logaritmos

a) $\log 10000 = \log 10^4 = 4\log 10 = 4$

b) $\log_2 64 = \log_2 2^6 = 6\log_2 2 = 6$

c) $\log_3 27 = \log_3 3^3 = 3\log_3 3 = 3$

d) $\log_{\frac{1}{3}} 27 = \log_{\frac{1}{3}} 3^3 = \log_{\frac{1}{3}}\left(\frac{1}{3}\right)^{-3} = -3\log_{\frac{1}{3}}\left(\frac{1}{3}\right) = -3$

e) $\log_5 \sqrt{\frac{1}{125}} = \log_5\left(\frac{1}{5^3}\right)^{\frac{1}{2}} = \log_5\left(5^{-3}\right)^{\frac{1}{2}} = \log_5 5^{-\frac{3}{2}} = -\frac{3}{2}\log_5 5 = -\frac{3}{2}$

f) $$\frac{\log_2 \sqrt[3]{2} - \log_2 \sqrt{2\sqrt{2}}}{\log_2 \sqrt{2}} - \frac{\log_2 \frac{1}{\sqrt{2}}}{1 - \log_2 \sqrt{\sqrt{2}}} = \frac{\log_2 2^{\frac{1}{3}} - \log_2 2^{\frac{3}{4}}}{\log_2 2^{\frac{1}{2}}} - \frac{\log_2 2^{-\frac{1}{2}}}{1 - \log_2 2^{\frac{1}{4}}} = \frac{\frac{1}{3} - \frac{3}{4}}{\frac{1}{2}} - \frac{-\frac{1}{2}}{1 - \frac{1}{4}} =$$

$$= \frac{\frac{1}{3} - \frac{3}{4}}{\frac{1}{2}} - \frac{-\frac{1}{2}}{1 - \frac{1}{4}} = -\frac{1}{6}$$

8. Calcula el valor del dato desconocido:

a) $\log_3 81 = x \Rightarrow 3^x = 81 \Rightarrow 3^x = 3^4 \Rightarrow x = 4$

b) $\log_5 a = 4 \Rightarrow 5^4 = a \Rightarrow a = 625$

c) $\log_a 1000 = 4 \Rightarrow a^4 = 10000 \Rightarrow a^4 = 10^4 \Rightarrow a = 10$

d) $\log_{32} \sqrt{2} = x \Rightarrow 32^x = \sqrt{2} \Rightarrow 2^{5x} = 2^{\frac{1}{2}} \Rightarrow 5x = \frac{1}{2} \Rightarrow x = \frac{1}{10}$

e) $\log_9 \frac{1}{81} = x \Rightarrow 9^x = \frac{1}{81} \Rightarrow 9^x = 9^{-2} \Rightarrow x = -2$

f) $\log_x 2 = \left(\frac{1}{5}\right)^{-1} \Rightarrow \log_x 2 = 5 \Rightarrow x^5 = 2 \Rightarrow \left(x^5\right)^{\frac{1}{5}} = 2^{\frac{1}{5}} \Rightarrow x^{\frac{5}{5}} = 2^{\frac{1}{5}} \Rightarrow x = \sqrt[5]{2}$

g) $\log_{\frac{1}{5}} x = -3 \Rightarrow \left(\frac{1}{5}\right)^{-3} = x \Rightarrow x = 5^3 = 125$

9. Escribe las siguientes expresiones como un solo logaritmo:

a) $\log(xy)-2\log\left(\frac{x}{y}\right)=\log(xy)-\log\left(\frac{x}{y}\right)^2=\log\left(\frac{xy}{\frac{x^2}{y^2}}\right)=\log\left(\frac{y^3}{x}\right)$

b) $2\ln(a-b)-\ln\left(a^2-b^2\right)=\ln(a-b)^2-\ln[(a+b)(a-b)]=$

$=\ln(a-b)^2-\ln[(a+b)(a-b)]=\ln\frac{(a-b)^{\cancel{2}}}{(a+b)\,\cancel{(a-b)}}=\ln\left(\frac{a-b}{a+b}\right)$

c) $4\log_2\frac{\sqrt{a-b}}{a}-\frac{1}{2}\log_2\left(\frac{a-b}{a}\right)^4=\log_2\left(\frac{\sqrt{a-b}}{a}\right)^4-\log_2\left[\left(\frac{a-b}{a}\right)^4\right]^{\frac{1}{2}}=$

$=\log_2\frac{(a-b)^2}{a^4}-\log_2\left(\frac{a-b}{a}\right)^2=\log_2\left[\frac{\frac{(a-b)^2}{a^4}}{\frac{(a-b)^2}{a^2}}\right]=\log_2\left[\frac{a^2(a-b)^2}{a^4(a-b)^2}\right]=\log_2\left(\frac{1}{a^2}\right)=$

$=\log_2\left(a^{-2}\right)$

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