EXAM 1_2 (Algebra - Polynomials)

Name:_____

1) Work out and simplify: (2.5 p)

a) $3(x-2)^2-2(x+3)(x-1)-(x+2)(x-2)=$

b) $\left[\frac{2x}{(x+2)^2}\div\frac{4x^2-4x}{3x+6}\right]\cdot\frac{x^2+x-2}{3x}=$

2) We know the expression $P(x)=x^4-px^2+2px-7$ has a remainder of 6 when divided by x + 1. Find the value of *p*. (1 p)

3) Factorise and simplify: (3 p)

a) $\frac{x^3-x^2-2x}{x^4-5x^2+4}$

b) $\frac{12a^2-3b^2}{24a^3-24a^2b+6ab^2}$

4) Find LCM and HCF of the polynomials: (3.5 p)

a) $P(x) = x^3 - 8x^2 + 20x - 16$ and $Q(x) = 2x^4 - x^3 - 5x^2 - 2x$

b) $P(x) = x^3 + 6x^2 + 9x$ and $Q(x) = 2x^4 - 18x^2$

SOLUTION

1) Work out and simplify:

a) $3(x-2)^2-2(x+3)(x-1)-(x+2)(x-2)=3\left(x^2-4x+4\right)-2\left(x^2+2x-3\right)-\left(x^2-4\right)=$

$=3x^2-12x+12-2x^2-4x+6-x^2+4=-16x+22$

b) $\left[\dfrac{2x}{(x+2)^2}\div\dfrac{4x^2-4x}{3x+6}\right]\cdot\dfrac{x^2+x-2}{3x}=\dfrac{2x\cdot(3x+6)}{(x+2)^2\left(4x^2-4x\right)}\cdot\dfrac{(x+2)(x-1)}{3x}=$

$=\dfrac{2x\cdot 3\cdot(x+2)(x+2)(x-1)}{4x\cdot(x+2)^2(x-1)\cdot 3x}=\dfrac{2}{4x}=\dfrac{1}{2x}$

$x^2+x-2=0$

$x=\dfrac{-1\pm\sqrt{1+8}}{2}=\dfrac{-1\pm 3}{2}=\begin{cases}1\\-2\end{cases}$

2) We know the expression $P(x)=x^4-px^2+2px-7$ has a remainder of 6 when divided by x + 1. Find the value of *p*.

Remainder Theorem: $P(-1)=(-1)^4-p(-1)^2+2p(-1)-7=1-p-2p-7=-3p-6=6$

$-3p-6=6\rightarrow -3p=12\rightarrow p=-4$

3) Factorise and simplify:

a) $\dfrac{x^3-x^2-2x}{x^4-5x^2+4}\rightarrow x^3-x^2-2x=x\left(x^2-x-2\right);\ x^2-x-2=0\rightarrow x=\dfrac{1\pm\sqrt{9}}{2}=\begin{cases}2\\-1\end{cases}$

$x^4-5x^2+4\rightarrow$ Factors of $4=1,-1,2,-2,4,-4$

$P(1)=1^4-5\cdot 1^2+4=0$

	1	0	-5	0	+4
1		+1	+1	-4	-4
	1	+1	-4	-4	0
-1		-1	0	+4	
	1	0	-4	0	

$Q(x)=x^3+x^2-4x-4$

$Q(1)=1^3+1^2-4-4\neq 0$

$Q(-1)=(-1)^3+(-1)^2-4(-1)-4=0$

$x^2-4=(x+2)(x-2)$

$\dfrac{x^3-x^2-2x}{x^4-5x^2+4}=\dfrac{x(x-2)(x+1)}{(x-1)(x+1)(x-2)(x+2)}=\dfrac{x}{(x-1)(x+2)}$

b) $\dfrac{12a^2-3b^2}{24a^3-24a^2b+6ab^2}=\dfrac{3\left(4a^2-b^2\right)}{6a\left(4a^2-4ab+b^2\right)}=\dfrac{3(2a+b)(2a-b)}{6a(2a-b)^2}=\dfrac{2a+b}{2a(2a-b)}$

4) Find LCM and HCF of the polynomials:

a) $P(x)=x^3-8x^2+20x-16$ and $Q(x)=2x^4-x^3-5x^2-2x$

$P(x)=x^3-8x^2+20x-16 \rightarrow \text{Factors}(-16)=\pm1,\pm2,\pm4,\pm8,\pm16$

$P(1)=1^3-8\cdot1^2+20-16\neq0$; $P(-1)=(-1)^3-8\cdot(-1)^2+20(-1)-16\neq0$

$P(2)=2^3-8\cdot2^2+20\cdot2-16=8-32+40-16=0$

	1	-8	+20	-16
2		2	-12	+16
	1	-6	+8	0

$x^2-6x+8=0 \rightarrow x=\dfrac{6\pm\sqrt{4}}{2}=\begin{cases}4\\2\end{cases}$

$Q(x)=2x^4-x^3-5x^2-2x=x\left(2x^3-x^2-5x-2\right) \rightarrow \text{Factors}(2)=\pm1,\pm2$

$Q(1)=2\cdot1^3-1^2-5\cdot1-2\neq0$; $Q(-1)=2(-1)^3-(-1)^2-5(-1)-2=0$

	2	-1	-5	-2
-1		-2	+3	+2
	2	-3	-2	0

$2x^2-3x-2=0 \rightarrow x=\dfrac{3\pm\sqrt{25}}{4}=\begin{cases}2\\-\dfrac{1}{2}\end{cases}$

$P(x)=x^3-8x^2+20x-16=(x-2)^2(x-4)$

$Q(x)=2x^4-x^3-5x^2-2x=2x(x+1)(x-2)\left(x+\dfrac{1}{2}\right)$

$\left.\begin{cases}LCM=2x(x-2)^2(x+1)(x-4)\left(x+\dfrac{1}{2}\right)\\HCF=(x-2)\end{cases}\right\}$

b) $P(x)=x^3+6x^2+9x$ and $Q(x)=2x^4-18x^2$

$P(x)=x^3+6x^2+9x=x\left(x^2+6x+9\right)=x(x+3)^2$

$Q(x)=2x^4-18x^2=2x^2\left(x^2-9\right)=2x^2(x+3)(x-3)$

$\left.\begin{cases}LCM=2x^2(x+3)^2(x-3)\\HCF=x(x+3)\end{cases}\right\}$