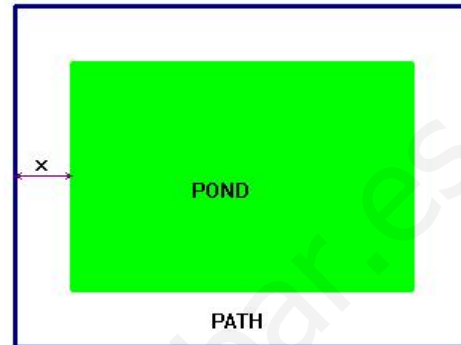


2nd TERM GENERAL EXAM

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)

1. A rectangular pond (6m x 4m), is surrounded by a uniform path of width x metres. The area of the path is equal to the area of the pond. Find x .



2. Solve the equation: $x + \sqrt{3x+10} = 6$

3. Solve: $\frac{x+5}{x+3} = 1 - \frac{x^2+3x+6}{x^2+2x-3}$

4. Solve by substitution and graphically : $\left. \begin{array}{l} 2x - y - 1 = 0 \\ y = -\frac{2}{x+1} \end{array} \right\}$

5. Solve the inequality: $\frac{(x-2)(x+2)}{4} - \frac{x-6}{2} \leq \frac{10+x}{5}$

6. Solve the system of inequalities: $\left. \begin{array}{l} 2x + \frac{3y}{4} \leq 5 \\ 10x - y > 6 \end{array} \right\}$

7. Sketch the graph of the compound function:

$$f(x) = \begin{cases} 1-x & x < -3 \\ 4 & -3 < x < 2 \\ 1 + \log_2 x & x \geq 2 \end{cases}$$

- a) Domain and range
b) Continuity

8. Calculate x in the following equations:

a) $3^{x-2} = \frac{1}{27}$

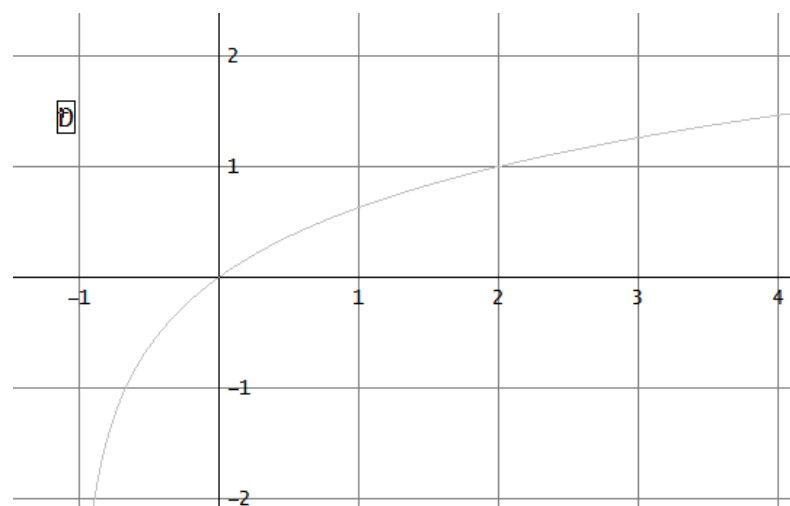
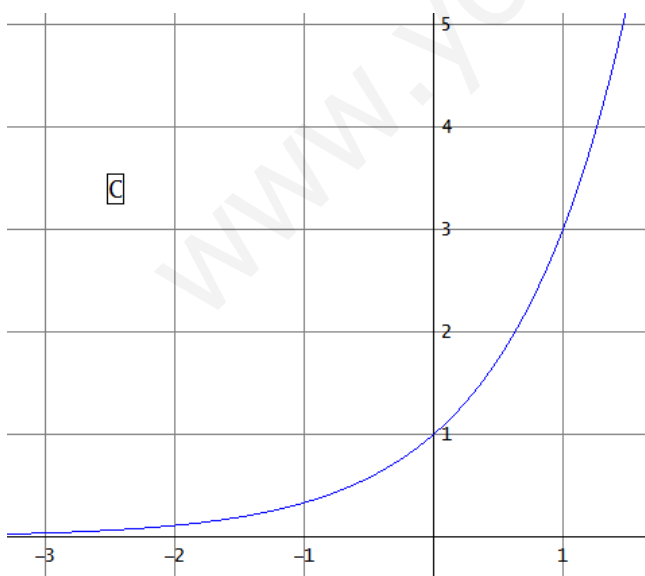
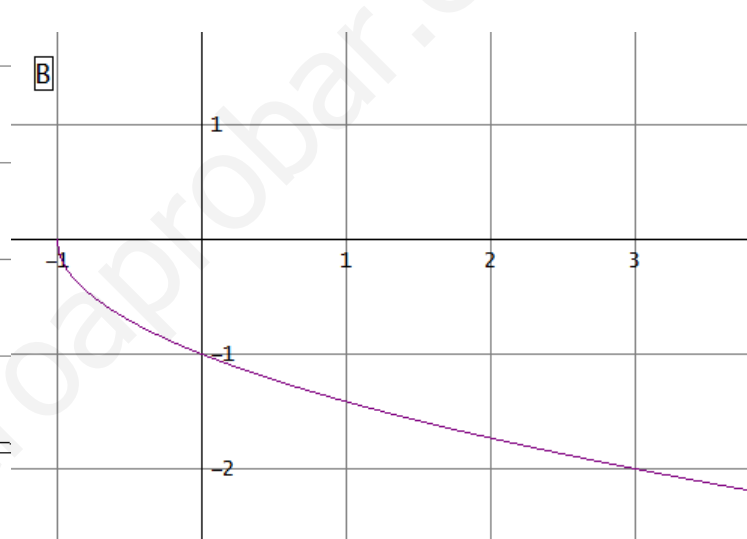
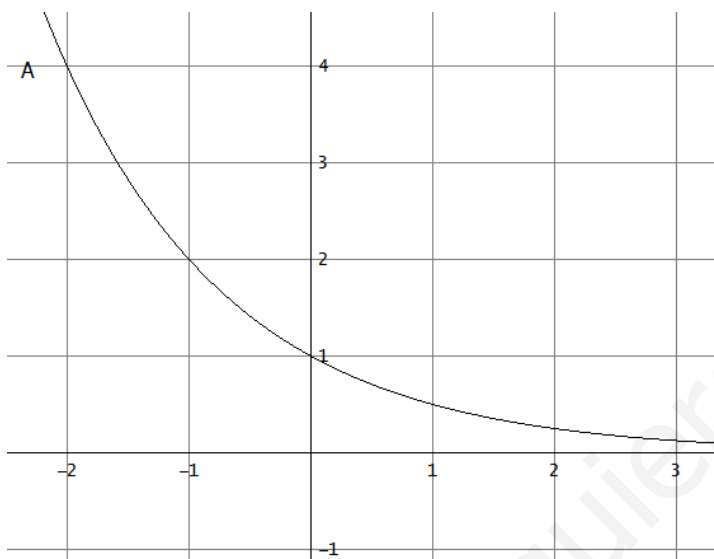
b) $\log_{(2x+1)} 49 = 2$

9. A ball is thrown in the air so that t seconds after it is thrown, its height h metres above its starting point is given by the function $h(t) = 25t - 5t^2$.

Draw the graph of the function for $0 \leq t \leq 6$ and find:

- The time when the ball is at its greatest height.
- The greatest height reached by the ball.
- The interval of time during which the ball is at a height of more than 30 m.

10. The diagrams show the graphs of four functions. Write their formulas (explaining why)



SOLUTION

1. Area of the pond $A = 6 \cdot 4 = 24\text{m}^2$

Area of the path 24m^2

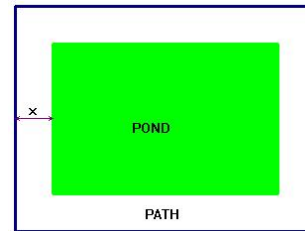
Area of the path+pond 48m^2

$A = (6 + 2x)(4 + 2x) = 48$

$24 + 12x + 8x + 4x^2 = 48$

$4x^2 + 20x - 24 = 0$

$$x = \frac{-20 \pm \sqrt{400 + 384}}{8} = \frac{-20 \pm 28}{8} = \begin{cases} 1 \\ -6 \end{cases} \text{ the width of the path is 1 metre}$$



2. $x + \sqrt{3x + 10} = 6 \rightarrow \sqrt{3x + 10} = 6 - x$

$(\sqrt{3x + 10})^2 = (6 - x)^2 \Rightarrow 3x + 10 = 36 - 12x + x^2 \rightarrow x^2 - 15x + 26 = 0$

$$x = \frac{15 \pm \sqrt{225 - 104}}{2} = \frac{15 \pm 11}{2} = \begin{cases} 13 \\ 2 \end{cases}$$

Check: $x = 13 \rightarrow 13 + \sqrt{39 + 10} = 6 \rightarrow 13 + 7 = 6$ NO

Solution: $x = 2$

$x = 2 \rightarrow 2 + \sqrt{6 + 10} = 6 \rightarrow 2 + 4 = 6$ ✓

3. $\frac{x+5}{x+3} = 1 - \frac{x^2 + 3x + 6}{x^2 + 2x - 3}$ factor: $x^2 + 2x - 3 = 0 \rightarrow x = \begin{cases} 1 \\ -3 \end{cases}$

$x^2 + 2x - 3 = (x - 1)(x + 3) \rightarrow \text{LCF} = (x - 1)(x + 3)$

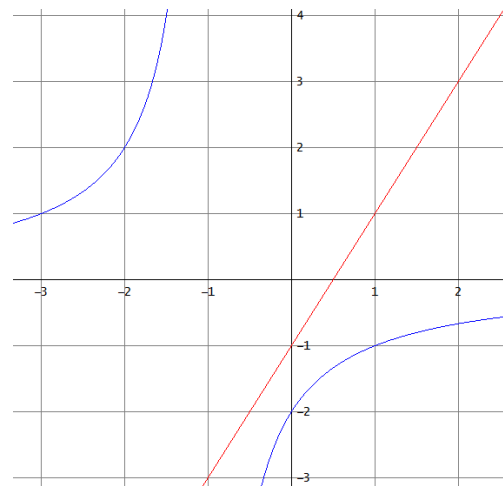
$$\frac{(x-1)(x+5)}{(x-1)(x+3)} = \frac{(x-1)(x+3)}{(x-1)(x+3)} - \frac{x^2 + 3x + 6}{(x-1)(x+3)} \rightarrow x^2 + 4x - 5 = x^2 + 2x - 3 - x^2 - 3x - 6$$

$$x^2 + 5x + 4 = 0 \rightarrow x = \frac{-5 \pm \sqrt{25 - 16}}{2} = \begin{cases} -1 \\ -4 \end{cases}$$

4.
$$\left. \begin{array}{l} 2x - y - 1 = 0 \\ y = -\frac{2}{x+1} \end{array} \right\} \rightarrow 2x + \frac{2}{x+1} - 1 = 0 \Rightarrow 2x(x+1) + 2 - (x+1) = 0$$

$2x^2 + x + 2 - x - 1 = 0 \Rightarrow 2x^2 + 1 = 0 \Rightarrow 2x^2 = -1$ No solution, inconsistent system

$$\left. \begin{array}{l} y = 2x - 1 \\ y = -\frac{2}{x+1} \end{array} \right\} \begin{array}{l} \text{straight line} \\ \text{hyperbola} \end{array}$$



$$5. \frac{(x-2)(x+2)}{4} - \frac{x-6}{2} \leq \frac{10+x}{5} \rightarrow \frac{x^2-4}{4} - \frac{x-6}{2} \leq \frac{10+x}{5}$$

$$\frac{5x^2-20}{20} - \frac{10x-60}{20} \leq \frac{40+4x}{20} \rightarrow 5x^2-20-10x+60 \leq 40+4x$$

$$5x^2 - 14x = 0 \rightarrow x(5x - 14) = 0 \rightarrow \begin{cases} x_1 = 0 \\ 5x - 14 = 0 \rightarrow x_2 = \frac{14}{5} \end{cases}$$

We study the sign:

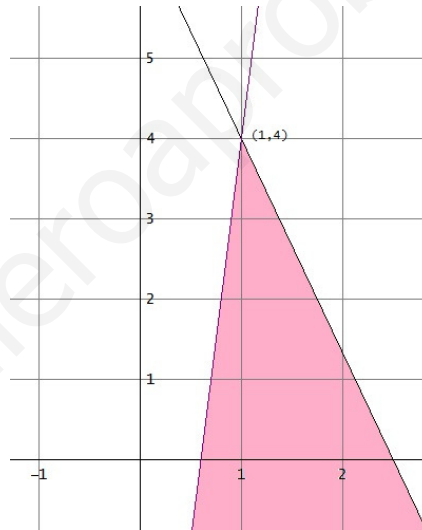


Solution: $\left[0, \frac{14}{5}\right]$

$$6. \left. \begin{array}{l} 2x + \frac{3y}{4} \leq 5 \\ 10x - y > 6 \end{array} \right\} \text{By graphing:}$$

$y = \frac{20-8x}{3}$	x	1	-2
	y	4	12

$y = 10x - 6$	x	1	0
	y	4	-6



$$7. f(x) = \begin{cases} 1-x & x < -3 \\ 4 & -3 < x < 2 \\ 1 + \log_2 x & x \geq 2 \end{cases}$$



Domain: $\mathbb{R} - \{-3\}$

Range: $[2, +\infty)$

Removable discontinuity
in $x = -3$ and jump
discontinuity in $x = 2$

8.

a) $3^{x-2} = \frac{1}{27} \Rightarrow 3^{x-2} = 3^{-3} \Rightarrow x-2 = -3 \Rightarrow x = -1$

b) $\log_{(2x+1)} 49 = 2 \Leftrightarrow (2x+1)^2 = 49 = 7^2 \Rightarrow 2x+1 = 7 \Rightarrow x = 3$

9. A ball is thrown in the air so that t seconds after it is thrown, its height h metres above its starting point is given by the function $h(t) = 25t - 5t^2$.

Draw the graph of the function for $0 \leq t \leq 6$ and find:

- The time when the ball is at its greatest height.
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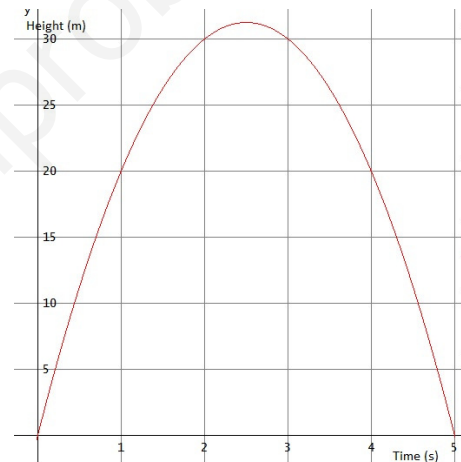
$$y = -5x^2 + 25x \text{ it is a parabola } \cap \text{ with vertex } \rightarrow x = -\frac{25}{-10} = \frac{5}{2}$$

So the ball is at its greatest height 2.5 seconds after it is thrown

The greatest height is for

$$x = \frac{5}{2} \rightarrow y = -5\left(\frac{5}{2}\right)^2 + 25 \cdot \frac{5}{2} = \frac{125}{4} = 31.25 \text{ m}$$

The ball is at a height of more than 30 metres between 2 and 3 seconds after the ball is thrown.



10. Graph A: It is an exponential function with base < 1 (decreasing) and it passes through the point $(-1, 2)$, so the function is $y = \left(\frac{1}{2}\right)^x$

Graph B: It is a radical function with Domain $[-1, +\infty)$ and negative. It passes through the point $(3, -2)$, so the function is $y = -\sqrt{x+1}$

Graph C: It is an exponential function with base > 1 (increasing). It passes through the point $(1, 3)$, so the function is $y = 3^x$

Graph D: It is a logarithmic function with Domain $(-1, +\infty)$ (Type $\log_a(x+1)$). It passes through the points $(0,0)$ and $(2,1)$, so $y = \log_2(x+1)$