EXAM 2_1 (Algebra: Inequalities, Word Problems)

1. Solve the following inequality and draw the solutions on the number line:

$$\frac{3-x}{4} - \frac{x+1}{6} < 2 - \frac{2-x}{3}$$ (1 point)

2. Solve the following systems of inequalities: (3 points)

a) $\left. \begin{array}{l} \dfrac{x-15}{2} \le 5 - 2x \\ 2 - x < \dfrac{1-x}{2} \end{array} \right\}$

b) $\left. \begin{array}{l} 2x - 3(x+1) \ge 2 \\ -2(x-2) + 5x < 1 \end{array} \right\}$

c) $\left. \begin{array}{l} x - 2y \ge 2 - y \\ -3x < 5 + y \end{array} \right\}$

3. Solve: (1.5 points)

$$\left. \begin{array}{l} \left(1 + x^2\right)y^2 = 5 \\ 4x - y = 0 \end{array} \right\}$$

4. Some kilograms of olive oil, which costs €5/kg, are combined with sunflower oil which costs €2.5/kg, to obtain 40 kg of a mixture which costs E3.25/kg How many kilograms of each type will we have to put into the mixture?   (1.5 points)

5. A furniture shop sold a total of 315 sofas and tables. A sofa sold for €2300 and a table for €890 euro. The total sales were € 401 610. How many tables were sold?
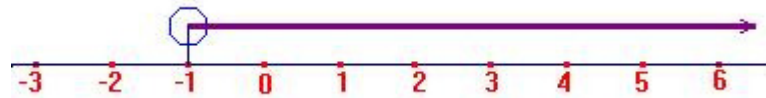   (1.5 points)

6. The area of a rectangle is 91 cm² and its perimeter is 40 cm. Find the base and the height of the rectangle.   (1.5 points)
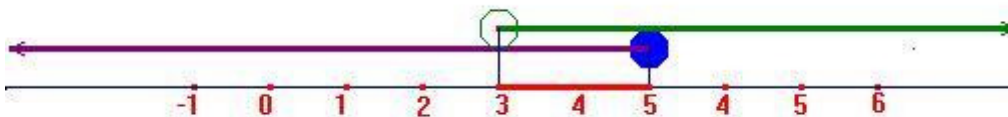
SOLUTION

1. $\dfrac{3-x}{4} - \dfrac{x+1}{6} < 2 - \dfrac{2-x}{3} \rightarrow \dfrac{9-3x}{12} - \dfrac{2x+2}{12} < \dfrac{24}{12} - \dfrac{8-4x}{12}$

$9 - 3x - 2x - 2 < 24 - 8 + 4x \rightarrow -9x < 9 \rightarrow 9x > -9 \rightarrow x > -1$

Solution:
$(-1, +\infty)$

2. a) $\left. \begin{array}{l} \dfrac{x-15}{2} \leq 5 - 2x \\[2mm] 2 - x < \dfrac{1-x}{2} \end{array} \right\} \rightarrow \left. \begin{array}{l} x - 15 \leq 10 - 4x \\ 4 - 2x < 1 - x \end{array} \right\} \rightarrow \left. \begin{array}{l} x + 4x \leq 10 + 15 \\ -2x + x < 1 - 4 \end{array} \right\} \rightarrow \left. \begin{array}{l} x \leq 5 \\ x > 3 \end{array} \right\}$

Solution: $(3, 5]$

b) $\left. \begin{array}{l} 2x - 3(x+1) \geq 2 \\ -2(x-2) + 5x < 1 \end{array} \right\} \rightarrow \left. \begin{array}{l} 2x - 3x - 3 \geq 2 \\ -2x + 4 + 5x < 1 \end{array} \right\} \rightarrow \left. \begin{array}{l} -x \geq 5 \\ 3x < -3 \end{array} \right\} \rightarrow \left. \begin{array}{l} x \leq -5 \\ x < -1 \end{array} \right\}$
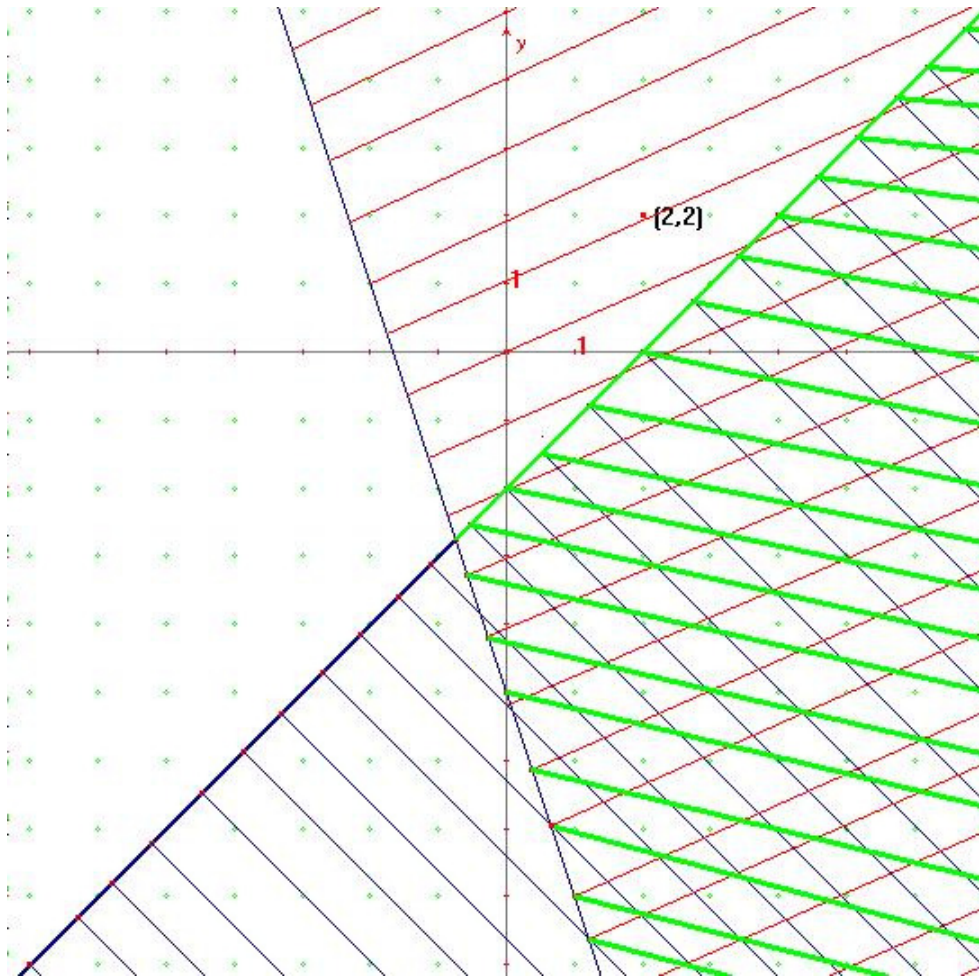

Solution: $(-\infty, -5]$

c) $\left. \begin{array}{l} x - 2y \geq 2 - y \\ -3x < 5 + y \end{array} \right\} \rightarrow \left\{ \begin{array}{l} x - 2y = 2 - y \\ -3x = 5 + y \end{array} \right. \rightarrow \left\{ \begin{array}{l} y = x - 2 \\ y = -3x - 5 \end{array} \right.$

$y = x - 2$	x	0	2
	y	-2	0

$y = -3x - 5$	x	0	-2
	y	-5	1

Point (2,2) $\left. \begin{array}{l} 2 - 2 \cdot 2 \geq 2 - 2 \\ -3 \cdot 2 < 5 + 2 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} -2 \geq 0 \rightarrow NO \\ -6 < 7 \rightarrow SI \end{array} \right.$

Solution: GREEN

3. $\left. \begin{array}{l} \left(1+x^2\right)y^2 = 5 \\ 4x - y = 0 \end{array} \right\} \rightarrow \left. \begin{array}{l} y^2 + x^2 y^2 = 5 \\ y = 4x \end{array} \right\} \rightarrow (4x)^2 + x^2(4x)^2 = 5$

$16x^4 + 16x^2 - 5 = 0 \rightarrow z = x^2 \rightarrow 16z^2 + 16z - 5 = 0$

$z = \dfrac{-16 \pm \sqrt{256 + 320}}{32} = \dfrac{-16 \pm 24}{32} = \begin{cases} \dfrac{1}{4} \rightarrow x = \pm\sqrt{\dfrac{1}{4}} = \pm\dfrac{1}{2} \\[3mm] -\dfrac{5}{4} \rightarrow x = \pm\sqrt{-\dfrac{5}{4}} \rightarrow NO \end{cases}$

$x = \dfrac{1}{2} \rightarrow y = 4 \cdot \dfrac{1}{2} = 2 \, ; \, x = -\dfrac{1}{2} \rightarrow y = 4 \cdot -\dfrac{1}{2} = -2$

Solution: $\begin{cases} x = \dfrac{1}{2}; \ y = 2 \\ x = -\dfrac{1}{2}; \ y = -2 \end{cases}$

4. Some kilograms of olive oil, which costs €5/kg, are combined with sunflower oil which costs €2.5/kg, to obtain 40 kg of a mixture which costs E3.25/kg How many kilograms of each type will we have to put into the mixture?

	Olive oil	Sunflower oil	Mixture
kilograms	x	40 – x	40
Price/kg	5	2.5	3.25

Equation:

$5x + 2.5(40 - x) = 3.25 \cdot 40 \Rightarrow 5x + 100 - 2.5x = 130 \rightarrow 2.5x = 30 \rightarrow x = 12$

Answer- We needed 12 kg of olive oil and 28 kg of sunflower oil

5. A furniture shop sold a total of 315 sofas and tables. A sofa sold for €2300 and a table for €890 euro. The total sales were € 401 610. How many tables were sold?

Number of sofas x; Number of tables y

$\left. \begin{array}{l} x + y = 315 \\ 2300x + 890y = 401610 \end{array} \right\} \rightarrow \left. \begin{array}{l} y = 315 - x \\ 230x + 89y = 40161 \end{array} \right\}$

$230x + 89(315 - x) = 40161 \rightarrow 230x - 89x = 40161 - 28035$

$141x = 12126 \rightarrow x = 86 \rightarrow y = 315 - x = 315 - 86 = 229$

Answer: They have sold 86 sofas and 229 tables

6. The area of a rectangle is 91 cm^2 and its perimeter is 40 cm. Find the base and the height of the rectangle.

Base x Height y

Equations: $\left. \begin{array}{l} x \cdot y = 91 \\ 2x + 2y = 40 \end{array} \right\} \rightarrow \left. \begin{array}{l} x \cdot y = 91 \\ y = \dfrac{40 - 2x}{2} = 20 - x \end{array} \right\} x(20 - x) = 91$

$x^2 - 20x + 91 = 0 \rightarrow x = \dfrac{20 \pm \sqrt{400 - 364}}{2} = \begin{cases} 13 \rightarrow y = 20 - 13 = 7 \\ 7 \rightarrow y = 20 - 7 = 13 \end{cases}$

Answer: Base 13 cm and height 7 cm