(Trigonometry- Equations)

Name:……….

1. Suppose that $\tan \alpha = \dfrac{3}{4}$ and α lies in quadrant III. Find the other trigonometric ratios for α. Draw the angle α.   (2 points)

2. In a right-angled triangle the length of a leg is twice the other. Calculate the trigonometric ratios of the smallest angle.   (1.75 points)

3. A woodcutter wants to determine the height of a tall tree. He stands at some distance from the tree and determines that the angle of elevation to the top of the tree is 40°. He moves 30 metres closer to the tree, and now the angle of elevation is 50°. If the woodcutter's eyes are 1.5m above the ground, how tall is the tree?   (2 points)

4. In an isosceles triangle, the base is 12 metres long and the congruent angles are 70° each. Find the length of the other sides and the area.   (2 points)

5. Solve: (2.25 points)

a) $x + \sqrt{3x + 10} = 6$

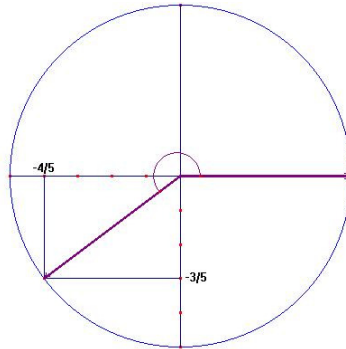b) $x + \dfrac{1}{x} = \dfrac{4}{\sqrt{3}}$

c) $\left(2x^4 - 3x^2 - 20\right)(3x - 2) = 0$

SOLUTION

1. Suppose that $\tan\alpha = \dfrac{3}{4}$ and α lies in quadrant III. Find the other

trigonometric ratios for α. Draw the angle α.

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \rightarrow 1 + \left(\frac{3}{4}\right)^2 = \frac{1}{\cos^2\alpha} \rightarrow \frac{25}{16} = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{16}{25}$$

$$\cos\alpha = \pm\sqrt{\frac{16}{25}} \rightarrow \cos\alpha = -\frac{4}{5} \rightarrow \tan\alpha = \frac{\sin\alpha}{\cos\alpha} \rightarrow \sin\alpha = \tan\alpha \cdot \cos\alpha$$

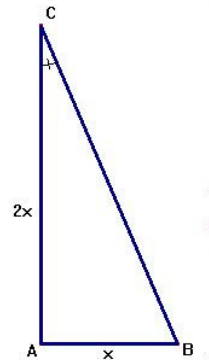$$\sin\alpha = \frac{3}{4} \cdot -\frac{4}{5} = -\frac{3}{5}$$

2. In a right-angled triangle the length of a leg is twice the
other. Calculate the trigonometric ratios of the smallest
angle. $h^2 = x^2 + (2x)^2 = 5x^2 \rightarrow h = \sqrt{5}x$

$$\sin\alpha = \frac{x}{\sqrt{5}x} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos\alpha = \frac{2x}{\sqrt{5}x} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan\alpha = \frac{x}{2x} = \frac{1}{2}$$

3. A woodcutter wants to determine the height of a tall tree. He stands at some
distance from the tree and determines that the angle of elevation to the top of
the tree is 40°. He moves 30 metres closer to the tree, and now the angle of
elevation is 50°. If the woodcutter's eyes are
1.5m above the ground, how tall is the tree?

$$\left. \begin{array}{l} \tan 40 = \dfrac{y}{x+30} \\ \tan 50 = \dfrac{y}{x} \end{array} \right\} \left. \begin{array}{l} y = (x+30)\tan 40 \\ y = x\tan 50 \end{array} \right\}$$

$x\tan 40 + 30\tan 40 = x\tan 50$
$x(\tan 50 - \tan 40) = 30\tan 40$
$x = 71.38m \rightarrow y = 71.38 \cdot \tan 50 = 85.07m$
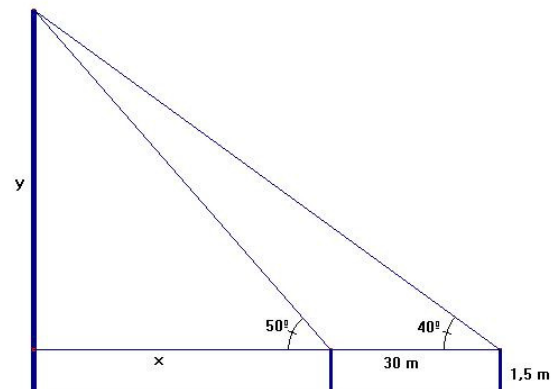
The tree is 85.07+1.5=86.57 metres tall

4. In an isosceles triangle, the base is 12 metres long and the congruent angles are 70° each. Find the length of the other sides and the area.
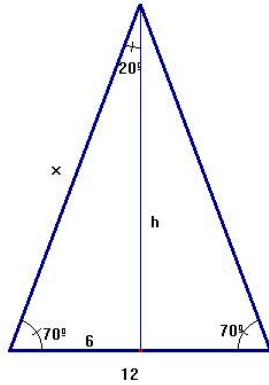
$$\cos 70 = \frac{6}{x} \Rightarrow x = \frac{6}{\cos 70} = 17.54\,\text{m}$$

The other sides are 17.54 metres long

$$\tan 70 = \frac{h}{6} \Rightarrow h = 6\tan 70 = 16.48\,\text{m}$$

Area: $A = \dfrac{12 \cdot 16.48}{2} = 98.9\,\text{m}^2$

5. Solve:

a) $x + \sqrt{3x + 10} = 6 \rightarrow \sqrt{3x + 10} = 6 - x \Rightarrow 3x + 10 = 36 - 12x + x^2$

$x^2 - 15x + 26 = 0 \rightarrow x = \dfrac{15 \pm \sqrt{121}}{2} = \begin{cases} 13 \\ 2 \end{cases}$

Checking: $\begin{array}{l} x = 13 \rightarrow \sqrt{39 + 10} = 6 - 13 \Rightarrow 7 = -7 \text{ NO} \\ x = 2 \rightarrow \sqrt{6 + 10} = 6 - 4 \Rightarrow 4 = 4 \text{ YES} \end{array}$ **Sol: x = 2**

b) $x + \dfrac{1}{x} = \dfrac{4}{\sqrt{3}} \rightarrow \sqrt{3}x^2 + \sqrt{3} = 4x \rightarrow \sqrt{3}x^2 - 4x + \sqrt{3} = 0$

$x = \dfrac{4 \pm \sqrt{16 - 4\sqrt{3}\sqrt{3}}}{2\sqrt{3}} = \dfrac{4 \pm 2}{2\sqrt{3}} \begin{cases} \dfrac{6}{2\sqrt{3}} = \dfrac{3}{\sqrt{3}} = \sqrt{3} \\ \dfrac{2}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} \end{cases}$

c) $\left(2x^4 - 3x^2 - 20\right)(3x - 12) = 0 \rightarrow \begin{cases} 2x^4 - 3x^2 - 20 = 0 \rightarrow (*) \\ 3x - 12 = 0 \rightarrow x = 4 \end{cases}$

$(*)\, 2x^4 - 3x^2 - 20 = 0,\, z = x^2 \rightarrow 2z^2 - 3z - 20 = 0 \rightarrow z = \dfrac{3 \pm \sqrt{169}}{4} = \begin{cases} 4 \\ -\dfrac{5}{2} \end{cases}$

$x^2 = z = \begin{cases} 4 \rightarrow x = \pm\sqrt{4} = \pm 2 \\ -\dfrac{5}{2} \rightarrow x = \pm\sqrt{-\dfrac{5}{2}} \text{ NO} \end{cases}$ **Solution: x = 4 , x = 2, x = -2**