

Relación – Ecuaciones

Ecuaciones de primer grado

Resolver las siguientes ecuaciones:

$$[1] \frac{5(x+1)}{24} = \frac{x+3}{3} - \frac{5x+9}{8} + \frac{3x+2}{6}$$

$$[2] 3 \cdot \{3 \cdot [3(3x-2) - 2] - 2\} - 2 = 1$$

$$[3] \frac{x+2}{2} - \frac{1-2x}{7} + \frac{3x+5}{4} = \frac{11-x}{14}$$

$$[4] 5x - 3(x-8) = 2(x+4)$$

$$[5] 4 - \frac{x+3}{6} = 2 + \frac{9-2x}{3}$$

$$[6] \frac{5x-3}{6} - \frac{7x-1}{4} = \frac{4x+2}{7} - 5$$

$$[7] 5 - 4\left(x - \frac{1}{2}\right) = 0'8\left(x - \frac{7}{4}\right)$$

$$[8] x + 2(x-1) = 4$$

$$[9] \frac{x-2}{8} - \frac{2(2x+6)}{3} + x = -4$$

$$[10] \frac{6-3x}{3} + \frac{(x-1)(x+2)}{4} = \frac{x^2}{4}$$

$$[11] \frac{x-3}{2} - \frac{x-8}{12} = \frac{5-x}{4} - \frac{x}{3}$$

$$[12] \frac{1}{6} \left[2x - 1 - 3 \left(\frac{5x}{3} - 1 \right) \right] + 2(x-3) + 6 = \frac{1}{3}$$

$$[13] \frac{3(x+4)}{4} - \left[\frac{x-1}{4} + \frac{2x-3}{3} - \frac{(x-1)(x+3)}{2} \right] + \frac{(x-3)(2-x)}{2} = 0$$

$$[14] \left(x + \frac{5}{2}\right) \left(x - \frac{3}{2}\right) - (x+5)(x-3) = 3 \left(3 + \frac{1}{4}\right)$$

$$[15] x + \frac{3x-1}{4} - \frac{x+1}{5} + 1 - \frac{x-2}{10} = 2(x+1)$$

$$[16] (x-2)^2 - (x-1)(x+1) - \frac{x-2}{3} = \frac{x+2}{2}$$

$$[17] \frac{x+2}{2} - \frac{1-2x}{7} = \frac{11-x}{14} - \frac{3x+5}{4}$$

Soluciones:

- | | | | | |
|----------------------|-----------------------|----------------------|--------------------|-----------------------|
| [1] Identidad | [2] 1 | [3] $-\frac{37}{45}$ | [4] Sin solución | [5] 3 |
| [6] 3 | [7] $\frac{7}{4}$ | [8] 2 | [9] $-\frac{6}{5}$ | [10] 2 |
| [11] $\frac{25}{12}$ | [12] 0 | [13] $\frac{3}{40}$ | [14] $\frac{3}{2}$ | [15] $-\frac{25}{11}$ |
| [16] $\frac{28}{29}$ | [17] $-\frac{37}{45}$ | | | |

Ecuaciones de segundo grado

1. Resolver las siguientes ecuaciones de segundo grado:

- | | |
|--|---|
| [1] $7x^2 = 63$ | [2] $x^2 - 24 = 120$ |
| [3] $\frac{x^2}{5} = x$ | [4] $5x^2 - 3 = 42$ |
| [5] $3x = 4x^2 - 2x$ | [6] $8x^2 - 56 = 34 - 2x^2$ |
| [7] $(x + 1)(x - 3) + 3 = 0$ | [8] $(x + 9)(x - 9) = 3(x - 27)$ |
| [9] $(2x + 3)(2x - 3) = 135$ | [10] $3(2x - 3)^2 = 4x(2x - 9) + 43$ |
| [11] $12x^2 - 18 = 0$ | [12] $3(x^2 - 2) = 21$ |
| [13] $4x^2 + 12 + 9 = 0$ | [14] $x^2 - 7x + 12 = 0$ |
| [15] $x^2 - 9x + 18 = 0$ | [16] $x^2 - 9x + 14 = 0$ |
| [17] $x^2 - 6x + 8 = 0$ | [18] $x^2 - 6x + 9 = 0$ |
| [19] $x^2 + 6x - 27 = 0$ | [20] $x^2 + x + \frac{1}{4} = 0$ |
| [21] $x(3x - 2) = 65$ | [22] $x(x - 3) + 1 = 5(x - 3)$ |
| [23] $x^2 - \frac{x}{2} = \frac{1}{3} - \frac{2x}{3}$ | [24] $\frac{x^2}{4} + 2 = \frac{3x}{2}$ |
| [25] $x + \frac{1}{x} = \frac{65}{8}$ | [26] $x - 2 = \frac{4x - 9}{x}$ |
| [27] $\frac{x}{2} + \frac{3}{x} = \frac{x + 13}{3x}$ | [28] $(x + 1)(x - 3) = 8x - 4(x - 1)$ |
| [29] $x(x + 1) + \frac{x + \frac{3}{2}}{4} = 0$ | [30] $8x + 11 - \frac{7}{x} = \frac{21 + 63x}{7}$ |
| [31] $\frac{x^2}{2} - \frac{3x}{5} = \frac{1}{5} - \frac{x}{6}$ | [32] $2x^2 + \frac{6}{5} = x \left(x + \frac{31}{5} \right)$ |
| [33] $\frac{5x^2}{8} - \frac{3x}{5} = \frac{x^2}{8} - \frac{x}{6} + \frac{1}{5}$ | [34] $\frac{5x^2}{6} - \frac{x}{2} + \frac{3}{4} = 8 - \frac{2x}{3} - x^2 + \frac{273}{12}$ |
| [35] $2x^2 + 5x + 3 = 0$ | [36] $x^2 = 5x + 6$ |
| [37] $x^2 - 5x + 4 = 5(8 - x)$ | [38] $5x^2 - 8x - \frac{3x^2}{4} = 0$ |
| [39] $(2x + 7)(2x - 7) = x^2 - 1$ | [40] $3x^2 = (4x + 8)(x - 2)$ |
| [41] $\frac{5x^2}{3} = x^2 + 24$ | [42] $8x^2 - 10x - \frac{3x^2}{7} - 43x = 0$ |

2. Resolver las siguientes ecuaciones de segundo grado:

$$[43] \frac{4x^2}{5} + x^2 - 5 = \frac{x^2}{5} + 10 + x^2$$

$$[44] x^2 + 6x = -9$$

$$[45] x^2 + 9 = 10x$$

$$[46] 2x^2 + 10x - 48 = 0$$

$$[47] x^2 - x = 20$$

$$[48] x^2 - 5x + 6 = 0$$

$$[49] x^2 + 8x + 15 = 0$$

$$[50] x^2 + 10x + 25 = 0$$

$$[51] 3x^2 - 39x + 108 = 0$$

$$[52] 6x^2 - 37x + 57 = 0$$

$$[53] 3x^2 + 2x = 8$$

$$[54] x^2 - 5x + 4 = 5(8 - x)$$

$$[55] 3x^2 = (4x + 8)(x - 2)$$

$$[56] 6[(x + 2)^2 + (x + 1)^2] = 13(x + 1)(x + 2)$$

$$[57] 9(x - 3) + 3(x + 4)(x - 3) = (4x + 7)(x - 3) + 9(7 - x)$$

$$[58] (2x + 1)(x + 1) = 5(x - 1)(x - 1)$$

$$[59] 76x(x + 3) = 9(x + 3)(4x + 7) - 171(x - 5)$$

$$[60] (x + 1)^2(x - 2) - (x + 12)(x - 1)(x - 2) - (x^2 - 1)(x - 2) + (x + 2)(x^2 - 1) = 0$$

$$[61] (2x - 1)(x - 1)(x + 2) - (x - 7)(x + 1)(x + 2) = 4(x^2 - 1)(x + 2) - (x^2 - 1)(3x - 1)$$

$$[62] (3x - 4)(6x - 11) = (4x + 1)(5x - 16)$$

$$[63] x(3 - x) = 10 - 4x$$

$$[64] 6x^2 + 6(x + 1)^2 = 13x(x + 1)$$

$$[65] 2(2x + 1)^2 - 3(2x - 1)^2 + 5(2x + 1)(2x - 1) = 0$$

$$[66] (x - 3)^2 - (x + 3)^2 = (x - 2)(x - 3)$$

$$[67] 2(6 - x)(6 + x) - 18(x - 4) = 3(x + 6)(x - 2)$$

Soluciones:

- | | | | | | | |
|-----------------------------------|----------------------------------|----------------------------------|-------------------------------|----------------------------------|--------------------------|-------------------------|
| [1] ± 3 | [2] ± 12 | [3] $0, 5$ | [4] ± 3 | [5] $0, \frac{5}{4}$ | [6] ± 3 | [7] $0, 2$ |
| [8] $0, 3$ | [9] ± 6 | [10] ± 2 | [11] $\pm \frac{\sqrt{6}}{2}$ | [12] ± 3 | [13] $-\frac{3}{2}$ | [14] $4, 3$ |
| [15] $6, 3$ | [16] $7, 2$ | [17] $4, 2$ | [18] 3 | [19] $3, -9$ | [20] $-\frac{1}{2}$ | [21] $5, -\frac{13}{3}$ |
| [22] 4 | [23] $\frac{1}{2}, -\frac{2}{3}$ | [24] $4, 2$ | [25] $8, \frac{1}{8}$ | [26] 3 | [27] $2, -\frac{4}{3}$ | [28] $7, -1$ |
| [29] $-\frac{1}{2}, -\frac{3}{4}$ | [30] $7, 1$ | [31] $\frac{6}{5}, -\frac{1}{3}$ | [32] $6, \frac{1}{5}$ | [33] $\frac{6}{5}, -\frac{1}{3}$ | [34] $4, -\frac{45}{11}$ | [35] $-\frac{3}{2}, -1$ |
| [36] $6, -1$ | [37] ± 6 | [38] $0, \frac{32}{17}$ | [39] ± 4 | [40] ± 4 | [41] ± 6 | [42] $0, 7$ |
| [43] ± 5 | [44] -3 | [45] $9, 1$ | [46] $3, -8$ | [47] $5, -4$ | [48] $3, 2$ | [49] $-3, -5$ |
| [50] -5 | [51] $9, 4$ | [52] $\frac{19}{6}, 3$ | [53] $\frac{4}{3}, -2$ | [54] ± 6 | [55] ± 4 | [56] $1, -4$ |
| [57] $21, 5$ | [58] $4, \frac{1}{3}$ | [59] $3, -\frac{87}{10}$ | [60] $\frac{6}{5}, 5$ | [61] $\frac{3}{2}, 1$ | [62] $6, -5$ | [63] $5, 2$ |
| [64] $2, -3$ | [65] $\frac{1}{4}, -\frac{3}{2}$ | [66] $-1, -6$ | [67] $-3 \pm 3\sqrt{5}$ | | | |

Ecuaciones biquadradas

Resolver las siguientes ecuaciones:

$$[1] 4x^4 + 8x^2 - 12 = 0 \quad [2] 5x^4 - 3x^2 + \frac{7}{16} = 0 \quad [3] 3x^4 + x^2 - 4 = 0$$

$$[4] 8x^4 - x^2 - 7 = 0 \quad [5] 5x^4 - 6x^2 - 351 = 0 \quad [6] x^4 + 5x^2 - 36 = 0$$

$$[7] x^4 + 3x^2 + 2 = 0 \quad [8] 4x^4 - 5x^2 - 9 = 0 \quad [9] x^4 - 7x^2 + 12 = 0$$

$$[10] x^4 - 29x^2 + 100 = 0 \quad [11] x^4 + 21x^2 - 100 = 0 \quad [12] 9x^4 + 16 = 40x^2$$

$$[13] x^4 - 7x^2 + 12 = 0 \quad [14] 34 - x^2 = \frac{225}{x^2} \quad [15] x^2 = \frac{12}{x^2 + 1}$$

$$[16] x^4 - 10x^2 + 9 = 0 \quad [17] 2x^4 + x^2 - 1 = 0 \quad [18] x^4 - 16 = 0$$

$$[19] 4x^4 - 9x^2 = 0 \quad [20] x^4 - 5x^2 + 4 = 0 \quad [21] x^4 - 3x^2 + 2 = 0$$

$$[22] x^2(2x - 5)(2x + 5) = 9(1 - x)(1 + x)$$

$$[23] 8(x^2 - 1) - 2(x^2 - 5)(x^2 - 1) = (x^2 - 9)(x^2 - 5)$$

$$[24] x^2(34 - x^2) = 225 \quad [25] 2x^2(x^2 - 1) + 3x^2(2 - x^2) = -45$$

$$[26] 2x^4 - 5x^2 + 97 = 0 \quad [27] x^4 + 3x^2 + 2 = 0 \quad [28] 2x^4 + x^2 - 1 = 0$$

$$[29] x^4 - 10x^2 + 9 = 0 \quad [30] 12[(x^2 + 1)(x^2 - 1) + x^2] = 19x^2(x^2 - 1)$$

Soluciones:

- | | | | | |
|----------------------------|---|---------------------------------------|---------------------------|---------------------------------------|
| [1] ± 1 | [2] $\pm \frac{1}{2}, \pm \frac{\sqrt{35}}{10}$ | [3] ± 1 | [4] ± 1 | [5] ± 3 |
| [6] ± 2 | [7] Sin solución | [8] $\pm \frac{3}{2}$ | [9] $\pm 2, \pm \sqrt{3}$ | [10] $\pm 5, \pm 2$ |
| [11] ± 2 | [12] $\pm 2, \pm \frac{2}{3}$ | [13] $\pm 2, \pm \sqrt{3}$ | [14] $\pm 5, \pm 3$ | [15] $\pm \sqrt{3}$ |
| [16] $\pm 3, \pm 1$ | [17] $\pm \frac{\sqrt{2}}{2}$ | [18] ± 2 | [19] $0, \pm \frac{3}{2}$ | [20] $\pm 1, \pm 2$ |
| [21] $\pm 1, \pm \sqrt{2}$ | [22] $\pm \frac{3\sqrt{2}}{2}$ | [23] $\pm 3, \pm \frac{\sqrt{21}}{3}$ | [24] $\pm 5, \pm 3$ | [25] ± 3 |
| [26] Sin solución | [27] Sin solución | [28] $\pm \frac{\sqrt{2}}{2}$ | [29] $\pm 3, \pm 1$ | [30] $\pm 2, \pm \frac{\sqrt{21}}{7}$ |

Sistemas de dos ecuaciones con dos incógnitas

Resolver los siguientes sistemas:

$$[1] \begin{cases} \frac{3(y+2x+2)}{4} = \frac{4x+y-1}{3} \\ \frac{x+y}{3} - \frac{x-y}{6} = \frac{y-1}{6} \end{cases}$$

$$[3] \begin{cases} \frac{4y-5x}{6} + \frac{3x-2y}{2} = 1 - \frac{2(x+y)}{9} \\ \frac{4y+x-8}{8} - x = \frac{2(y-2x)}{3} \end{cases}$$

$$[5] \begin{cases} \frac{5x}{6} + \frac{3y}{7} = 2 \\ \frac{x}{2} - \frac{y}{7} = 2 \end{cases}$$

$$[7] \begin{cases} x+y = 0'3 \\ 0'1x + 0'2y = 0'04 \end{cases}$$

$$[9] \begin{cases} 4(x+1) - y = 0 \\ 5x - y - 1 = 0 \end{cases}$$

$$[11] \begin{cases} 3 - y(x-1) - x(3-y) = 0 \\ xy - x(y-1) = 2 \end{cases}$$

$$[13] \begin{cases} x-y = 2 \\ xy = 48 \end{cases}$$

$$[15] \begin{cases} 5x + 7y = 61 \\ xy = 8 \end{cases}$$

$$[17] \begin{cases} x + xy + y = 1 \\ xy = 6 \end{cases}$$

$$[19] \begin{cases} x^2 + y^2 = 290 \\ x + y = 4 \end{cases}$$

$$[21] \begin{cases} x - 2y^2 = 0 \\ y + 5 = 3x \end{cases}$$

$$[23] \begin{cases} x^2 + y^2 = 40 \\ xy = 12 \end{cases}$$

$$[25] \begin{cases} x^2 + y^2 = 61 \\ xy = 30 \end{cases}$$

$$[2] \begin{cases} \frac{3-2y}{4} - \frac{1}{4} = \frac{1-2x}{6} \\ \frac{25}{8} - 1 = \frac{x+3}{2} - \frac{3(1+y)}{8} \end{cases}$$

$$[4] \begin{cases} \frac{3(x-y)}{4} = \frac{2+y}{4} - \frac{5x-y}{6} \\ 1 + \frac{2y-7x}{12} = \frac{x-y}{2} + \frac{x}{2} \end{cases}$$

$$[6] \begin{cases} x-y = 1 \\ \frac{2x}{5} + \frac{3y}{4} = 5 \end{cases}$$

$$[8] \begin{cases} 4x - 3y = 0 \\ 2(x-1) = y + 2 \end{cases}$$

$$[10] \begin{cases} 3(x+2) - (x-2) = 2y \\ x+y = 8 \end{cases}$$

$$[12] \begin{cases} x+2y = 10 \\ x^2 + y^2 = 25 \end{cases}$$

$$[14] \begin{cases} 2x^2 - y^2 = 5 \\ 3x^2 + 4y^2 = 57 \end{cases}$$

$$[16] \begin{cases} 6x - 5y = 14 \\ xy = 72 \end{cases}$$

$$[18] \begin{cases} xy + 2y = 4 \\ 3x - y = 5 \end{cases}$$

$$[20] \begin{cases} x^2 + y^2 = 9 \\ 2x + y = 3 \end{cases}$$

$$[22] \begin{cases} 2x^2 - 3y^2 = -6 \\ 4x^2 - y^2 = 8 \end{cases}$$

$$[24] \begin{cases} 3x^2 - 4y^2 = 16 \\ xy = 4 \end{cases}$$

Soluciones: En los pares que siguen, la primera coordenada es la x y la segunda la y .

[1] $(39, -20)$

[2] $(5, 4)$

[3] $\left(\frac{4}{7}, -\frac{31}{7}\right)$ [4] $\left(\frac{20}{19}, 1\right)$

[5] $\left(\frac{24}{7}, -2\right)$

[6] $(5, 4)$

[7] $(0'2, 0'1)$ [8] $(6, 8)$

[9] $(5, 24)$

[10] $(2, 6)$

[11] $(2, 3)$ [12] $\{(0, 5), (4, 3)\}$

[13] $\{(8, 6), (-6, -8)\}$ [14] $\{(\sqrt{7}, 3), (-\sqrt{7}, 3), (\sqrt{7}, -3), (-\sqrt{7}, -3)\}$

[15] $\{(1, 8), \left(\frac{56}{5}, \frac{5}{7}\right)\}$

[16] $\{(9, 8), \left(-\frac{20}{3}, -\frac{54}{5}\right)\}$

[17] $\{(-2, -3), (-3, -2)\}$

[18] $\{(2, 1), \left(-\frac{7}{3}, -12\right)\}$ [19] $\{(13, 11), (11, 13)\}$ [20] $\{(0, 3), \left(\frac{12}{5}, -\frac{9}{5}\right)\}$

[21] $\{(2, 1), \left(\frac{25}{18}, -\frac{5}{6}\right)\}$ [22] $\{(\sqrt{3}, 2), (-\sqrt{3}, 2), (\sqrt{3}, -2), (-\sqrt{3}, -2)\}$

[23] $\{(2, 6), (6, 2), (-6, -2), (-2, -6)\}$

[24] $\{(2\sqrt{2}, \sqrt{2}), (2\sqrt{2}, -\sqrt{2}), (-2\sqrt{2}, \sqrt{2}), (-2\sqrt{2}, -\sqrt{2})\}$

[25] $\{(6, 5), (5, 6), (-5, -6), (-6, -5)\}$

Sistemas de tres ecuaciones con tres incógnitas

Resolver los siguientes sistemas:

[1] $\begin{cases} x + y + z = 31 \\ x + y - z = 25 \\ x - y - z = 9 \end{cases}$

[2] $\begin{cases} 2x + y - z = 11 \\ 3x - 4y - 2z = 2 \\ x + 5y + 3z = 5 \end{cases}$

[3] $\begin{cases} x + y + z = 4 \\ x - 2y + 3z = 13 \\ x + 3y + 4z = 11 \end{cases}$

[4] $\begin{cases} x + y - 2z = 13 \\ x - 3y - z = -3 \\ x - y + 4z = -17 \end{cases}$

[5] $\begin{cases} x - y - z = 1 \\ 2x - 5y + 10z = 5 \\ 3x + 2y - 11z = 10 \end{cases}$

[6] $\begin{cases} 4x + 2y - z = 5 \\ 5x - 3y + z = 2 \\ 2x - y + z = 3 \end{cases}$

[7] $\begin{cases} 4x - 3y + 2z = 8 \\ 5x + y - z = 16 \\ 6x - 2y - 3z = 11 \end{cases}$

[8] $\begin{cases} 2x - y + z = 12 \\ 4y - 3x - z = -18 \\ x + 3y - 4z = -20 \end{cases}$

[9] $\begin{cases} 2x + 4y - 3z = 22 \\ 4x - 2y + 5z = 18 \\ 6x + 7y - z = 63 \end{cases}$

Soluciones: En las ternas que siguen, la primera coordenada es la x , la segunda la y , y la tercera es la z

[1] $(20, 8, 3)$ [2] $(2, 3, -4)$ [3] $(2, -1, 3)$ [4] $(2, 3, -4)$ [5] $(5, 3, 1)$
[6] $(1, 2, 3)$ [7] $(3, 2, 1)$ [8] $(3, -1, 5)$ [9] $(3, 7, 4)$

Problemas

- La suma de los catetos de un triángulo rectángulo es de 49 cm. Determinar los lados del triángulo y el área sabiendo que la hipotenusa es 32 cm. mayor que el cateto

menor.

Solución: lados = 9, 40, 41 cm., Área = 180 cm^2 .

2. ¿Cuál es el número cuyos $\frac{3}{4}$ más 9 multiplicados por los $\frac{3}{4}$ menos 9 dan por resultado 1 008.

Solución: ± 44

3. Hallar un número cuyo cuadrado lo sobrepease en 13 110 unidades.

Solución: 115, -114

4. La suma de dos números es 42 y la diferencia de sus cuadrados es 504. ¿Cuáles son estos números?.

Solución: 27, 15

5. El perímetro de un triángulo rectángulo mide 60 cm. y la hipotenusa 26 cm. Calcular las longitudes de los dos catetos.

Solución: 24, 10

6. Un librero vende 84 libros a dos precios distintos: unos a 45 pts. y otros a 36 pts., obteniendo por la venta 3 105 pts. ¿Cuántos libros vendió de cada clase?.

Solución: 9 libros de 45 pts., 75 libros de 36 pts.

7. Una habitación rectangular tiene una superficie de 28 m^2 y su perímetro es de 22 m. Hallar las dimensiones de la habitación.

Solución: 4, 7 m.

8. La diagonal de un rectángulo mide 55 cm. ¿Cuánto miden sus lados, si la base es los $\frac{3}{4}$ de la altura?.

Solución: 44, 33 m.

9. El perímetro de un rectángulo es de 30 m. y su área 54 m^2 . ¿Cuánto miden sus lados?.

Solución: 9, 6 m.

10. El área de un rectángulo es de 12 m^2 y su perímetro es 16 m. Hallar las dimensiones.

Solución: 6, 2 m.

11. El área de un rectángulo es 620 m^2 . Si cada dimensión aumenta en 1 m., el área aumenta en 52 m^2 . Hallar las dimensiones.

Solución: 20, 31 m.

12. Hallar la altura de un cono sabiendo que su generatriz mide 13 cm. y el área de su base $25\pi \text{ cm}^2$.

Solución: 12 cm.

13. Hallar la altura y el radio de la base de un cono, sabiendo que la generatriz mide 50 m., y que la altura es 10 m. más larga que el radio de la base.

Solución: altura = 40 m., radio = 30 m.

14. La suma de dos números es 8 y su producto es 15. Calcular dichos números.

Solución: 3, 5

15. La hipotenusa de un triángulo mide 5 cm. ¿Cuánto miden los catetos sabiendo que uno es 1 cm. mayor que el otro?.

Solución: 3, 4 cm.

16. La hipotenusa de un triángulo rectángulo vale 34 cm. Hallar las longitudes de los catetos sabiendo que uno de ellos es 14 cm. mayor que el otro.

Solución: 16, 30 cm.

17. Hallar las dimensiones de un rectángulo cuyo perímetro es 50 cm. y su área 150 cm².

Solución: 10, 15 cm.

18. Hallar un número sabiendo que la suma del triple del mismo con el doble de su recíproco es igual a 5.

Solución: 1 ó $\frac{2}{3}$.

19. Hallar dos números positivos sabiendo que uno de ellos es igual al triple del otro más 5 y que el producto de ambos es 68.

Solución: 4, 17

Ecuaciones de grado superior e irracionales

1. Resolver las siguientes ecuaciones:

$$[1] x^3 + 2x^2 + 2x + 1 = 0$$

$$[2] x^3 + 3x^2 - x - 3 = 0$$

$$[3] x^5 - 32 = 0$$

$$[4] x^4 - x^3 - 16x^2 - 20x = 0$$

$$[5] x^4 - 6x^3 - 11x^2 + 96x - 80 = 0$$

$$[6] 6x^3 + x^2 - 26x - 21 = 0$$

$$[7] x^3 + 6x^2 + 3x - 10 = 0$$

$$[8] 2x^4 - 5x^3 + 5x - 2 = 0$$

$$[9] 6x^3 + 7x^2 - 9x + 2 = 0$$

$$[10] x^3 + 3x^2 - 4x - 12 = 0$$

$$[11] x^3 + 2x^2 - x - 2 = 0$$

$$[12] x^3 - 5x^2 + 6 = 0$$

$$[13] 8x^4 - 14x^3 - 9x^2 + 11x - 2 = 0$$

$$[14] 4x^5 + 4x^4 + 3x^3 + 3x^2 - x - 1 = 0$$

$$[15] x^4 + x^3 - 16x^2 - 4x + 48 = 0$$

Soluciones:

$$[1] -1$$

$$[2] \{1, -1, -3\}$$

$$[3] 2$$

$$[4] \{0, 5, -2\}$$

$$[5] \{1, 5, 4, -4\}$$

$$[6] \{-1, \frac{7}{3}, -\frac{3}{2}\}$$

$$[7] \{1, -2, -5\}$$

$$[8] \{1, -1, 2, \frac{1}{2}\}$$

$$[9] \{-2, \frac{1}{2}, \frac{1}{3}\}$$

$$[10] \{2, -2, -3\}$$

$$[11] \{1, -1, -2\}$$

$$[12] \{-1, 3 \pm \sqrt{3}\}$$

$$[13] \{\frac{1}{4}, -1, 2, \frac{1}{2}\}$$

$$[14] \{-1, \frac{1}{2}, -\frac{1}{2}\}$$

$$[15] \{2, -2, 3, -4\}$$

2. Resolver las siguientes ecuaciones:

- | | |
|--|---|
| [1] $\sqrt{3x - 2} = 0$ | [2] $\sqrt{2x + 1} = x - 1$ |
| [3] $\sqrt{7 - 3x} - x = 7$ | [4] $3\sqrt{6x + 1} - 5 = 2x$ |
| [5] $\sqrt{x + 4} = 3 - \sqrt{x - 1}$ | [6] $\sqrt{2x - 1} + \sqrt{x + 4} = 6$ |
| [7] $\sqrt{x^2 + 3x + 7} = 5$ | [8] $\sqrt{x^2 - x + 2} = 2$ |
| [9] $2\sqrt{2x - 1} = \sqrt{6x - 5} + \sqrt{2x - 9}$ | [10] $\sqrt{2x - 2} = x - 1$ |
| [11] $\sqrt{4x + 1} = 3 - 3x$ | [12] $\sqrt{2x + 7} = \sqrt{x} + 2$ |
| [13] $\sqrt{2x^2 - 7} - x = 3$ | [14] $3 - 2\sqrt{x} = x$ |
| [15] $4 + \sqrt{x + 2} = \frac{4x}{7}$ | [16] $5\sqrt{x} - 2 = 2x$ |
| [17] $\sqrt{2x - 5} + 6 = x + 2$ | [18] $\sqrt{x} + 1 = \sqrt{x - 3} + 2$ |
| [19] $\sqrt{2x + 10} = 1 + \sqrt{2x + 3}$ | [20] $\sqrt{2x + 5} - \sqrt{3x + 4} = 1$ |
| [21] $\sqrt{2(x - 4)} - \sqrt{x - 3} = \sqrt{3(x - 5)}$ | [22] $\sqrt{x - 4} + \sqrt{x + 4} = \sqrt{2x}$ |
| [23] $\sqrt{x + 6} + \sqrt{x + 1} = \sqrt{7x + 4}$ | [24] $\sqrt{3\sqrt{15 - x}} = \sqrt{2x - 3}$ |
| [25] $\sqrt{5 + 2x} = x + 1$ | [26] $\sqrt{2x + 1} - \sqrt{x} = 1$ |
| [27] $\sqrt{4x - 1} + \sqrt{2x - 3} = 1$ | [28] $2\sqrt{2x - 1} = \sqrt{6x - 5} + \sqrt{2x - 9}$ |
| [29] $\frac{\sqrt{4x + 20}}{4 + \sqrt{x}} = \frac{4 - \sqrt{x}}{\sqrt{x}}$ | [30] $\sqrt{\sqrt{x + 16} - \sqrt{x}} = 2$ |
| [31] $\sqrt{x^2 - 6x + 9} - 8 = 0$ | [32] $\sqrt{x} + 1 = \sqrt{3(x - 1)}$ |
| [33] $3 - x = \sqrt{x} + 1$ | [34] $\sqrt{4x + 1} = 5\sqrt{3x - 2} - 7$ |
| [35] $x + \sqrt{x} = 6$ | [36] $x + 5 = \sqrt{x^2 + 10x + 5\sqrt{5(x^2 + 4)}}$ |
| [37] $(\sqrt{x - 1} - 1)^2 + 2 = 2\sqrt{x - 1}$ | [38] $\sqrt{x - 85} = \sqrt[4]{3x - 47}$ |

Soluciones:

- | | | | | | |
|-------------------|-----------------------|-------------------|--------------------------|--------------------|--------------|
| [1] $\frac{2}{3}$ | [2] 4 | [3] -3 | [4] $\{8, \frac{1}{2}\}$ | [5] $\frac{13}{9}$ | [6] 5 |
| [7] {3, -6} | [8] {2, -1} | [9] 5 | [10] {3, 1} | [11] $\frac{4}{9}$ | [12] {9, 1} |
| [13] {8, -2} | [14] 1 | [15] 14 | [16] {4, $\frac{1}{4}\}$ | [17] 7 | [18] 4 |
| [19] 3 | [20] $6 - 2\sqrt{13}$ | [21] 5 | [22] 4 | [23] 3 | [24] 6 |
| [25] 2 | [26] {0, 4} | [27] Sin solución | [28] 5 | [29] 4 | [30] 0 |
| [31] {11, -5} | [32] 4 | [33] 1 | [34] 2 | [35] 4 | [36] {1, -1} |
| [37] {10, 2} | [38] 101 | | | | |

Inecuaciones

1. Resolver las siguientes desigualdades:

$$[1] x - 2 > 0$$

$$[2] 3 - x > 0$$

$$[3] x + 3 > 4$$

$$[4] 3 - x \leq 6$$

$$[5] 3(x - 2) < 6$$

$$[6] 2(x + 3) > 3(x + 2)$$

$$[7] \frac{x - 1}{4} - \frac{x + 2}{3} > \frac{3x - 1}{6} - x \quad [8] (x - 3)^2 - (x + 2)^2 < 5$$

$$[9] (4x - 3)(2 + x) > (3 - 2x)^2 \quad [10] 3 \left[x - 2 \left(\frac{x(x - 1)}{4} - 5 \right) \right] < \frac{3}{2}x(4 - x)$$

$$[11] \frac{3 - \left[\frac{x - 2}{4} + x \left(\frac{x - 3}{2} - x \right) \right]}{2 - \frac{3}{2}} \leq (x - 2)(x - 3)$$

$$[12] 3x + 2 > 6x - 3$$

$$[13] 2(3x - 7) \leq -3(2x + 4) + 1$$

$$[14] 7x - 6 \geq -2x + 5(x + 3) - 1 \quad [15] 4x^2 - (2x - 2) < (2x + 3)^2$$

$$[16] \frac{3 - \frac{x}{3}}{3 + \frac{1}{2}} - x \geq \frac{3x - \frac{5}{2}}{1 - \frac{2}{3}}$$

Soluciones:

$$[1] x > 2 \quad [2] x < 3 \quad [3] x > 1 \quad [4] x \geq -3$$

$$[5] x < 4 \quad [6] x < 0 \quad [7] x > \frac{9}{5} \quad [8] x > 0$$

$$[9] x > \frac{15}{17} \quad [10] x > 20 \quad [11] x \leq -\frac{2}{15} \quad [12] x < \frac{5}{3}$$

$$[13] x \leq \frac{1}{4} \quad [14] x \geq 5 \quad [15] x > -\frac{1}{2} \quad [16] x \leq \frac{351}{424}$$

2. Resolver las siguientes desigualdades:

$$[1] \begin{cases} 2x - 3 > x - 2 \\ 3x - 7 < x - 1 \end{cases}$$

$$[2] \begin{cases} \frac{x}{3} + \frac{x}{5} < 8 \\ \frac{x}{2} - \frac{4x}{9} < 5 \end{cases}$$

$$[3] \begin{cases} (x-1)^2 + (x+2)^2 > \frac{(2x-3)^2}{2} \\ (2x+1)^2 - (x-3)^2 < 3(x+2)^2 \\ \frac{x-1}{3} + 1 > x \end{cases}$$

$$[4] \begin{cases} 2x - 1 < 3x + 7 \\ x + 2 \geq 2x - 5 \end{cases}$$

$$[5] \begin{cases} 5(2x+1) < 3x + 19 \\ 2x(x+1) < 3 + 2(x-1)^2 \end{cases}$$

$$[6] \begin{cases} 6x + 5(2-x) > 3x - 8(x+4) \\ x(7-2x) > 2x(5-x) + 10x \end{cases}$$

$$[7] \begin{cases} (3x+4)(2x-1) + 11 \leq (2x-1)^2 + x(2x+3) \\ 7(x-2) + 7 > 2(4x+9) - 3(5x+5) \end{cases}$$

$$[8] \begin{cases} \frac{2x-2}{5} + \frac{5-2x}{3} < 1 \\ \frac{x+2}{3} - \frac{2x-3}{4} > \frac{3}{4} \end{cases} [9] \begin{cases} \frac{13x-2}{12} - 1 < \frac{3x-2}{10} + \frac{x+1}{5} \\ (2x+1)^2 - 8 \leq (2x-1)^2 \\ (x+1)(x-1) > (x-2)^2 - 3 \end{cases}$$

$$[10] \begin{cases} \frac{3x}{2} + \frac{1}{2} - x + 3 > \frac{1-3x}{5} \\ \frac{3x+1}{4} + \frac{1-3x}{2} + \frac{1-x}{3} > 0 \end{cases}$$

Soluciones:

- | | | | | |
|------------------|------------------|-----------------------------|------------------------------|-----------------------|
| [1] $1 < x < 3$ | [2] $x < 15$ | [3] $-\frac{1}{16} < x < 1$ | [4] $-8 < x \leq 7$ | [5] $x < \frac{5}{6}$ |
| [6] $-7 < x < 0$ | [7] Sin solución | [8] $1 < x < 4$ | [9] $\frac{1}{2} < x \leq 1$ | [10] $-3 < x < 1$ |

3. Resolver las siguientes desigualdades:

$$[1] x^2 - 5x + 6 > 0$$

$$[2] (x+1)^2 + 6x + 2 \geq 2(x+3)(x-2) + 4x$$

$$[3] x^2 + 5x + 6 > 0$$

$$[4] x^2 - 3x + 2 < 0$$

$$[5] x^2 - 2x - 8 \geq 0$$

$$[6] x^2 - x - 20 \leq 0$$

$$[7] x^2 + 2x + 1 > 0$$

$$[8] x^2 - 6x + 9 < 0$$

$$[9] x^2 - 4x + 4 \geq 0$$

$$[10] x^2 + 10x + 25 \leq 0$$

$$[11] x^2 + 2x + 2 > 0$$

$$[12] x^2 - 4x + 9 \geq 0$$

$$[13] x^2 - 4x + 9 < 0$$

$$[14] x^2 - 8x + 25 \leq 0$$

$$[15] 2(x+1)^2 - 2x + 5 > x(x+4) + 10$$

$$[16] (x+3)(x-1) + x \leq 5x - 4$$

$$[17] (3x-4)^2 - (5x+3)(x+1) \geq 4x + 13 \quad [18] (2x+3)^2 + 4x - 5 < 8(x+2)^2 + 10$$

Soluciones:

- | | | | |
|------------------------------|------------------------|---------------------------|-------------------|
| [1] $x < 2$ ó $x > 3$ | [2] $-3 \leq x \leq 5$ | [3] $x < -3$ ó $x > -2$ | [4] $1 < x < 2$ |
| [5] $x \leq -2$ ó $x \geq 4$ | [6] $-4 \leq x \leq 5$ | [7] $\mathbb{R} - \{-1\}$ | [8] Sin solución |
| [9] \mathbb{R} | [10] $x = -5$ | [11] \mathbb{R} | [12] \mathbb{R} |
| [13] Sin solución | [14] Sin solución | [15] $x < -1$ ó $x > 3$ | [16] $x = 1$ |
| [17] $x \leq 0$ ó $x \geq 9$ | [18] \mathbb{R} | | |

4. Resolver las siguientes desigualdades:

[1] $\frac{x+3}{x-2} \leq 0$	[2] $\frac{3x+1}{2x-3} \geq 0$
[3] $\frac{6x+5}{3x+2} > 0$	[4] $\frac{9x-7}{5-2x} \leq 0$
[5] $\frac{3-4x}{2x+5} \geq 0$	[6] $\frac{x}{(x+3)(x-2)} < 0$
[7] $2x^3 - 7x^2 + 7x - 2 < 0$	[8] $\frac{2x^2 + x - 3}{x+1} \geq 0$
[9] $2x - \frac{3}{5} > \frac{x}{3} + \frac{1}{6}$	[10] $\frac{1}{2} \left(x - \frac{1}{3} \right) + x < \frac{x+2}{4}$
[11] $\frac{3(x-1)}{5} + \frac{x}{2} > 2x - \frac{5}{8}$	[12] $\frac{5(6x-2)}{8} - \frac{7}{3} \left(1 - \frac{2x}{3} \right) < 4x + \frac{2}{3} \left(\frac{x}{2} - \frac{5}{12} \right)$
[13] $\frac{x-2}{x^3-x} \geq 0$	[14] $4^{-\frac{1}{2}}(x-1) - 8^{-\frac{2}{3}}(x+1) \leq -2^{-2}(1-3x)$

Soluciones:

- | | |
|--|---|
| [1] $x \in [-3, 2]$ | [2] $x \in]-\infty, -\frac{1}{3}] \cup]\frac{3}{2}, +\infty[$ |
| [3] $x \in]-\infty, -\frac{5}{6}] \cup]-\frac{2}{3}, +\infty[$ | [4] $x \in]-\infty, \frac{7}{9}] \cup]\frac{5}{2}, +\infty[$ |
| [5] $x \in]-\frac{5}{2}, \frac{3}{4}]$ | [6] $x \in]-\infty, -3] \cup]0, 2[$ |
| [7] $x \in]-\infty, \frac{1}{2}[\cup]1, 2[$ | [8] $x \in]-\frac{3}{2}, -1] \cup [1, +\infty[$ |
| [9] $x \in]\frac{23}{50}, +\infty[$ | [10] $x \in]-\infty, \frac{8}{15}[$ |
| [11] $x \in]-\infty, \frac{1}{36}[$ | [12] $x \in]-\infty, \frac{17}{5}[$ |
| [13] $x \in]-\infty, -1] \cup]0, 1[\cup]2, +\infty[$ | [14] $x \in [-1, +\infty[$ |