

3<sup>rd</sup> TERM GENERAL EXAM

Name: \_\_\_\_\_

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)

1. Work out and simplify:

a)  $\frac{2}{3} \cdot \frac{6}{5} - \frac{6}{5} \cdot \frac{3}{2} =$  (0,5p)

b)  $\frac{2}{3} \cdot \left( \frac{6}{5} - \frac{6}{5} \div \frac{3}{2} \right) =$  (0,5p)

2. Calculate, using the rules for powers :

a)  $\left( \frac{2^5 \cdot 4^2}{8^3} \right)^2 =$  (0,5p)

b)  $\frac{x^{-2}}{x^{-1}} \cdot \left( \frac{x^2 y}{x^3 y^2} \right)^2 =$  (0,5p)

3. Solve:

a)  $(2x - 1)^2 = (x - 1)^2 + 8$  (0,5p)

b)  $\frac{5x - 16}{6} - \frac{x + 1}{3} + \frac{x + 8}{12} = 0$  (0,5p)

4. Work out and simplify:

a)  $\frac{2}{x - 2} - \frac{2}{x + 2} =$  (0,5p)

b)  $\left( \frac{1}{x} \div \frac{1}{x + 1} \right) \cdot \frac{x}{2} =$  (0,5p)

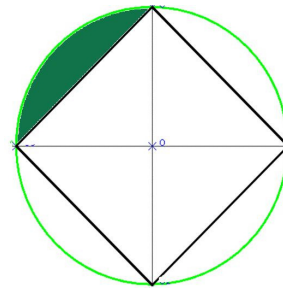
5. Calculate the sum to 50 terms of an arithmetic progression whose first and ninth terms are 3 and 35.

 6. A cone of radius 6 cm has a total surface area of  $324\pi \text{ cm}^2$ . Calculate the volume of the cone.

7. A square of side 10cm is drawn inside a circle.

a) Find the diameter of the circle.

b) Find the shaded area.


 8. Halla las dimensiones de un rectángulo del que conocemos su perímetro, 34 m y su área  $60 \text{ m}^2$ .

9. Hace dos años compré un juego de ordenador y un equipo de música por 260€.

Los acabo de vender por un total de 162 €, habiendo perdido el 30% con el juego y el 40% con el equipo de música ¿Cuánto me costó cada cosa?

10. Solve by graphing and using another method the simultaneous equation:

$$\begin{cases} 3x + 2y = 7 \\ 5x - y = 16 \end{cases}$$

**SOLUTION**

1. Work out and simplify:

$$a) \frac{2}{3} \cdot \frac{6}{5} - \frac{6}{5} \cdot \frac{3}{2} = \frac{2 \cdot 6}{3 \cdot 5} - \frac{6 \cdot 3}{5 \cdot 2} = \frac{2 \cdot 2}{5} - \frac{3 \cdot 3}{5} = \frac{4}{5} - \frac{9}{5} = -\frac{5}{5} = -1$$

$$b) \frac{2}{3} \cdot \left( \frac{6}{5} - \frac{6}{5} \div \frac{3}{2} \right) = \frac{2}{3} \cdot \left( \frac{6}{5} - \frac{6 \cdot 2}{5} \right) = \frac{2}{3} \cdot \left( \frac{6}{5} - \frac{2 \cdot 2}{5} \right) = \frac{2}{3} \cdot \frac{2}{5} = \frac{10 - 6}{15} = \frac{4}{15}$$

2. Calculate, using the rules for powers :

$$a) \left( \frac{2^5 \cdot 4^2}{8^3} \right)^2 = \left( \frac{2^5 \cdot 2^4}{2^9} \right)^2 = (2^0)^2 = 2^0 = 1$$

$$b) \frac{x^{-2}}{x^{-1}} \cdot \left( \frac{x^2 y}{x^3 y^2} \right)^2 = \frac{x}{x^2} \cdot \frac{x^4 y^2}{x^6 y^4} = \frac{x^5 y^2}{x^8 y^4} = \frac{1}{x^3 y^2}$$

3. Solve:

$$a) (2x-1)^2 - (x-1)^2 = 8 \rightarrow 4x^2 - 4x + 1 - (x^2 - 2x + 1) = 8$$

$$4x^2 - 4x + 1 - x^2 + 2x - 1 = 8 \rightarrow 3x^2 - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{4+96}}{6} = \frac{2 \pm 10}{6} = \begin{cases} \frac{2}{6} = \frac{1}{3} \\ -\frac{8}{6} = -\frac{4}{3} \end{cases}$$

$$b) \frac{5x-16}{6} - \frac{x+1}{3} + \frac{x+8}{12} = 0 \rightarrow \frac{2(5x-16)}{12} - \frac{4(x+1)}{12} + \frac{x+8}{12} = 0$$

$$10x - 32 - 4x - 4 + x + 8 = 0 \rightarrow 7x = 32 + 4 - 8 \rightarrow 7x = 28 \rightarrow x = 4$$

4. Work out and simplify:

$$a) \frac{2}{x-2} - \frac{2}{x+2} = \frac{2(x+2)}{(x+2)(x-2)} - \frac{2(x-2)}{(x+2)(x-2)} = \frac{2x+4}{x^2-4} - \frac{2x-4}{x^2-4} = \frac{8}{x^2-4}$$

$$b) \left( \frac{1}{x} \div \frac{1}{x+1} \right) \cdot \frac{x}{2} = \frac{x+1}{x} \cdot \frac{x}{2} = \frac{(x+1)x}{2x} = \frac{x+1}{2}$$

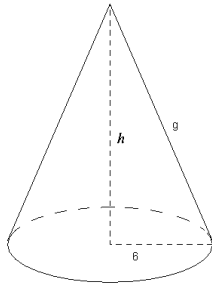
5. Calculate the sum to 50 terms of an arithmetic progression whose first and ninth terms are 3 and 35.

$$a_1 = 3; \quad a_9 = 35 \rightarrow a_9 = a_1 + 8d \Rightarrow 35 = 3 + 8d \rightarrow 8d = 32 \rightarrow d = 4$$

$$S_{50} = \frac{(a_1 + a_{50}) \cdot 50}{2}, \text{ we need to calculate } a_{50} = a_1 + 49d = 3 + 49 \cdot 4 = 199$$

$$6. \quad S_{50} = \frac{(a_1 + a_{50}) \cdot 50}{2} = \frac{(3 + 199) \cdot 50}{2} = 202 \cdot 25 = 5050$$

6. A cone of radius 6 cm has a total surface area of  $324\pi \text{ cm}^2$ . Calculate the volume of the cone.



$$A = \pi r^2 + \pi r g \rightarrow 324\pi = 36\pi + 6\pi g \rightarrow 324 = 36 + 6g$$

$$324 - 36 = 6g \rightarrow 6g = 288 \rightarrow g = \frac{288}{6} = 48$$

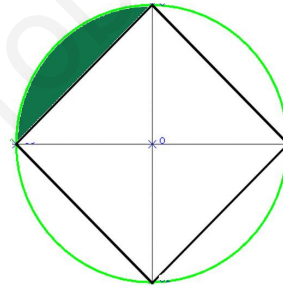
$$\text{Pitágoras: } g^2 = h^2 + r^2 \rightarrow h^2 = 48^2 - 6^2 = 2268$$

$$h = \sqrt{2268} = \sqrt{2^2 \cdot 3^4 \cdot 7} = 18\sqrt{7} \text{ cm}$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi 6^2 \cdot 18\sqrt{7} = 36\pi \cdot 6\sqrt{7} = 216\sqrt{7} \pi \text{ cm}^3$$

7. A square of side 10cm is drawn inside a circle.

- Find the diameter of the circle.
- Find the shaded area.



a) Pythagoras:  $d^2 = 10^2 + 10^2 \rightarrow d^2 = 100 + 100 = 200$

$$d = \sqrt{200} = 10\sqrt{2} \text{ cm} \rightarrow r = \frac{d}{2} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

b)  $A = \frac{1}{4} (A_{\text{Circle}} - A_{\text{Square}}) = \frac{1}{4} (\pi r^2 - l^2)$

$$A = \frac{1}{4} (\pi (5\sqrt{2})^2 - 10^2) = \frac{50\pi - 100}{4} = \left( \frac{25}{2} \pi - 25 \right) \text{ cm}^2$$

8. Halla las dimensiones de un rectángulo del que conocemos su perímetro, 34 m y su área  $60 \text{ m}^2$ . Largo x, Ancho y

$$\left. \begin{array}{l} 2x + 2y = 34 \\ x \cdot y = 60 \end{array} \right\} \rightarrow 2y = 34 - 2x \rightarrow y = 17 - x \Rightarrow x(17 - x) = 60$$

$$17x - x^2 = 60 \rightarrow x^2 - 17x + 60 = 0 \rightarrow x = \frac{17 \pm \sqrt{289 - 240}}{2} = \left\{ \begin{array}{l} \frac{17 + 7}{2} = 12 \\ \frac{17 - 7}{2} = 5 \end{array} \right.$$

Si  $x = 12$   $y = 5$ , y si  $x = 5$   $y = 12$

Solución: las dimensiones son 12 m de largo por 5 m de ancho

9. Hace dos años compré un juego de ordenador y un equipo de música por 260€. Los acabo de vender por un total de 162 €, habiendo perdido el 30% con el juego y el 40% con el equipo de música ¿Cuánto me costó cada cosa?

Precio juego  $x$ , Precio equipo de música  $y$

$$\left. \begin{array}{l} x + y = 260 \\ \frac{70}{100}x + \frac{60}{100}y = 162 \end{array} \right\} \rightarrow \left. \begin{array}{l} y = 260 - x \\ 7x + 6y = 1620 \end{array} \right\} \rightarrow 7x + 6(260 - x) = 1620$$

$$7x + 1560 - 6x = 1620 \Rightarrow x = 1620 - 1560 = 60$$

$$y = 260 - x = 260 - 60 = 200$$

Solución: El juego costó 60 euros y el equipo de música 200 euros

10. Solve by graphing and using another method the simultaneous equation:

$$\left. \begin{array}{l} 3x + 2y = 7 \\ 5x - y = 16 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} y = \frac{7 - 3x}{2} \\ y = 5x - 16 \end{array} \right. \rightarrow \frac{7 - 3x}{2} = 5x - 16 \rightarrow 7 - 3x = 10x - 32$$

$$13x = 39 \rightarrow x = \frac{39}{13} = 3 \Rightarrow y = 5x - 16 = 5 \cdot 3 - 16 = 15 - 16 = -1 \quad \text{Solution: } x=3, y=-1$$

Graphically: We plot both straight lines

