Trigonometría y Complejos

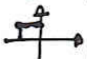
1) Demostrar que: $\dfrac{1+\mathrm{tg}^2 x}{\mathrm{tg}^2 x} = \mathrm{cosec}^2 x$
2) Resolver: $\cos 2x + \cos x = 0$
3) Resolver un triángulo del que conocemos $a = 6$, $b = 8$ y $C = 60^o$.
4) Calcular $(-\sqrt{3} + i)^{11}$, dando el resultado final en forma polar, trigonómetrica, binómica y cartesiana.
5) Calcular las raíces sextas de $-i$.

1) $\boxed{\dfrac{1+tg^2x}{tg^2x}} = \dfrac{1/\cos^2x}{\frac{sen^2x}{\cos^2x}} = \dfrac{\cos^2x}{sen^2x\cos^2x} = \dfrac{1}{sen^2x} = \boxed{cosec^2x}$

2) $\cos 2x + \cos x = 0 \Rightarrow \cos^2x - sen^2x + \cos x = 0 \Rightarrow \cos^2x - (1-\cos^2x) + \cos x = 0$
$\Rightarrow \cos^2x - 1 + \cos^2x + \cos x = 0 \Rightarrow 2\cos^2x + \cos x - 1 = 0 \Rightarrow \cos x = \dfrac{-1 \pm \sqrt{1+8}}{4} =$
$= \dfrac{-1 \pm 3}{4}$ $\begin{cases} = -1 \Rightarrow \boxed{x = 180^\circ + 360^\circ k,\ k \in \mathbb{Z}} \\ = \frac{2}{4} = \frac{1}{2} \Rightarrow \boxed{x = \pm 60^\circ + 360^\circ k,\ k \in \mathbb{Z}} \end{cases}$

3) T. coseno: $\boxed{c = \sqrt{a^2+b^2-2ab\cos C}} = \sqrt{36+64-2\cdot 6\cdot 8\cdot \frac{1}{2}} = \sqrt{100-48} = \boxed{\sqrt{52} \simeq 7,21}$
En el caso de 2 lados y el ángulo comprendido hay sol. única. Buscamos la solución por el T. del coseno: $b^2 = a^2+c^2-2ac\cos B \Rightarrow 2ac\cos B = a^2+c^2-b^2 \Rightarrow$
$\Rightarrow \cos B = \dfrac{a^2+c^2-b^2}{2ac} = \dfrac{36+52-64}{2\cdot 6\cdot\sqrt{52}} \Rightarrow \boxed{B = 73,90^\circ = 73^\circ 53' 52,4''}$
$\Rightarrow \boxed{A} = 180^\circ - B - C = \boxed{46,10^\circ = 46^\circ 6' 7,61''}$

4) $tg\,x = \frac{1}{\sqrt{3}} \Rightarrow x = 30^\circ \Rightarrow \alpha = 150^\circ$



2° cuad.

Llamamos $z = -\sqrt{3} + i \Rightarrow |z| = \sqrt{3+1} = 2$;
$\Rightarrow z = 2_{150^\circ} \Rightarrow \boxed{z^{11}} = (2_{150^\circ})^{11} = (2^{11})_{150\cdot 11} = 2048_{1650^\circ} = \boxed{2048_{210^\circ}}$ Polar

Trigonométrica: $z^{11} = 2048(\cos 210 + i\, sen\, 210)$
Binómica: $z^{11} = 2048\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) = -1024\sqrt{3} - 1024i$
Cartesiana: $z^{11} = (-1024\sqrt{3}, -1024)$

5) $-i = 1_{270^\circ} \Rightarrow$ Módulo de las raíces: $\sqrt[6]{1} = 1$. Argumentos:

$\beta_1 = \frac{270}{6} = 45 \Rightarrow 1_{45^\circ}$
$\beta_2 = \frac{270}{6} + \frac{360}{6} = 105 \Rightarrow 1_{105^\circ}$
$\beta_3 = \frac{270}{6} + 2\cdot\frac{360}{6} = 165 \Rightarrow 1_{165^\circ}$
$\beta_4 = \frac{270}{6} + 3\cdot\frac{360}{6} = 225 \Rightarrow 1_{225^\circ}$
$\beta_5 = \frac{270}{6} + 4\cdot\frac{360}{6} = 285 \Rightarrow 1_{285^\circ}$
$\beta_6 = \frac{270}{6} + 5\cdot\frac{360}{6} = 345 \Rightarrow 1_{345^\circ}$