

Trigonometría y Complejos

- 1) Demostrar que: $\cos x + \operatorname{sen} x \operatorname{tg} x = \sec x$
- 2) Resolver la ecuación: $2 \operatorname{tg} x \sec x - \operatorname{tg} x = 0$
- 3) Resolver un triángulo del que conocemos: $a = 13$, $b = 5$ y $C = 100^\circ$
- 4) Hallar $(-2 + 2i)^6$, dando los resultados en polar, trigonométrica, binómica y cartesiana.
- 5) Hallar todos los complejos que son resultados de $\sqrt[5]{-1}$

$$\textcircled{1} \quad \boxed{\cos x + \operatorname{sen} x \operatorname{tg} x} = \cos x + \operatorname{sen} x \frac{\operatorname{sen} x}{\cos x} = \cos x + \frac{\operatorname{sen}^2 x}{\cos x} = \frac{\cos^2 x + \operatorname{sen}^2 x}{\cos x} = \frac{1}{\cos x} = \boxed{\sec x}$$

$$\textcircled{2} \quad 2 \operatorname{tg} x \sec x - \operatorname{tg} x = 0 \Rightarrow \operatorname{tg} x (2 \sec x - 1) = 0 \Rightarrow \begin{cases} \operatorname{tg} x = 0 \\ 2 \sec x - 1 = 0 \end{cases}$$

Si $\operatorname{tg} x = 0 \Rightarrow \boxed{x = 0^\circ + 180^\circ k, k \in \mathbb{Z}}$

Si $2 \sec x - 1 = 0 \Rightarrow 2 \sec x = 1 \Rightarrow \sec x = \frac{1}{2} \Rightarrow \frac{1}{\cos x} = \frac{1}{2} \Rightarrow 2 = \cos x$ } No es posible

$$\textcircled{3} \quad \text{2 lados y ángulo comprendidos} \Rightarrow \text{T. coseno y solución única.}$$

$$\boxed{c = \sqrt{a^2 + b^2 - 2ab \cos C} = \sqrt{169 + 25 - 130 \cos 100^\circ} = 14,72}$$

$$b^2 = a^2 + c^2 - 2ac \cos B \Rightarrow 2ac \cos B = a^2 + c^2 - b^2 \Rightarrow \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\Rightarrow \boxed{B = 19,55^\circ = 19^\circ 32' 52,2''} \Rightarrow \boxed{A = 180^\circ - 100^\circ - B = 60,45^\circ = 60^\circ 27' 25''}$$

$$\textcircled{4} \quad z = -2 + 2i \Rightarrow |z| = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2} \quad \operatorname{tg} x = \frac{2}{-2} = -1 \Rightarrow x = 45^\circ$$

$\begin{matrix} \nearrow \\ \text{2º cuadrante} \\ \searrow \end{matrix}$

$$\Rightarrow z = (2\sqrt{2})_{135^\circ} \Rightarrow \boxed{z^6 = [(2\sqrt{2})_{135^\circ}]^6} = (2\sqrt{2})_{135^\circ \cdot 6}^6 = [2^6 \cdot (\sqrt{2})^6]_{810^\circ} = (2^6 \cdot 2^3)_{90^\circ} = (2^9)_{90^\circ} = \sqrt{512}_{90^\circ}$$

Polár. $\boxed{z^6 = 512(\cos 90^\circ + i \operatorname{sen} 90^\circ)}$ Trigonometría.

Binómica $\boxed{z^6 = 512(0 + i(+1)) = +512i}$ Cartesiana

$$\textcircled{5} \quad z = -1 = 1_{180^\circ} \Rightarrow \sqrt[5]{-1} = \sqrt[5]{1_{180^\circ}}. \text{ Tiene 5 resultados, de módulo } \sqrt[5]{1} = 1 \text{ y argumentos:}$$

$$\beta_1 = \frac{180}{5} = 36^\circ \Rightarrow 1_{36^\circ}$$

$$\beta_2 = \frac{180}{5} + \frac{360}{5} = 108^\circ \Rightarrow 1_{108^\circ}$$

$$\beta_3 = \frac{180}{5} + \frac{360}{5} \cdot 2 = 180^\circ \Rightarrow 1_{180^\circ} = -1$$

$$\beta_4 = \frac{180}{5} + \frac{360}{5} \cdot 3 = 252^\circ \Rightarrow 1_{252^\circ}$$

$$\beta_5 = \frac{180}{5} + \frac{360}{5} \cdot 4 = 324^\circ \Rightarrow 1_{324^\circ}$$