

Discuta, según los valores del parámetro m , el sistema:

$$\begin{cases} x + (m-3)y + mz = 1 \\ (m-3)y + (m^2 - m)z = 1 \\ x + m^2z = 0 \end{cases}$$

Discuta, según los valores del parámetro m , el sistema:

$$\begin{cases} x + 2y = m \\ my + 3z = 1 \\ x + (m+2)y + (m+1)z = m+1 \end{cases}$$

Discuta, según los valores del parámetro m , el sistema:

$$\begin{cases} mx + y + z = 2m \\ mx + (m+1)y + z = 1 \\ mx + (m+1)y + 2z = m+1 \end{cases}$$

Resuélvelos para todos los casos en los que sean compatibles

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$$1) \quad A^* = \left(\begin{array}{ccc|c} 1 & u-3 & u & 1 \\ 0 & u-3 & u^2-u & 1 \\ 1 & 0 & u^2 & 0 \end{array} \right) \quad A = \begin{pmatrix} 1 & u-3 & u \\ 0 & u-3 & u^2-u \\ 1 & 0 & u^2 \end{pmatrix}$$

[Nota: como $C_2 = (u-3) \cdot C_1 \Rightarrow \text{rg } A = \text{rg } A^*$]

$$1^\circ) |A| = u^2(u-3) + (u^2-u)(u-3) - u \cdot (u-3) = 2u^2(u-3) - 2u(u-3) = 2u(u-3)(u-1)$$

$$2^\circ) |A| = 0 \Leftrightarrow \begin{cases} u=0 \\ u=3 \\ u=1 \end{cases}$$

por lo tto, de $u \neq 0, 1, 3 \Rightarrow |A| \neq 0 \Rightarrow \text{rg } A = 3 = \text{rg } A^* \Rightarrow$ SCD
Única Solución

3º) Estudiamos los casos nos que se anula $|A|$ e calculamos en cada caso, $\text{rg } A$ e $\text{rg } A^*$.

$$u=0$$

$$A^* \ni \begin{vmatrix} 1 & -3 \\ 0 & -3 \end{vmatrix} = -3 \neq 0$$

menor orde 2 $\neq 0$
de A \downarrow
 $\text{rg } A = 2$

$$A^* = \begin{pmatrix} 1 & -3 & 0 & 1 & 1 \\ 0 & -3 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -3 & 0 \\ 0 & -3 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^* \ni \begin{vmatrix} 1 & -3 & 1 \\ 0 & -3 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0 \quad \dots \quad \begin{matrix} \text{todos os menores de orde 3 son cero} \\ \text{(2ª fila } = C_2 : (-3)) \\ \text{e } C_3 = 0 \end{matrix} \text{ de } A^*$$

$$\Rightarrow \text{rg } A^* = 2$$

$$\Rightarrow \text{rg } A = 2 = \text{rg } A^* < 3 = \text{n}^\circ \text{ incógnitas} \Rightarrow \text{SCI} \text{ Inf. Soluciones}$$

$$\underline{u=3} \quad A^* = \begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 0 & 6 & 1 \\ 1 & 0 & 9 & 0 \end{pmatrix} \quad A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 0 & 6 \\ 1 & 0 & 9 \end{pmatrix}$$

A.)

$$\exists \text{ menor orde } 2 \text{ de } A : \begin{vmatrix} 1 & 3 \\ 0 & 6 \end{vmatrix} = 6 \neq 0 \Rightarrow \text{rg } A = 2$$

A*.)

$$\exists \begin{vmatrix} 1 & 3 & 1 \\ 0 & 6 & 1 \\ 1 & 9 & 0 \end{vmatrix} = 3 - 6 - 9 \neq 0 \Rightarrow \text{rg } A^* = 3$$

menor orde 3
de A^*

$$\Rightarrow \text{rg } A = 2 \neq 3 = \text{rg } A^* \Rightarrow \text{S. I. No hai soluci\u00f3n}$$

u=1

$$A^* = \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

A.)

$$\begin{pmatrix} C_2 = -2C_4 \\ \text{rg } A = \text{rg } A^* \end{pmatrix}$$

\exists menor orde

$$2 \text{ de } A \quad \begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} = -2 \neq 0 \Rightarrow \text{rg } A = 2$$

A*.) todos os menores de orde 3 de A^* son iguais a zero

$$\text{p.e. } \begin{cases} C_1 = C_3 \\ C_2 = -2C_4 \end{cases}$$

$$\Rightarrow \text{rg } A^* = 2$$

$$\Rightarrow \text{rg } A = 2 = \text{rg } A^* < 3 = n^{\circ} \text{ / u\u00f3nitas} \Rightarrow \text{S. I. Infinitas soluci\u00f3ns}$$

Conclusión

$$m \text{ ludo's} = 3$$

valor de m	$\text{rg } A$	$\text{rg } A^*$	clasificación
$m=0$	2	2	SCI
$m=1$	2	2	SCI
$m=3$	2	3	SJ
$m \neq 0, 1, 3$	3	3	SCD

Resolvemos los casos posibles:

$m=0$ SCI

$$\begin{cases} x - 3y = 1 & \text{---} \\ -3y = 1 \Rightarrow y = -1/3 \\ x = 0 \Rightarrow x = 0 \end{cases} \Rightarrow \left\{ \left(0, -\frac{1}{3}, \alpha \right); \alpha \in \mathbb{R} \right\}$$

α cq valor

$m=1$ SCI

$$\begin{cases} x - 2y + z = 1 & \text{---} \\ -2y = 1 \Rightarrow y = -1/2 \\ x + z = 0 \Rightarrow z = -x \end{cases} \Rightarrow \left\{ \left(+\alpha, -\frac{1}{2}, -\alpha \right), \alpha \in \mathbb{R} \right\}$$

α cq vr.

$m \neq 0, 1, 3$ SCD
Regra de Cramer

$$x = \frac{\begin{vmatrix} 1 & m-3 & m \\ 1 & m-3 & m^2 m \\ 0 & 0 & m^2 \end{vmatrix}}{|A|} = \frac{m^2(m-3) - m^2(m-3)}{|A|} = 0$$

$$y = \frac{\begin{vmatrix} 1 & 1 & m \\ 0 & 1 & m^2 m \\ 1 & 0 & m^2 \end{vmatrix}}{|A|} = \frac{m^2 + m^2 m - m}{|A|} = \frac{2m^2 - 2m}{|A|} = \frac{2m(m-1)}{2m(m-3)(m-1)} = \frac{1}{m-3}$$

$$z = \frac{\begin{vmatrix} 1 & m-3 & 1 \\ 1 & m-3 & 1 \\ 0 & 0 & 0 \end{vmatrix}}{|A|} = 0 \quad \text{de } \left\{ \left(0, \frac{1}{m-3}, 0 \right) \right\}$$

②

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & u & 3 \\ 1 & u+2 & u+1 \end{pmatrix}$$

$$A^* = \begin{pmatrix} 1 & 2 & 0 & | & u \\ 0 & u & 3 & | & 1 \\ 1 & u+2 & u+1 & | & u+1 \end{pmatrix}$$

$$|A| = u(u+1) + 6 - 3(u+2) = u^2 - 2u$$

$$|A| = 0 \Leftrightarrow \begin{cases} u = 0 \\ u = 2 \end{cases}$$

Polos tto, de $u \neq 0, 2$, $|A| \neq 0 \Rightarrow \text{rg } A = 3 = \text{rg } A^* \Rightarrow$ SCD
 "u" lue d'guitas Sol. Única

Estudamos os casos nos que $|A| = 0$.

$$\underline{u = 0}$$

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \\ 1 & 2 & 1 \end{pmatrix} \quad A^* = \begin{pmatrix} 1 & 2 & 0 & | & 0 \\ 0 & 0 & 3 & | & 1 \\ 1 & 2 & 1 & | & 1 \end{pmatrix}$$

$$A? \exists \text{ menor orde 2 en } A \quad \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6 \neq 0 \Rightarrow \text{rg } A = 2$$

$$A^*? \exists \text{ menor orde 3 en } A^* \quad \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 3 - 1 = 2 \neq 0 \Rightarrow \text{rg } A^* = 3$$

$$\Rightarrow \text{rg } A = 2 \neq 3 = \text{rg } A^* \Rightarrow \text{S.I. Non hai solución}$$

$$\underline{u = 2}$$

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 4 & 3 \end{pmatrix} \quad A^* = \begin{pmatrix} 1 & 2 & 0 & | & 2 \\ 0 & 2 & 3 & | & 1 \\ 1 & 4 & 3 & | & 3 \end{pmatrix} \quad \begin{cases} F_3 = F_1 + F_2 \\ \Rightarrow \text{rg } A^* \leq 2 \end{cases}$$

$$A? \exists \text{ menor orde 2 en } A: \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 \neq 0 \Rightarrow \text{rg } A = 2$$

$$A^*? \text{ todos os menores de orde 3 de } A^* \text{ son iguais a } 0$$

$$\Rightarrow \text{rg } A^* = 2$$

$$\text{Polos tto, } \text{rg } A = 2 = \text{rg } A^* < 3 = \text{u}^{\circ} \text{ lue d'guitas} \Rightarrow \text{SCT} / \text{luf. solucións}$$

Resumo n° de equações = 3

Valor de m	$\text{rg } A$	$\text{rg } A^*$	Classificação
$m = 0$	2	3	SI - não há sol
$m = 2$	2	2	SCI - inf. sol.
$m \neq 0, 2$	3	3	SCD - única sol.

Resolvemos os casos possíveis:

$m = 2$

$$\begin{cases} x + 2y = 2 \\ 2y + 3z = 1 \\ x + 4y + 3z = 3 \end{cases} \rightarrow \begin{cases} x = 2 - 2y = 2 - (1 - 3z) = 1 + 3z \\ y = \frac{1 - 3z}{2} \\ z \text{ c.q. real.} \end{cases}$$

$$\Rightarrow \left\{ \left(1 + 3z, \frac{1 - 3z}{2}, z \right); z \in \mathbb{R} \right\}$$

$m \neq 0, 2$ Regra de Cramer

$$\begin{aligned} x &= \frac{\begin{vmatrix} m & 2 & 0 \\ 1 & m & 3 \\ m+1 & m+2 & m+1 \end{vmatrix}}{|A|} = \frac{m^2(m+1) + 6(m+1) - 3m(m+2) - 2(m+1)}{|A|} \\ &= \frac{m^3 + m^2 + 6m + 6 - 3m^2 - 6m - 2m - 2}{|A|} = \frac{m^3 - 2m^2 - 2m + 4}{|A|} \\ &= \frac{(m-2)(m^2-2)}{m(m-2)} = \frac{m-2}{m} \end{aligned}$$

$$\begin{aligned} y &= \frac{\begin{vmatrix} 1 & m & 0 \\ 1 & 1 & 3 \\ 1 & m+2 & m+1 \end{vmatrix}}{|A|} = \frac{m+1 + 3m - 3(m+1)}{|A|} = \frac{m-2}{m(m-2)} = \frac{1}{m} \end{aligned}$$

$$z = \frac{\begin{vmatrix} 1 & 2 & m \\ 0 & m & 1 \\ 1 & m+2 & m+1 \end{vmatrix}}{|A|} = \frac{m(m+1) + 2 - m^2 - (m+2)}{|A|} = 0$$

$$\text{Sol } \left\{ \left(\frac{m-2}{m}, \frac{1}{m}, 0 \right) \right\}$$

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③

$$A = \begin{pmatrix} u & 1 & 1 \\ u & u+1 & 1 \\ u & u+1 & 2 \end{pmatrix} \quad A^* = \begin{pmatrix} u & 1 & 1 & 2u \\ u & u+1 & 1 & 1 \\ u & u+1 & 2 & u+1 \end{pmatrix}$$

$$|A| = 2u(u+1) + u(u+1) + u - u(u+1) - u(u+1) - 2u = \\ = u(u+1) - u = u^2$$

$$|A| = 0 \Leftrightarrow u = 0$$

→ Pelo tto, de $u \neq 0$, $|A| \neq 0 \Rightarrow \text{rg } A = 3 = \text{rg } A^* \Rightarrow$ SCD
Sol. Única

Se $u = 0$

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} \quad A^* = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{pmatrix}$$

$$A) \text{ } \oint \text{ menor de orde } 2 : \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1 \neq 0 \Rightarrow \text{rg } A = 2$$

$$A^*) \text{ } \oint \text{ menor de orde } 3 : \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 1 + 1 - 2 - 1 = -1 \neq 0 \Rightarrow \\ = \text{Drg } A^* = 3$$

$\Rightarrow \text{Drg } A = 2 \neq 3 = \text{rg } A^* \Rightarrow$ S. J
Não há solução

Resumo

• $u \neq 0$ $\text{rg } A = 3 = \text{rg } A^* \Rightarrow$ SCD

• $u = 0$ $\text{rg } A = 2 \neq 3 = \text{rg } A^* \Rightarrow$ S J

Resolvendo no caso $u \neq 0$

$$x = \frac{1}{|A|} \cdot \begin{vmatrix} 2u & 1 & 1 \\ 1 & u+1 & 1 \\ u+1 & u+1 & 2 \end{vmatrix} = \frac{4u(u+1) + (u+1) + u+1 - (u+1)^2 - 2u(u+1) - 2}{|A|} = \\ = \frac{4u^2 + 4u + 2u + 2 - u^2 - 2u - 1 - 2u^2 - 2u - 2}{u^2} = \\ = \frac{u^2 + 2u - 1}{u^2}$$

$$y = \frac{1}{|A|} \cdot \begin{vmatrix} u & 2u & 1 \\ u & 1 & 1 \\ u & u+1 & 2 \end{vmatrix} = \frac{2u + u(u+1) + 2u^2 - u - u(u+1) - 2u \cdot 2u}{|A|}$$

$$= \frac{2u + u^2 + u^2 + \cancel{u} - \cancel{u} - u^2 - u - 4u^2}{u^2} = \frac{-2u^2 + u}{u^2} = \frac{u-2}{u}$$

$$z = \frac{1}{|A|} \cdot \begin{vmatrix} u & 1 & 2u \\ u & u+1 & 1 \\ u & u+1 & u+1 \end{vmatrix} = \frac{u(u+1)^2 + 2u^2(u+1) + u - 2u^2(u+1)}{|A|}$$

$$= \frac{u(u^2 + 2u + 1) + 2u^3 + 2u^2 + u - 2u^3 - 2u^2 - 2u^2 - 2u}{|A|}$$

$$= \frac{u^3 + 2u^2 + u^2 + u - 2u^2 - 2u}{u^2} = u$$

$$\text{Sol. } \left\{ \left(\frac{u^2 + 2u - 1}{u^2}, \frac{u-2}{u}, u \right) \right\}$$

$$\left/ \frac{2u - \frac{1-2u}{u} - u}{u} = \frac{2u^2 - 1 + 2u - u^2}{u^2} = \frac{u^2 + 2u - 1}{u^2} \right/$$