

- Calcule las siguientes integrales

$$\int \frac{\operatorname{sen} 3x}{e^x} dx$$

$$\int \frac{\operatorname{sen} x}{5 - 2\cos x} dx$$

$$\int \frac{\ln x}{\sqrt{x}} dx$$

$$\int \frac{x}{\cos^2 x} dx$$

$$\int \frac{x^2}{4+x^6} dx$$

$$\int \sqrt{x^4 - 5x^2} dx$$

$$\int \frac{2x^3 - 9x^2 + 9x + 6}{x^2 - 5x + 6} dx$$

- Calcule las siguientes integrales:

$$\int \frac{\sin 3x}{e^x} dx = \int e^{-x} \cdot \sin 3x dx \rightarrow \text{por partes}$$

$$u = \sin 3x \rightarrow du = 3 \cos 3x dx$$

$$dv = e^{-x} dx \rightarrow v = \int e^{-x} dx = -e^{-x}$$

$$\int e^{-x} \sin 3x dx = -e^{-x} \cdot \sin 3x - \int -e^{-x} 3 \cos 3x dx =$$

$$\Rightarrow u = \cos 3x \rightarrow du = -3 \sin 3x dx$$

$$dv = e^{-x} dx \rightarrow v = \int e^{-x} dx = -e^{-x}$$

$$= -e^{-x} \cdot \sin 3x + 3 \left[-e^{-x} \cdot \cos 3x - \int +e^{-x} \cdot 3 \sin 3x dx = \right.$$

$$\Rightarrow \int e^{-x} \sin 3x dx = -e^{-x} \sin 3x - 3e^{-x} \cos 3x - 9 \cdot I$$

$$I \cdot \int e^{-x} \sin 3x dx \Rightarrow$$

$$I = \int e^{-x} \sin 3x dx$$

$$I = -e^{-x} \sin 3x - 3e^{-x} \cos 3x - 9 \cdot I$$

$$9I + I = -e^{-x} \sin 3x - 3e^{-x} \cos 3x$$

$$I = \frac{-e^{-x} (\sin 3x + 3 \cos 3x)}{10} \Rightarrow$$

$$\int e^{-x} \sin 3x dx = \frac{-e^{-x} (\sin 3x + 3 \cos 3x)}{10} + C$$

$$\begin{aligned} \int \frac{\sin x}{5-2\cos x} dx &= \frac{1}{2} \int \frac{2\sin x}{5-2\cos x} dx = \\ &= \frac{1}{2} \ln |5-2\cos x| + C \end{aligned}$$

$$\int \frac{3x-8}{x^3+4x} dx = \int \frac{3x-8}{x(x^2+4)} = \int \frac{-2}{x} dx + \int \frac{2x+3}{x^2+4} dx =$$

$$\frac{3x-8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$3x-8 = A(x^2+4) + Bx(x-2) + Cx(x+2)$$

$$\text{SP } x=2 \rightarrow 6-8 = 8A + 8C$$

$$\text{SP } x=-2 \rightarrow -6-8 = 8A + 8B$$

$$x=0 \rightarrow -8 = 4A \rightarrow \boxed{A = -2}$$

$$\rightarrow B = \frac{-14+16}{8} = \frac{2}{8} = \boxed{\frac{1}{4}}$$

$$C = \frac{-2+16}{8} = \frac{14}{8} = \boxed{\frac{7}{4}}$$

$$= \int \frac{-2}{x} + \int \frac{1}{4x+8} dx + \int \frac{7}{4x-2} dx =$$

$$= -2 \ln |x| +$$

$$\bullet \int \frac{\ln x}{\sqrt{x}} dx = \left[\begin{array}{l} \sqrt{x} = t \Rightarrow x = t^2 \\ \frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2\sqrt{x} dt \\ dx = 2t dt. \end{array} \right.$$

$$\Rightarrow \int \frac{\ln t^2}{t} 2t dt = \int \frac{2 \ln t \cdot 2t dt}{t} = 4 \int \ln t dt =$$

$$= 4(t \ln t - t) + C = 4(\sqrt{x} \ln \sqrt{x} - \sqrt{x}) + C$$

$$= 4 \left(\frac{\sqrt{x} \ln x}{2} - \sqrt{x} \right) + C = \boxed{2\sqrt{x} \ln x - 4\sqrt{x} + C}$$

$$\bullet \int \frac{x}{\cos^2 x} dx = \left[\begin{array}{l} u = x \Rightarrow du = dx \\ dv = \frac{1}{\cos^2 x} \Rightarrow v = \operatorname{tg} x \end{array} \right.$$

$$\Rightarrow x \cdot \operatorname{tg} x - \int \operatorname{tg} x \cdot dx = x \operatorname{tg} x - \int \frac{\operatorname{sen} x}{\cos x} dx =$$

$$= x \operatorname{tg} x - (-\ln(\cos x)) + C = x \operatorname{tg} x + \ln(\cos x) + C$$

$$\bullet \int \frac{x^2}{4+x^6} dx = \int \frac{x^2}{4+(x^3)^2} dx = \int \frac{\frac{x^2}{4}}{\frac{4}{4} + \frac{(x^3)^2}{4}} dx = \int \frac{\frac{x^2}{4}}{1 + \left(\frac{x^3}{2}\right)^2} dx =$$

$$= \frac{1}{4} \int \frac{x^2}{1 + \left(\frac{x^3}{2}\right)^2} dx = \frac{1}{4} \cdot \frac{2}{3} \int \frac{3x^2/2}{1 + \left(\frac{x^3}{2}\right)^2} dx = \frac{1}{6} \operatorname{arctg} \frac{x^3}{2} + C$$

$$\begin{aligned}
 \int \sqrt{x^4 - 5x^2} dx &= \int \sqrt{x^2(x^2 - 5)} dx = \int x \sqrt{x^2 - 5} dx = \\
 &= \frac{1}{2} \int 2x(x^2 - 5)^{1/2} dx = \frac{1}{2} \frac{(x^2 - 5)^{1/2 + 1}}{\frac{1}{2} + 1} + C = \\
 &= \frac{1}{2} \frac{(x^2 - 5)^{3/2}}{3/2} + C = \frac{\sqrt{(x^2 - 5)^3}}{3} + C = \frac{(x^2 - 5)\sqrt{x^2 - 5}}{3} + C
 \end{aligned}$$

$$\bullet \int \frac{2x^3 - 9x^2 + 9x + 6}{x^2 - 5x + 6} dx \Rightarrow$$

$$2x^3 - 9x^2 + 9x + 6 = (2x + 1)(x^2 - 5x + 6) + 2x$$

$$\Rightarrow \int (2x + 1) dx + \int \frac{2x}{x^2 - 5x + 6}$$

$$\frac{2x}{x^2 - 5x + 6} = \frac{A}{x - 2} + \frac{B}{x - 3} = \frac{A(x - 3) + B(x - 2)}{(x - 2)(x - 3)} \left. \vphantom{\frac{2x}{x^2 - 5x + 6}} \right\}$$

$$\Rightarrow \left. \begin{aligned} A + B &= 2 \\ -3A - 2B &= 0 \end{aligned} \right\} \begin{aligned} A &= -4 \\ B &= 6 \end{aligned}$$

$$\Rightarrow \int (2x + 1) dx - 4 \int \frac{1}{x - 2} + 6 \int \frac{1}{x - 3} dx =$$

$$x^2 + x - 4 \ln|x - 2| + 6 \ln|x - 3| + C$$