

RELACIÓN DE EJERCICIOS DE INTEGRALES.

1. $\int x^5 dx$

$$\int x^5 dx = \frac{x^{5+1}}{5+1} + k = \frac{x^6}{6} + k$$

2. $\int (x + \sqrt{x}) dx$

$$\begin{aligned} \int (x + \sqrt{x}) dx &= \int (x + x^{1/2}) dx = \frac{x^{1+1}}{1+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + k = \frac{x^2}{2} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{x^2}{2} + \frac{2\sqrt{x^3}}{3} + k = \\ &= \frac{x^2}{2} + \frac{2x\sqrt{x}}{3} + k \end{aligned}$$

3. $\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4} \right) dx$

Sol: $6\sqrt{x} - \frac{1}{10}x^2\sqrt{x} + k$

$$\begin{aligned} \int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4} \right) dx &= \int \left(3x^{-\frac{1}{2}} - \frac{1}{4}x^{\frac{3}{2}} \right) dx = 3 \cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - \frac{1}{4} \cdot \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + k = 3 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{4} \cdot \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + k = \\ &= 6\sqrt{x} - \frac{1}{2} \frac{\sqrt{x^5}}{5} + k = 6\sqrt{x} - \frac{1}{10}x^2\sqrt{x} + k \end{aligned}$$

4. $\int \frac{x^2 dx}{\sqrt{x}}$

Sol: $\frac{2}{5}x^2\sqrt{x} + k$

$$\int \frac{x^2 dx}{\sqrt{x}} = \int \frac{x^2}{x^{\frac{1}{2}}} dx = \int x^{\frac{3}{2}} dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + k = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + k = \frac{2\sqrt{x^5}}{5} + k = \frac{2x^2\sqrt{x}}{5} + k$$

5. $\int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2 \right) dx$

Sol: $-\frac{1}{x} - \frac{8}{\sqrt{x}} + 2x + k$

$$\begin{aligned} \int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2 \right) dx &= \int \left(x^{-2} + 4x^{-\frac{3}{2}} + 2 \right) dx = \frac{x^{-2+1}}{-2+1} + 4 \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + 2x + k = \\ &= \frac{x^{-1}}{-1} + 4 \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 2x + k = -\frac{1}{x} - 8 \cdot \frac{1}{x^{\frac{1}{2}}} + 2x + k = -\frac{1}{x} - \frac{8}{\sqrt{x}} + 2x + k \end{aligned}$$

6. $\int \frac{dx}{\sqrt[4]{x}}$ Sol: $\frac{4}{3}\sqrt[4]{x^3} + k$

$$\int \frac{dx}{\sqrt[4]{x}} = \int x^{-\frac{1}{4}} dx = \frac{x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1} + k = \frac{x^{\frac{3}{4}}}{\frac{3}{4}} + k = \frac{4x^{\frac{3}{4}}}{3} + k = \frac{4}{3}\sqrt[4]{x^3} + k$$

7. $\int \left(x^2 + \frac{1}{\sqrt[3]{x}}\right)^2 dx$ Sol: $\frac{x^5}{5} + \frac{3}{4}x^2\sqrt[3]{x^2} + 3\sqrt[3]{x} + k$

$$\begin{aligned} \int \left(x^2 + \frac{1}{\sqrt[3]{x}}\right)^2 dx &= \int \left(x^2 + x^{-\frac{1}{3}}\right)^2 dx = \int (x^4 + 2x^{\frac{5}{3}} + x^{-\frac{2}{3}}) dx = \frac{x^5}{5} + 2 \cdot \frac{x^{\frac{8}{3}}}{\frac{8}{3}} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + K = \\ &= \frac{1}{5} \cdot x^5 + \frac{3}{4} \cdot x^{\frac{8}{3}} + 3 \cdot x^{\frac{1}{3}} + K = \frac{1}{5} \cdot x^5 + \frac{3}{4} \cdot \sqrt[3]{x^8} + 3 \cdot \sqrt[3]{x} + K = \\ &= \frac{1}{5} \cdot x^5 + \frac{3}{4} \cdot x^2 \cdot \sqrt[3]{x^2} + 3 \cdot \sqrt[3]{x} + K \end{aligned}$$

8. $\int \frac{Lx}{x} \cdot dx$ Sol: $\frac{1}{2}L^2x + k$

$$\int \frac{\text{Ln}(x)}{x} \cdot dx = \int \text{Ln}(x) \cdot \frac{1}{x} \cdot dx = \left\{ \int f^a \cdot f' dx \right\} = \frac{(\text{Ln}(x))^2}{2} + k$$

9. $\int \text{tg } x \cdot \sec^2 x \cdot dx$ Sol: $\frac{1}{2} \cdot \text{tg}^2 x + k$

$$\int \text{tg } x \cdot \sec^2 x \cdot dx = \left\{ \int f^1 \cdot f' dx \right\} = \frac{(\text{tg } x)^2}{2} + k$$

10. $\int \text{sen}^2 x \cdot \cos x \cdot dx$ Sol: $\frac{\text{sen}^3 x}{3} + k$

$$\int \text{sen}^2 x \cdot \cos x \cdot dx = \left\{ \int f^2 \cdot f' dx \right\} = \frac{\text{sen}^3 x}{3} + k$$

11. $\int \cos^3 x \cdot \text{sen } x \cdot dx$ Sol: $-\frac{\cos^4 x}{4} + k$

$$\int \cos^3 x \cdot \text{sen } x \cdot dx = - \int \underbrace{\cos^2 x}_{f^2} \cdot \underbrace{(-\text{sen } x)}_{f'} \cdot dx = \frac{\cos^4 x}{4} + k$$

12. $\int x\sqrt{x^2+1} \cdot dx$ Sol: $\frac{1}{3}\sqrt{(x^2+1)^3} + k$

$$\int x\sqrt{x^2+1} \cdot dx = \frac{1}{2} \int \underbrace{2x}_{f'} \cdot \underbrace{(x^2+1)^{\frac{1}{2}}}_{f^{1/2}} dx = \frac{1}{2} \cdot \frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{\sqrt{(x^2+1)^3}}{3} + k$$

13. $\int \frac{xdx}{\sqrt{2x^2+3}}$ Sol: $\frac{1}{2}\sqrt{2x^2+3} + k$

$$\int \frac{xdx}{\sqrt{2x^2+3}} = \frac{1}{4} \int \underbrace{4x}_{f'} \cdot \underbrace{(2x^2+3)^{-\frac{1}{2}}}_{f^{-1/2}} dx = \frac{1}{4} \cdot \frac{(2x^2+3)^{\frac{1}{2}}}{\frac{1}{2}} + k = \frac{1}{2}\sqrt{2x^2+3} + k$$

$$\int \frac{xdx}{\sqrt{2x^2+3}} = \frac{2}{4} \int \frac{4xdx}{2\sqrt{2x^2+3}} = \left\{ \int \frac{f'}{2\sqrt{f}} dx = \sqrt{f} + k \right\} = \frac{1}{2}\sqrt{2x^2+3} + k$$

14. $\int \frac{x^2 dx}{\sqrt{x^3+1}}$ Sol: $\frac{2}{3}\sqrt{x^3+1} + k$

$$\int \frac{x^2 dx}{\sqrt{x^3+1}} = \int x^2(x^3+1)^{-\frac{1}{2}} dx = \frac{1}{3} \int 3x^2(x^3+1)^{-\frac{1}{2}} dx = \frac{1}{3} \cdot \frac{(x^3+1)^{\frac{1}{2}}}{\frac{1}{2}} + k = \frac{2}{3}\sqrt{x^3+1} + k$$

$$\int \frac{x^2 dx}{\sqrt{x^3+1}} = \frac{2}{3} \int \frac{3x^2 dx}{2\sqrt{x^3+1}} = \left\{ \int \frac{f'}{2\sqrt{f}} dx = \sqrt{f} + k \right\} = \frac{2}{3}\sqrt{x^3+1} + k$$

15. $\int \frac{\cos x}{\text{sen}^2 x} dx$ Sol: $-\frac{1}{\text{sen } x} + k$

$$\int \frac{\cos x}{\text{sen}^2 x} dx = \int \underbrace{\cos x}_{f'} \cdot \underbrace{\text{sen}^{-2} x}_{f^{-2}} dx = \frac{\text{sen}^{-1} x}{-1} + k = -\frac{1}{\text{sen } x} + k$$

16. $\int x \cdot (x^2+1)^4 dx$ Sol: $\frac{(x^2+1)^5}{10} + k$

$$\int x \cdot (x^2+1)^4 dx = \frac{1}{2} \int \underbrace{2x}_{f'} \cdot \underbrace{(x^2+1)^4}_{f^4} dx = \frac{1}{2} \cdot \frac{(x^2+1)^5}{5} + k = \frac{(x^2+1)^5}{10} + k$$

17. $\int \frac{\text{sen } x}{\cos^3 x} \cdot dx$ Sol: $\frac{1}{2\cos^2 x} + k$

$$\int \frac{\text{sen } x}{\cos^3 x} \cdot dx = - \int \underbrace{-\text{sen } x}_{f'} \cdot \underbrace{\cos^{-3} x}_{f^{-3}} dx = -\frac{\cos^{-2} x}{-2} + k = \frac{1}{2\cos^2 x} + k$$

18. $\int \frac{\operatorname{tg} x}{\cos^2 x} \cdot dx$ Sol: $\frac{\operatorname{tg}^2 x}{2} + k$

$$\int \frac{\operatorname{tg} x}{\cos^2 x} \cdot dx = \int \underbrace{\operatorname{tg} x}_{f'} \cdot \underbrace{\frac{1}{\cos^2 x}}_{f'} \cdot dx = \frac{\operatorname{tg}^2 x}{2} + k$$

19. $\int \frac{\operatorname{cotg} x}{\operatorname{sen}^2 x} \cdot dx$ Sol: $-\frac{\operatorname{cotg}^2 x}{2} + k$

$$\int \frac{\operatorname{cotg} x}{\operatorname{sen}^2 x} \cdot dx = - \int \operatorname{cotg} x \cdot \frac{-1}{\operatorname{sen}^2 x} \cdot dx = -\frac{\operatorname{cotg}^2 x}{2} + k$$

20. $\int \frac{1}{\cos^2 x \sqrt{\operatorname{tg} x - 1}} \cdot dx$ Sol: $2\sqrt{\operatorname{tg} x - 1} + k$

$$\int \frac{1}{\cos^2 x \sqrt{\operatorname{tg} x - 1}} \cdot dx = \int \frac{1}{\cos^2 x} \cdot (\operatorname{tg} x - 1)^{-\frac{1}{2}} \cdot dx = \frac{(\operatorname{tg} x - 1)^{\frac{1}{2}}}{\frac{1}{2}} + k = 2\sqrt{\operatorname{tg} x - 1} + k$$

21. $\int \frac{L(x+1)}{x+1} dx$ Sol: $\frac{L^2(x+1)}{2} + k$

$$\int \frac{L(x+1)}{x+1} dx = \int \underbrace{L(x+1)}_{f'} \cdot \underbrace{\frac{1}{x+1}}_{f'} dx = \frac{L^2(x+1)}{2} + k$$

22. $\int \frac{\cos x}{\sqrt{2\operatorname{sen} x + 1}} dx$ Sol: $\sqrt{2\operatorname{sen} x + 1} + k$

$$\int \frac{\cos x}{\sqrt{2\operatorname{sen} x + 1}} dx = \frac{1}{2} \int \underbrace{2\cos x}_{f'} \cdot \underbrace{(2\operatorname{sen} x + 1)^{-\frac{1}{2}}}_{f^{-1/2}} dx = \frac{1}{2} \cdot \frac{(2\operatorname{sen} x + 1)^{\frac{1}{2}}}{\frac{1}{2}} + k = \sqrt{2\operatorname{sen} x + 1} + k$$

23. $\int \frac{\operatorname{sen} 2x}{(1 + \cos 2x)^2} dx$ Sol: $\frac{1}{2(1 + \cos 2x)} + k$

$$\int \frac{\operatorname{sen} 2x}{(1 + \cos 2x)^2} dx = -\frac{1}{2} \int -2\operatorname{sen} 2x \cdot (1 + \cos 2x)^{-2} dx = -\frac{1}{2} \cdot \frac{(1 + \cos 2x)^{-1}}{-1} + k =$$

$$= \frac{1}{2(1 + \cos 2x)} + k$$

24. $\int \frac{\text{sen}2x}{\sqrt{1+\text{sen}^2x}} dx$ Sol: $2\sqrt{1+\text{sen}^2x} + k$

$$\int \frac{\text{sen}2x}{\sqrt{1+\text{sen}^2x}} dx = \int \text{sen}2x(1+\text{sen}^2x)^{-\frac{1}{2}} dx = \int \underbrace{2\text{sen}x \cdot \text{cos}x}_{f'} \underbrace{(1+\text{sen}^2x)^{-\frac{1}{2}}}_{f^{-1/2}} dx =$$

$$= \frac{(1+\text{sen}^2x)^{\frac{1}{2}}}{\frac{1}{2}} + k = 2\sqrt{1+\text{sen}^2x} + k$$

25. $\int \frac{\sqrt{\text{tg}x+1}}{\text{cos}^2x} \cdot dx$ Sol: $\frac{2}{3}\sqrt{(\text{tg}x+1)^3} + k$

$$\int \frac{\sqrt{\text{tg}x+1}}{\text{cos}^2x} \cdot dx = \int (\text{tg}x+1)^{\frac{1}{2}} \cdot \frac{1}{\text{cos}^2x} \cdot dx = \frac{(\text{tg}x+1)^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{2}{3}\sqrt{(\text{tg}x+1)^3} + k$$

26. $\int \frac{\text{cos}2x}{(2+3\text{sen}2x)^3} dx$ Sol: $-\frac{1}{12} \frac{1}{(2+3\text{sen}2x)^2} + k$

$$\int \frac{\text{cos}2x}{(2+3\text{sen}2x)^3} dx = \frac{1}{6} \int 6\text{cos}2x \cdot (2+3\text{sen}2x)^{-3} dx = \frac{1}{6} \cdot \frac{(2+3\text{sen}2x)^{-2}}{-2} + k =$$

$$= -\frac{1}{12} \frac{1}{(2+3\text{sen}2x)^2} + k$$

27. $\int \frac{\text{sen}3x}{\sqrt[3]{\text{cos}^43x}} dx$ Sol: $\frac{1}{\sqrt[3]{\text{cos}3x}} + k$

$$\int \frac{\text{sen}3x}{\sqrt[3]{\text{cos}^43x}} dx = -\frac{1}{3} \int \underbrace{-3\text{sen}3x}_{f'} \underbrace{\text{cos}^{-\frac{4}{3}}3x}_{f^{-4/3}} dx = -\frac{1}{3} \cdot \frac{\text{cos}^{-\frac{1}{3}}3x}{-\frac{1}{3}} + k = \frac{1}{\sqrt[3]{\text{cos}3x}} + k$$

28. $\int \frac{\text{Ln}^2x}{x} dx$ Sol: $\frac{\text{Ln}^3x}{3} + k$

$$\int \frac{\text{Ln}^2x}{x} dx = \int \text{Ln}^2x \cdot \frac{1}{x} dx = \frac{\text{Ln}^3x}{3} + k$$

$$29. \int \frac{\arcsen x \, dx}{\sqrt{1-x^2}} \quad \text{Sol: } \frac{\arcsen^2 x}{2} + k$$

$$\int \frac{\arcsen x \, dx}{\sqrt{1-x^2}} = \int \arcsen x \cdot \frac{1}{\sqrt{1-x^2}} \, dx = \frac{\arcsen^2 x}{2} + k$$

$$30. \int \frac{\arccos^2 x \, dx}{\sqrt{1-x^2}} \quad \text{Sol: } -\frac{\arccos^3 x}{3} + k$$

$$\int \frac{\arccos^2 x \, dx}{\sqrt{1-x^2}} = -\int \arccos^2 x \cdot \frac{-1}{\sqrt{1-x^2}} \, dx = -\frac{\arccos^3 x}{3} + k$$

$$31. \int \frac{\arctg x}{1+x^2} \, dx \quad \text{Sol: } \frac{\arctg^2 x}{2} + k$$

$$\int \frac{\arctg x}{1+x^2} \, dx = \int \arctg x \cdot \frac{1}{1+x^2} \, dx = \frac{\arctg^2 x}{2} + k$$

$$32. \int \frac{\text{arctg} x}{1+x^2} \, dx \quad \text{Sol: } -\frac{\text{arctg}^2 x}{2} + k$$

$$\int \frac{\text{arctg} x}{1+x^2} \, dx = -\int \text{arctg} x \cdot \frac{-1}{1+x^2} \, dx = -\frac{\text{arctg}^2 x}{2} + k$$

$$33. \int \frac{x}{x^2+1} \, dx \quad \text{Sol: } \frac{1}{2} \text{Ln}(x^2+1) + k$$

$$\int \frac{x}{x^2+1} \, dx = \frac{1}{2} \int \frac{2x}{x^2+1} \, dx = \left\{ \int \frac{f'}{f} \, dx = \text{Ln}|f| + k \right\} = \frac{1}{2} \text{Ln}(x^2+1) + k$$

$$34. \int \frac{dx}{1-x} \quad \text{Sol: } -\text{Ln}|1-x| + k$$

$$\int \frac{dx}{1-x} = -\int \frac{-1}{1-x} \cdot dx = \left\{ \int \frac{f'}{f} \, dx = \text{Ln}|f| + k \right\} = -\text{Ln}|1-x| + k$$

$$35. \int \frac{dx}{3x-7} \quad \text{Sol: } \frac{1}{3} \text{L}|3x-7| + k$$

$$\int \frac{dx}{3x-7} = \frac{1}{3} \int \frac{3}{3x-7} \cdot dx = \left\{ \int \frac{f'}{f} \, dx = \text{Ln}|f| + k \right\} = \frac{1}{3} \text{Ln}|3x-7| + k$$

$$36. \int \frac{dx}{5-2x} \quad \text{Sol: } -\frac{1}{2} \text{L} |5-2x| + k$$

$$\int \frac{dx}{5-2x} = -\frac{1}{2} \int \frac{-2}{5-2x} \cdot dx = \left\{ \int \frac{f'}{f} dx = \text{Ln} |f| + k \right\} = -\frac{1}{2} \text{Ln} |5-2x| + k$$

$$37. \int \frac{x+1}{x^2+2x+3} dx \quad \text{Sol: } \frac{1}{2} \text{L} |x^2+2x+3| + k$$

$$\int \frac{x+1}{x^2+2x+3} dx = \frac{1}{2} \int \frac{2(x+1)}{x^2+2x+3} dx = \frac{1}{2} \int \frac{2x+2}{x^2+2x+3} dx = \frac{1}{2} \text{L} |x^2+2x+3| + k$$

$$38. \int \frac{dx}{x \cdot \text{Ln} x} \quad \text{Sol: } \text{Ln} | \text{Ln} x | + k$$

$$\int \frac{dx}{x \cdot \text{Ln} x} = \int \frac{\frac{1}{x} dx}{\text{Ln} x} = \left\{ \int \frac{f'}{f} dx = \text{Ln} |f| + k \right\} = \text{Ln} | \text{Ln} x | + k$$

$$39. \int \text{tg} x dx \quad \text{Sol: } -\text{Ln} | \cos x | + k$$

$$\int \text{tg} x \cdot dx = \int \frac{\text{sen} x}{\cos x} \cdot dx = - \int \frac{-\text{sen} x}{\cos x} \cdot dx = -\text{Ln} | \cos x | + K$$

$$40. \int \text{tg} 2x dx \quad \text{Sol: } -\frac{1}{2} \text{L} | \cos 2x | + k$$

$$\int \text{tg} 2x \cdot dx = \int \frac{\text{sen} 2x}{\cos 2x} \cdot dx = -\frac{1}{2} \int \frac{-2\text{sen} 2x}{\cos 2x} \cdot dx = -\frac{1}{2} \text{Ln} | \cos 2x | + K$$

$$41. \int \text{ctg} x dx \quad \text{Sol: } \text{Ln} | \text{sen} x | + k$$

$$\int \text{ctg} x \cdot dx = \int \frac{\cos x}{\text{sen} x} \cdot dx = \text{Ln} | \text{sen} x | + K$$

$$42. \int \text{ctg} (5x-7) dx \quad \text{Sol: } \frac{1}{5} \text{L} | \text{sen} (5x-7) | + k$$

$$\int \text{ctg} (5x-7) \cdot dx = \int \frac{\cos(5x-7)}{\text{sen}(5x-7)} \cdot dx = \frac{1}{5} \int \frac{5\cos(5x-7)}{\text{sen}(5x-7)} \cdot dx = \frac{1}{5} \text{Ln} | \text{sen}(5x-7) | + K$$

$$43. \int \frac{dx}{\text{ctg} 3x} \quad \text{Sol: } -\frac{1}{3} \text{L} | \cos 3x | + k$$

$$\int \frac{dx}{\text{ctg} 3x} = \int \text{tg} 3x \cdot dx = \int \frac{\text{sen} 3x}{\cos 3x} \cdot dx = -\frac{1}{3} \int \frac{-3\text{sen} 3x}{\cos 3x} \cdot dx = -\frac{1}{3} \text{Ln} | \cos 3x | + K$$

44. $\int \operatorname{ctg} \frac{x}{3} dx$ Sol: $3L \left| \operatorname{sen} \frac{x}{3} \right| + k$

$$\int \operatorname{ctg} \frac{x}{3} \cdot dx = \int \frac{\cos \frac{x}{3}}{\operatorname{sen} \frac{x}{3}} \cdot dx = 3 \int \frac{\frac{1}{3} \cos \frac{x}{3}}{\operatorname{sen} \frac{x}{3}} \cdot dx = 3 \cdot \operatorname{Ln} \left| \operatorname{sen} \frac{x}{3} \right| + K$$

45. $\int (\operatorname{ctg} e^x) e^x dx$ Sol: $L \left| \operatorname{sen} e^x \right| + k$

$$\int (\operatorname{ctg} e^x) e^x \cdot dx = \int \frac{(\operatorname{cose}^x) e^x}{\operatorname{sen} e^x} \cdot dx = \operatorname{Ln} \left| \operatorname{sen} e^x \right| + K$$

46. $\int \left(\operatorname{tg} 4x - \operatorname{ctg} \frac{x}{4} \right) dx$ Sol: $\operatorname{Sol} : -\frac{1}{4} \operatorname{Ln} \left| \cos 4x \right| - 4 \operatorname{Ln} \left| \operatorname{sen} \frac{x}{4} \right| + k$

$$\begin{aligned} \int \left(\operatorname{tg} 4x - \operatorname{ctg} \frac{x}{4} \right) dx &= \int \left(\frac{\operatorname{sen} 4x}{\cos 4x} - \frac{\cos \frac{x}{4}}{\operatorname{sen} \frac{x}{4}} \right) dx = \int \frac{\operatorname{sen} 4x}{\cos 4x} dx - \int \frac{\cos \frac{x}{4}}{\operatorname{sen} \frac{x}{4}} dx = \\ &= -\frac{1}{4} \int \frac{-4 \operatorname{sen} 4x}{\cos 4x} dx - 4 \int \frac{\frac{1}{4} \cos \frac{x}{4}}{\operatorname{sen} \frac{x}{4}} dx = -\frac{1}{4} \operatorname{Ln} \left| \cos 4x \right| - 4 \operatorname{Ln} \left| \operatorname{sen} \frac{x}{4} \right| + k \end{aligned}$$

47. $\int \frac{\cos x}{2 \operatorname{sen} x + 3} dx$ Sol: $\frac{1}{2} \operatorname{Ln} (2 \operatorname{sen} x + 3) + k$

$$\int \frac{\cos x}{2 \operatorname{sen} x + 3} dx = \frac{1}{2} \int \frac{2 \cos x}{2 \operatorname{sen} x + 3} dx = \frac{1}{2} \operatorname{Ln} (2 \operatorname{sen} x + 3) + k$$

48. $\int \frac{dx}{(1+x^2) \operatorname{arc} \operatorname{tg} x}$ Sol: $\operatorname{Ln} \left| \operatorname{arc} \operatorname{tg} x \right| + k$

$$\int \frac{dx}{(1+x^2) \operatorname{arc} \operatorname{tg} x} = \int \frac{1}{\frac{1+x^2}{\operatorname{arc} \operatorname{tg} x}} dx = \operatorname{Ln} \left| \operatorname{arc} \operatorname{tg} x \right| + k$$

49. $\int \frac{dx}{\cos^2 x (3 \operatorname{tg} x + 1)}$ Sol: $\frac{1}{3} \operatorname{Ln} (3 \operatorname{tg} x + 1) + k$

$$\int \frac{dx}{\cos^2 x (3 \operatorname{tg} x + 1)} = \int \frac{1}{\frac{\cos^2 x}{3 \operatorname{tg} x + 1}} dx = \frac{1}{3} \int \frac{3}{\frac{\cos^2 x}{3 \operatorname{tg} x + 1}} dx = \frac{1}{3} \operatorname{Ln} (3 \operatorname{tg} x + 1) + k$$

$$50. \int \frac{dx}{\sqrt{1-x^2} \arcsen x} \quad \text{Sol: } \text{Ln} |\arcsen x| + k$$

$$\int \frac{dx}{\sqrt{1-x^2} \arcsen x} = \int \frac{1}{\arcsen x} \frac{dx}{\sqrt{1-x^2}} = \text{Ln} |\arcsen x| + k$$

$$51. \int \frac{\cos 2x}{2+3\sen 2x} dx \quad \text{Sol: } \frac{1}{6} \text{Ln} |2+3\sen 2x| + k$$

$$\int \frac{\cos 2x}{2+3\sen 2x} dx = \frac{1}{6} \int \frac{6 \cos 2x}{2+3\sen 2x} dx = \frac{1}{6} \text{Ln} |2+3\sen 2x| + k$$

$$52. \int e^{2x} dx \quad \text{Sol: } \frac{1}{2} e^{2x} + k$$

$$\int e^{2x} dx = \frac{1}{2} \int 2e^{2x} dx = \left\{ \int f' e^f dx = e^f + k \right\} = \frac{1}{2} e^{2x} + k$$

$$53. \int e^{\frac{x}{2}} dx \quad \text{Sol: } 2e^{\frac{x}{2}} + k$$

$$\int e^{\frac{x}{2}} dx = 2 \int \frac{1}{2} e^{\frac{x}{2}} dx = \left\{ \int f' e^f dx = e^f + k \right\} = 2e^{\frac{x}{2}} + k$$

$$54. \int e^{\sen x} \cos x dx \quad \text{Sol: } e^{\sen x} + k$$

$$\int e^{\sen x} \cos x dx = \left\{ \int f' e^f dx = e^f + k \right\} = e^{\sen x} + k$$

$$55. \int a^{x^2} \cdot x \cdot dx \quad \text{Sol: } \frac{a^{x^2}}{2 \text{Ln } a} + k$$

$$\int a^{x^2} \cdot x \cdot dx = \frac{1}{2 \text{Ln } a} \int \underbrace{a^{x^2} 2x \cdot \text{Ln } a}_{D(a^{x^2})} dx = \frac{1}{2 \text{Ln } a} a^{x^2} + k$$

$$56. \int e^{\frac{x}{a}} dx \quad \text{Sol: } a e^{\frac{x}{a}} + k$$

$$\int e^{\frac{x}{a}} dx = a \int \frac{1}{a} e^{\frac{x}{a}} dx = a e^{\frac{x}{a}} + k$$

$$57. \int (e^{2x})^2 dx \quad \text{Sol: } \frac{1}{4} e^{4x} + k$$

$$\int (e^{2x})^2 dx = \int e^{4x} dx = \frac{1}{4} \int 4e^{4x} dx = \frac{1}{4} e^{4x} + k$$

$$58. \int e^{-3x} dx \quad \text{Sol: } -\frac{1}{3} e^{-3x} + k$$

$$\int e^{-3x} dx = -\frac{1}{3} \int (-3)e^{-3x} dx = -\frac{1}{3} e^{-3x} + k$$

$$59. \int 5^x e^x dx \quad \text{Sol: } \frac{5^x e^x}{\text{Ln } 5 + 1} + k$$

$$\int 5^x e^x dx = \int (5e)^x dx = \frac{1}{\text{Ln}(5e)} \int (5e)^x \text{Ln}(5e) dx = \frac{1}{\text{Ln}(5e)} (5e)^x + k = \frac{5^x e^x}{\text{Ln } 5 + 1} + k$$

$$60. \int (e^{5x} + a^{5x}) dx \quad \text{Sol: } \frac{1}{5} \left(e^{5x} + \frac{a^{5x}}{\text{L } a} \right) + k$$

$$\begin{aligned} \int (e^{5x} + a^{5x}) dx &= \frac{1}{5} \int 5e^{5x} dx + \frac{1}{5 \text{Ln } a} \int 5a^{5x} \text{Ln } a dx = \frac{1}{5} e^{5x} + \frac{1}{5 \text{Ln } a} \cdot a^{5x} + k = \\ &= \frac{1}{5} \left(e^{5x} + \frac{a^{5x}}{\text{L } a} \right) + k \end{aligned}$$

$$61. \int e^{x^2+4x+3} (x+2) dx \quad \text{Sol: } \frac{1}{2} e^{x^2+4x+3} + k$$

$$\int e^{x^2+4x+3} (x+2) dx = \frac{1}{2} \int e^{x^2+4x+3} 2(x+2) dx = \frac{1}{2} e^{x^2+4x+3} + k$$

$$62. \int \frac{(a^x - b^x)^2}{a^x b^x} dx \quad \text{Sol: } \frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\text{L } a - \text{L } b} - 2x + k$$

$$\begin{aligned} \int \frac{(a^x - b^x)^2}{a^x b^x} dx &= \int \frac{a^{2x} - 2a^x b^x + b^{2x}}{a^x b^x} dx = \int \left(\frac{a^{2x}}{a^x b^x} + \frac{b^{2x}}{a^x b^x} - 2 \right) dx = \int \left(\frac{a^x}{b^x} + \frac{b^x}{a^x} - 2 \right) dx = \\ &= \int \left[\left(\frac{a}{b}\right)^x + \left(\frac{b}{a}\right)^x - 2 \right] dx = \frac{1}{\text{Ln}\left(\frac{a}{b}\right)} \cdot \left(\frac{a}{b}\right)^x + \frac{1}{\text{Ln}\left(\frac{b}{a}\right)} \cdot \left(\frac{b}{a}\right)^x - 2x + k = \end{aligned}$$

$$= \frac{1}{\ln a - \ln b} \cdot \left(\frac{a}{b}\right)^x + \frac{1}{\ln b - \ln a} \cdot \left(\frac{b}{a}\right)^x - 2x + k =$$

$$= \frac{\left(\frac{a}{b}\right)^x}{\ln a - \ln b} - \frac{\left(\frac{b}{a}\right)^x}{\ln a - \ln b} - 2x + k = \frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\ln a - \ln b} - 2x + k$$

63. $\int \frac{e^x}{3+4e^x} dx$ Sol: $\frac{1}{4} \ln(3+4e^x) + k$

$$\int \frac{e^x}{3+4e^x} dx = \frac{1}{4} \int \frac{4e^x}{3+4e^x} dx = \frac{1}{4} \ln(3+4e^x) + k$$

64. $\int \cos 5x dx$ Sol: $\frac{1}{5} \operatorname{sen} 5x + k$

$$\int \cos 5x dx = \frac{1}{5} \int 5 \cos 5x dx = \left\{ \int f'(x) \cdot \cos f(x) dx = \operatorname{sen} f(x) + k \right\} = \frac{1}{5} \operatorname{sen} 5x + k$$

65. $\int \operatorname{sen} \frac{x}{3} dx$ Sol: $-3 \cos \frac{x}{3} + k$

$$\int \operatorname{sen} \frac{x}{3} dx = 3 \int \frac{1}{3} \operatorname{sen} \frac{x}{3} dx = \left\{ \int f'(x) \cdot \operatorname{sen} f(x) dx = -\cos f(x) + k \right\} = 3(-\cos \frac{x}{3}) + k =$$

$$= -3 \cos \frac{x}{3} + k$$

66. $\int \sec^2(7x+2) dx$ Sol: $\frac{1}{7} \operatorname{tg}(7x+2) + k$

$$\int \sec^2(7x+2) dx = \frac{1}{7} \int 7 \sec^2(7x+2) dx = \left\{ \int f'(x) \sec^2 f(x) dx = \operatorname{tg} f(x) + k \right\} =$$

$$= \frac{1}{7} \operatorname{tg}(7x+2) + k$$

67. $\int x \cos 3x^2 dx$ Sol: $\frac{1}{6} \operatorname{sen} 3x^2 + k$

$$\int x \cos(3x^2) dx = \frac{1}{6} \int 6x \cos(3x^2) dx = \frac{1}{6} \operatorname{sen}(3x^2) + k$$

68. $\int \operatorname{tg}^2 x \, dx$ Sol: $\operatorname{tg} x - x + k$

Por trigonometría sabemos que $\operatorname{tg}^2 x + 1 = \sec^2 x \Rightarrow \operatorname{tg}^2 x = \sec^2 x - 1$, entonces

$$\int \operatorname{tg}^2 x \, dx = \int (\sec^2 x - 1) dx = \int \sec^2 x \, dx - \int dx = \operatorname{tg} x - x + k$$

69. $\int \frac{\cos(\operatorname{Ln}(x))}{x} dx$ Sol: $\operatorname{sen}(\operatorname{Ln}(x)) + k$

$$\int \frac{\cos(\operatorname{Ln}(x))}{x} dx = \int \cos(\operatorname{Ln}(x)) \frac{1}{x} dx = \operatorname{sen}(\operatorname{Ln}(x)) + k$$

70. $\int \operatorname{tg}^3 x \, dx$ Sol: $\frac{\operatorname{tg}^2 x}{2} + \operatorname{Ln}|\cos x| + k$

$$\int \operatorname{tg}^3 x \, dx = \int \operatorname{tg} x \cdot \operatorname{tg}^2 x \, dx = \int \operatorname{tg} x \cdot (\sec^2 x - 1) dx = \int \underbrace{\operatorname{tg} x}_{f'} \cdot \underbrace{\sec^2 x}_f dx - \int \operatorname{tg} x \, dx =$$

$$= \frac{\operatorname{tg}^2 x}{2} - \int \frac{\operatorname{sen} x}{\cos x} dx = \frac{\operatorname{tg}^2 x}{2} + \int \frac{-\operatorname{sen} x}{\cos x} dx = \frac{\operatorname{tg}^2 x}{2} + \operatorname{Ln}|\cos x| + k$$

71. $\int \cos \sqrt{x} \frac{dx}{\sqrt{x}}$ Sol: $2 \operatorname{sen} \sqrt{x} + k$

$$\int \cos \sqrt{x} \frac{dx}{\sqrt{x}} = \int \cos \sqrt{x} \frac{1}{\sqrt{x}} dx = 2 \int \cos \sqrt{x} \frac{1}{2\sqrt{x}} dx = 2 \operatorname{sen} \sqrt{x} + k$$

72. $\int \frac{x}{\sqrt{1-x^4}} dx$ Sol: $\frac{1}{2} \operatorname{arc} \operatorname{sen} x^2 + k$

$$\int \frac{x}{\sqrt{1-x^4}} dx = \int \frac{x}{\sqrt{1-(x^2)^2}} dx = \left\{ \int \frac{f'(x)}{\sqrt{1-(f(x))^2}} dx = \left\{ \begin{array}{l} \operatorname{arc} \operatorname{sen} f(x) + k \\ -\operatorname{arccos} f(x) + k \end{array} \right\} =$$

$$= \frac{1}{2} \int \frac{2x}{\sqrt{1-(x^2)^2}} dx = \left\{ \int \frac{f'(x) dx}{\sqrt{1-(f(x))^2}} \right\} = \frac{1}{2} \operatorname{arc} \operatorname{sen}(x^2) + k$$

73. $\int \frac{dx}{\sqrt{1-4x^2}}$ Sol: $\frac{1}{2} \operatorname{arc} \operatorname{sen}(2x) + k$

$$\int \frac{dx}{\sqrt{1-4x^2}} = \int \frac{dx}{\sqrt{1-(2x)^2}} = \frac{1}{2} \int \frac{2dx}{\sqrt{1-(2x)^2}} = \left\{ \int \frac{f'(x) dx}{\sqrt{1-(f(x))^2}} \right\} = \frac{1}{2} \operatorname{arc} \operatorname{sen}(2x) + k$$

74. $\int \frac{dx}{\sqrt{9-4x^2}}$ Sol: $\frac{1}{2} \arcsen\left(\frac{2x}{3}\right) + k$

$$\begin{aligned} \int \frac{dx}{\sqrt{9-4x^2}} &= \int \frac{dx}{\sqrt{9\left(1-\frac{4x^2}{9}\right)}} = \int \frac{dx}{3\sqrt{1-\left(\frac{2x}{3}\right)^2}} = \frac{1}{3} \int \frac{dx}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} = \frac{1}{3} \cdot \frac{3}{2} \int \frac{\frac{2}{3}dx}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} = \\ &= \frac{1}{2} \int \frac{\frac{2}{3}dx}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} = \left\{ \int \frac{f'(x)dx}{\sqrt{1-(f(x))^2}} \right\} = \frac{1}{2} \arcsen\left(\frac{2x}{3}\right) + k \end{aligned}$$

75. $\int \frac{dx}{\sqrt{a^2-b^2x^2}}$ Sol: $\frac{1}{b} \arcsen\left(\frac{bx}{a}\right) + k$

$$\begin{aligned} \int \frac{dx}{\sqrt{a^2-b^2x^2}} &= \int \frac{dx}{\sqrt{a^2\left(1-\frac{b^2x^2}{a^2}\right)}} = \int \frac{dx}{a\sqrt{1-\left(\frac{bx}{a}\right)^2}} = \frac{1}{a} \int \frac{dx}{\sqrt{1-\left(\frac{bx}{a}\right)^2}} = \frac{1}{a} \cdot \frac{a}{b} \int \frac{\frac{b}{a}dx}{\sqrt{1-\left(\frac{bx}{a}\right)^2}} = \\ &= \frac{1}{b} \int \frac{\frac{b}{a}dx}{\sqrt{1-\left(\frac{bx}{a}\right)^2}} = \left\{ \int \frac{f'(x)dx}{\sqrt{1-(f(x))^2}} \right\} = \frac{1}{b} \arcsen\left(\frac{bx}{a}\right) + k \end{aligned}$$

76. $\int \frac{e^x}{3+4e^x} dx$ Sol: $\frac{1}{4} \text{Ln}(3+4e^x) + k$

$$\int \frac{e^x}{3+4e^x} dx = \frac{1}{4} \int \frac{4e^x}{3+4e^x} dx = \left\{ \int \frac{f'(x)}{f(x)} dx \right\} = \frac{1}{4} \text{Ln}(3+4e^x) + k$$

77. $\int \frac{e^{2x}}{2+e^{2x}} dx$ Sol: $\frac{1}{2} \text{Ln}(2+e^{2x}) + k$

$$\int \frac{e^{2x}}{2+e^{2x}} dx = \frac{1}{2} \int \frac{2e^{2x}}{2+e^{2x}} dx = \left\{ \int \frac{f'(x)}{f(x)} dx \right\} = \frac{1}{2} \text{Ln}(2+e^{2x}) + k$$

78. $\int \frac{e^x}{1+e^{2x}} dx$ Sol: $\arcsen\left(\frac{e^x}{\sqrt{1+e^{2x}}}\right) + k$

$$\int \frac{e^x}{1+e^{2x}} dx = \int \frac{e^x}{1+(e^x)^2} dx = \left\{ \int \frac{f'(x)}{1+(f(x))^2} dx = \arcsen f(x) + k \right\} = \arcsen\left(\frac{e^x}{\sqrt{1+e^{2x}}}\right) + k$$

$$79. \int \frac{dx}{1+2x^2} \quad \text{Sol: } \frac{1}{\sqrt{2}} \arctan(\sqrt{2}x) + k$$

$$\int \frac{dx}{1+2x^2} = \int \frac{dx}{1+(\sqrt{2}x)^2} = \frac{1}{\sqrt{2}} \int \frac{\sqrt{2} dx}{1+(\sqrt{2}x)^2} = \left\{ \int \frac{f'(x)}{1+(f(x))^2} dx \right\} = \frac{1}{\sqrt{2}} \arctan(\sqrt{2}x) + k$$

$$80. \int \frac{dx}{4+x^2} \quad \text{Sol: } \frac{1}{2} \arctan\left(\frac{x}{2}\right) + k$$

$$\int \frac{dx}{4+x^2} = \int \frac{dx}{4\left(1+\frac{x^2}{4}\right)} = \frac{1}{4} \int \frac{dx}{1+\left(\frac{x}{2}\right)^2} = \frac{1}{4} \cdot 2 \int \frac{\frac{1}{2} dx}{1+\left(\frac{x}{2}\right)^2} = \frac{1}{2} \arctan\left(\frac{x}{2}\right) + k$$

$$81. \int \frac{xdx}{x^4+a^4} \quad \text{Sol: } \frac{1}{2a^2} \arctan\left(\frac{x^2}{a^2}\right) + k$$

$$\begin{aligned} \int \frac{xdx}{x^4+a^4} &= \int \frac{xdx}{a^4\left(\frac{x^4}{a^4}+1\right)} = \frac{1}{a^4} \int \frac{xdx}{1+\frac{x^4}{a^4}} = \frac{1}{a^4} \int \frac{xdx}{1+\left(\frac{x^2}{a^2}\right)^2} = \frac{1}{a^4} \cdot \frac{a^2}{2} \int \frac{\frac{2x}{a^2} dx}{1+\left(\frac{x^2}{a^2}\right)^2} = \\ &= \frac{1}{2a^2} \int \frac{\frac{2x}{a^2} dx}{1+\left(\frac{x^2}{a^2}\right)^2} = \frac{1}{2a^2} \arctan\left(\frac{x^2}{a^2}\right) + k \end{aligned}$$

$$82. \int \frac{\cos x dx}{a^2 + \sen^2 x} \quad \text{Sol: } \frac{1}{a} \arctan\left(\frac{\sen x}{a}\right) + k$$

$$\begin{aligned} \int \frac{\cos x dx}{a^2 + \sen^2 x} &= \int \frac{\cos x dx}{a^2\left(1+\frac{\sen^2 x}{a^2}\right)} = \frac{1}{a^2} \int \frac{\cos x dx}{1+\frac{\sen^2 x}{a^2}} = \frac{1}{a^2} \int \frac{\cos x dx}{1+\left(\frac{\sen x}{a}\right)^2} = \\ &= \frac{1}{a^2} \cdot a \int \frac{\frac{1}{a} \cos x dx}{1+\left(\frac{\sen x}{a}\right)^2} = \frac{1}{a} \int \frac{\frac{1}{a} \cos x dx}{1+\left(\frac{\sen x}{a}\right)^2} = \frac{1}{a} \arctan\left(\frac{\sen x}{a}\right) + k \end{aligned}$$

$$83. \int \frac{dx}{x\sqrt{1-\ln^2(x)}} \quad \text{Sol: } \arcsen(\ln(x)) + k$$

$$\int \frac{dx}{x\sqrt{1-\ln^2(x)}} = \int \frac{\frac{1}{x} dx}{\sqrt{1-(\ln(x))^2}} = \left\{ \int \frac{f'(x) dx}{\sqrt{1-(f(x))^2}} \right\} = \arcsen(\ln(x)) + k$$

$$84. \int \frac{\arccos x - x}{\sqrt{1-x^2}} dx \quad \text{Sol: } -\frac{1}{2}(\arccos(x))^2 + \sqrt{1-x^2} + k$$

$$\begin{aligned} \int \frac{\arccos x - x}{\sqrt{1-x^2}} dx &= \int \frac{\arccos x}{\sqrt{1-x^2}} dx + \int \frac{-x}{\sqrt{1-x^2}} dx = - \int \underbrace{\arccos x}_{f'} \cdot \underbrace{\frac{-1}{\sqrt{1-x^2}}}_{f'} dx + \int \frac{-2x}{2\sqrt{1-x^2}} dx = \\ &= \left\{ \int f' \cdot f' dx + \int \frac{f'}{2\sqrt{f}} dx \right\} = -\frac{1}{2}(\arccos(x))^2 + \sqrt{1-x^2} + k \end{aligned}$$

$$85. \int \frac{x - \arctg x}{1+x^2} dx \quad \text{Sol: } \frac{1}{2}\ln(1+x^2) - \frac{1}{2}(\arctg x)^2 + k$$

$$\begin{aligned} \int \frac{x - \arctg x}{1+x^2} dx &= \int \frac{x}{1+x^2} dx - \int \frac{\arctg x}{1+x^2} dx = \frac{1}{2} \int \frac{2x}{1+x^2} dx - \int \arctg x \cdot \frac{1}{1+x^2} dx = \\ &= \left\{ \int \frac{f'(x)}{f(x)} dx - \int f' \cdot f' dx \right\} = \frac{1}{2}\ln(1+x^2) - \frac{1}{2}(\arctg x)^2 + k \end{aligned}$$

$$86. \int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx \quad \text{Sol: } \frac{4}{3}\sqrt{(1+\sqrt{x})^3} + k$$

$$\begin{aligned} \int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx &= \int \frac{1}{\sqrt{x}} \sqrt{1+\sqrt{x}} dx = 2 \int \frac{1}{2\sqrt{x}} (1+\sqrt{x})^{\frac{1}{2}} dx = \left\{ \int f^{\frac{1}{2}} \cdot f' dx \right\} = \\ &= 2 \cdot \frac{(1+\sqrt{x})^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{4}{3}\sqrt{(1+\sqrt{x})^3} + k \end{aligned}$$

Veamos como podemos realizar esta misma integral por el método de sustitución o cambio de variable.

Haciendo el cambio $1 + \sqrt{x} = t$, calculamos dx : $\frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2\sqrt{x} dt$ y

sustituimos en nuestra integral:

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx = \int \frac{\sqrt{t}}{\sqrt{x}} \cdot 2\sqrt{x} dt = 2 \int \sqrt{t} dt = 2 \int t^{\frac{1}{2}} dt = 2 \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{4}{3} t^{\frac{3}{2}} + k = \frac{4}{3} \sqrt{t^3} + k =$$

una vez realizada la integral hay que deshacer el cambio de variable y volver a la variable x , con lo que nos quedará:

$$= \frac{4}{3} \sqrt{(1+\sqrt{x})^3} + k$$

87. $\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}} dx$ Sol: $4\sqrt{1+\sqrt{x}} + k$

$$\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}} dx = \int \frac{1}{\sqrt{x}} \frac{1}{\sqrt{1+\sqrt{x}}} dx = 2 \int \frac{1}{2\sqrt{x}} (1+\sqrt{x})^{-\frac{1}{2}} dx = \left\{ \int f^{-\frac{1}{2}} \cdot f' dx \right\} =$$

$$= 2 \cdot \frac{(1+\sqrt{x})^{\frac{1}{2}}}{\frac{1}{2}} + k = 4\sqrt{1+\sqrt{x}} + k$$

Veamos como podemos realizar esta misma integral por el método de sustitución o cambio de variable.

Haciendo el cambio $1+\sqrt{x} = t$, calculamos dx : $\frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2\sqrt{x} dt$ y

sustituimos en nuestra integral:

$$\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}} dx = \int \frac{1}{\sqrt{x}\sqrt{t}} \cdot 2\sqrt{x} dt = 2 \int \frac{1}{\sqrt{t}} dt = 2 \int t^{-\frac{1}{2}} dt = 2 \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + k = 4t^{\frac{1}{2}} + k = 4\sqrt{t} + k =$$

una vez realizada la integral hay que deshacer el cambio de variable y volver a la variable x , con lo que nos quedará:

$$= 4\sqrt{1+\sqrt{x}} + k$$

88. $\int x\sqrt{x-1} dx$ Sol: $\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + k$

Hacemos la sustitución $x-1 = t^2 \Rightarrow x = t^2 + 1$

Calculamos la diferencial de x : $dx = 2t \cdot dt$ y sustituimos en la integral que deseamos calcular. Tendremos:

$$\int x \cdot \sqrt{x-1} \cdot dx = \int (t^2 + 1) \cdot \sqrt{t^2} \cdot 2t dt = 2 \int (t^2 + 1) \cdot t^2 \cdot dt = 2 \int (t^4 + t^2) \cdot dt = 2 \left(\frac{t^5}{5} + \frac{t^3}{3} \right) + k =$$

$$= \frac{2}{5} \cdot t^5 + \frac{2}{3} \cdot t^3 + k = \frac{2}{5} \cdot (x-1)^{\frac{5}{2}} + \frac{2}{3} \cdot (x-1)^{\frac{3}{2}} + k$$

89. $\int x(5x^2 - 3)^7 dx$ Sol: $\frac{1}{80}(5x^2 - 3)^8 + k$

Directamente:

$$\int x(5x^2 - 3)^7 dx = \frac{1}{10} \int 10x(5x^2 - 3)^7 dx = \left\{ \int f^7 \cdot f' dx \right\} = \frac{1}{10} \cdot \frac{(5x^2 - 3)^8}{8} + k =$$

$$= \frac{1}{80}(5x^2 - 3)^8 + k$$

Por sustitución:

Hacemos $5x^2 - 3 = t \Rightarrow 10x dx = dt \Rightarrow dx = \frac{dt}{10x}$ y sustituimos en nuestra integral

$$\int x(5x^2 - 3)^7 dx = \int xt^7 \frac{dt}{10x} = \frac{1}{10} \int t^7 dt = \frac{1}{10} \cdot \frac{t^8}{8} + k = \frac{1}{80}(5x^2 - 3)^8 + k$$

90. $\int x(2x + 5)^{10} dx$ Sol: $\frac{1}{4} \left[\frac{(2x + 5)^{12}}{12} - \frac{5(2x + 5)^{11}}{11} \right] + k$

Por sustitución:

Hacemos $2x + 5 = t \Rightarrow x = \frac{t-5}{2} \Rightarrow dx = \frac{1}{2} dt$ y sustituimos en nuestra integral

$$\int x(2x + 5)^{10} dx = \int \frac{t-5}{2} \cdot t^{10} \cdot \frac{1}{2} dt = \frac{1}{4} \int (t-5)t^{10} dt = \frac{1}{4} \int (t^{11} - 5t^{10}) dt =$$

$$= \frac{1}{4} \left[\frac{t^{12}}{12} - 5 \cdot \frac{t^{11}}{11} \right] + k = \frac{1}{4} \left[\frac{(2x + 5)^{12}}{12} - \frac{5(2x + 5)^{11}}{11} \right] + k$$

91. $\int xe^x dx$ Sol: $e^x(x - 1) + k$

Por el método de integración por partes:

$$\int xe^x dx = \left\{ \begin{array}{l} u = x \Rightarrow du = dx \\ dv = e^x dx \Rightarrow v = e^x \end{array} \right\} = xe^x - \int e^x dx = xe^x - e^x + k = (x-1)e^x + k$$

$$92. I = \int (x^2 - 3x + 5)e^x dx \quad \text{Sol: } e^x(x^2 - 5x + 10) + k$$

Por el método de integración por partes:

$$I = \int (x^2 - 3x + 5)e^x dx = \left\{ \begin{array}{l} u = x^2 - 3x + 5 \Rightarrow du = (2x - 3)dx \\ dv = e^x dx \Rightarrow v = e^x \end{array} \right\} =$$

$$= (x^2 - 3x + 5)e^x - \int e^x(2x - 3)dx =$$

La integral que nos ha quedado es del mismo tipo que la que pretendemos calcular, por lo que nuevamente aplicaremos el método de integración de partes:

Hacemos $\left\{ \begin{array}{l} u = 2x - 3 \Rightarrow du = 2dx \\ dv = e^x dx \Rightarrow v = e^x \end{array} \right\}$ y sustituimos:

$$I = (x^2 - 3x + 5)e^x - \int e^x(2x - 3)dx = (x^2 - 3x + 5)e^x - \left[(2x - 3)e^x - \int 2e^x dx \right] =$$

$$= (x^2 - 3x + 5)e^x - (2x - 3)e^x + 2 \int e^x dx = (x^2 - 3x + 5)e^x - (2x - 3)e^x + 2e^x + k =$$

$$= [(x^2 - 3x + 5) - (2x - 3) + 2]e^x + k = e^x(x^2 - 5x + 10) + k$$

$$93. \int x \text{Ln}(x) dx \quad \text{Sol: } \frac{1}{2}x^2 \left(\text{Ln}(x) - \frac{1}{2} \right) + k$$

Por el método de integración por partes:

$$\int x \text{Ln}(x) dx = \left\{ \begin{array}{l} u = \text{Ln}(x) \Rightarrow du = \frac{1}{x} dx \\ dv = x dx \Rightarrow v = \frac{1}{2}x^2 \end{array} \right\} = \frac{1}{2}x^2 \text{Ln}(x) - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx =$$

$$= \frac{1}{2}x^2 \text{Ln}(x) - \frac{1}{2} \int x dx = \frac{1}{2}x^2 \text{Ln}(x) - \frac{1}{2} \cdot \frac{1}{2}x^2 + k = \frac{1}{2}x^2 \left(\text{Ln}(x) - \frac{1}{2} \right) + k$$

$$94. \int \text{Ln}(x) dx \quad \text{Sol: } x(\text{Ln}(x) - 1) + k$$

Por el método de integración por partes:

$$\int \text{Ln}(x) dx = \left\{ \begin{array}{l} u = \text{Ln}(x) \Rightarrow du = \frac{1}{x} dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x \text{Ln}(x) - \int x \cdot \frac{1}{x} dx =$$

$$= x \operatorname{Ln}(x) - \int dx = x \operatorname{Ln}(x) - x + k = x(\operatorname{Ln}(x) - 1) + k$$

95. $\int x \operatorname{sen} x \, dx$ Sol: $\operatorname{sen} x - x \cos x + k$

Por el método de integración por partes:

$$\int x \operatorname{sen} x \, dx = \left\{ \begin{array}{l} u = x \Rightarrow du = dx \\ dv = \operatorname{sen} x \, dx \Rightarrow v = -\cos x \end{array} \right\} = -x \cos x - \int -\cos x \, dx =$$

$$= -x \cos x + \int \cos x \, dx = -x \cos x + \operatorname{sen} x + k$$

96. $\int x \cos^2 x \, dx$ Sol: $\frac{x^2}{4} + \frac{1}{4} x \operatorname{sen} 2x + \frac{1}{8} \cos 2x + k$

Por el método de integración por partes, hacemos $u = x \Rightarrow du = dx$ y $dv = \cos^2 x \, dx$

Para calcular el valor de v recurrimos a las razones trigonométricas del ángulo mitad y

tendremos que $\cos^2 x = \frac{1 + \cos 2x}{2}$. Por tanto,

$$v = \int \cos^2 x \, dx = \int \frac{1 + \cos 2x}{2} \, dx = \frac{1}{2} \int (1 + \cos 2x) \, dx = \frac{1}{2} \left(x + \frac{1}{2} \operatorname{sen} 2x \right)$$

En consecuencia:

$$\int x \cos^2 x \, dx = x \cdot \frac{1}{2} \left(x + \frac{1}{2} \operatorname{sen} 2x \right) - \int \frac{1}{2} \left(x + \frac{1}{2} \operatorname{sen} 2x \right) \, dx =$$

$$= \frac{1}{2} \left(x^2 + \frac{1}{2} x \operatorname{sen} 2x \right) - \frac{1}{2} \int \left(x + \frac{1}{2} \operatorname{sen} 2x \right) \, dx = \frac{1}{2} \left(x^2 + \frac{1}{2} x \operatorname{sen} 2x \right) - \frac{1}{2} \cdot \left(\frac{x^2}{2} - \frac{1}{4} \cos 2x \right) + k =$$

$$= \frac{1}{2} x^2 + \frac{1}{4} x \operatorname{sen} 2x - \frac{x^2}{4} + \frac{1}{8} \cos 2x + k = \frac{x^2}{4} + \frac{1}{4} x \operatorname{sen} 2x + \frac{1}{8} \cos 2x + k$$

97. $\int e^{-x} \cos x \, dx$ Sol: $\frac{1}{2} e^{-x} (\operatorname{sen} x - \cos x) + k$

$$I = \int e^{-x} \cos x \, dx = \left\{ \begin{array}{l} u = e^{-x} \Rightarrow du = -e^{-x} \, dx \\ dv = \cos x \, dx \Rightarrow v = \operatorname{sen} x \end{array} \right\} = e^{-x} \operatorname{sen} x - \int -e^{-x} \operatorname{sen} x \, dx =$$

$$= e^{-x} \operatorname{sen} x + \int e^{-x} \operatorname{sen} x \, dx$$

Al aplicar el método de partes nos ha quedado una integral del mismo tipo que la que pretendemos calcular, por lo que volvemos a aplicar el mismo método. En ella hacemos:

$$\begin{aligned} u = e^{-x} &\Rightarrow du = -e^{-x} dx \\ dv = \operatorname{sen} x dx &\Rightarrow v = -\cos x \end{aligned}$$

Sustituyendo en la expresión anterior nos queda:

$$\begin{aligned} I = e^{-x} \operatorname{sen} x + \int e^{-x} \operatorname{sen} x dx &= e^{-x} \operatorname{sen} x + \left[-\cos x \cdot e^{-x} - \int -\cos x \cdot (-e^{-x}) dx \right] = \\ &= e^{-x} \operatorname{sen} x - \cos x \cdot e^{-x} - \int \cos x \cdot e^{-x} dx \end{aligned}$$

es decir, volvemos a la misma integral que pretendemos calcular. Entonces:

$$I = e^{-x} \operatorname{sen} x - \cos x \cdot e^{-x} - I \Rightarrow 2I = e^{-x} \operatorname{sen} x - \cos x \cdot e^{-x} \Rightarrow I = \frac{e^{-x} (\operatorname{sen} x - \cos x)}{2}$$

En consecuencia:

$$I = \int e^{-x} \cos x dx = \frac{e^{-x} (\operatorname{sen} x - \cos x)}{2} + k$$

98. $\int \operatorname{Ln}(1-x) dx$

Sol: $-x - (1-x) \operatorname{Ln}(1-x) + k$

$$\begin{aligned} \int \operatorname{Ln}(1-x) dx &= \left\{ \begin{array}{l} u = \operatorname{Ln}(1-x) \Rightarrow du = \frac{-1}{1-x} dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x \operatorname{Ln}(1-x) - \int x \cdot \frac{-1}{1-x} dx = \\ &= x \operatorname{Ln}(1-x) - \int \frac{-x}{1-x} dx = x \operatorname{Ln}(1-x) - \int \frac{1-x-1}{1-x} dx = x \operatorname{Ln}(1-x) - \int \left(1 + \frac{-1}{1-x} \right) dx = \\ &= x \operatorname{Ln}(1-x) - (x + \operatorname{Ln}(1-x)) + k = x \operatorname{Ln}(1-x) - x - \operatorname{Ln}(1-x) + k = \\ &= -x - (1-x) \operatorname{Ln}(1-x) + k \end{aligned}$$

$$99. \int x^n \operatorname{Ln}(x) dx \quad \text{Sol: } \frac{x^{n+1}}{n+1} \left(\operatorname{Ln}(x) - \frac{1}{n+1} \right) + k$$

Por el método de integración por partes:

$$\begin{aligned} \int x^n \operatorname{Ln}(x) dx &= \left\{ \begin{array}{l} u = \operatorname{Ln}(x) \Rightarrow du = \frac{1}{x} dx \\ dv = x^n dx \Rightarrow v = \frac{1}{n+1} x^{n+1} \end{array} \right\} = \frac{1}{n+1} x^{n+1} \operatorname{Ln}(x) - \int \frac{1}{n+1} x^{n+1} \cdot \frac{1}{x} dx = \\ &= \frac{1}{n+1} x^{n+1} \operatorname{Ln}(x) - \frac{1}{n+1} \int x^n dx = \frac{1}{n+1} x^{n+1} \operatorname{Ln}(x) - \frac{1}{n+1} \cdot \frac{1}{n+1} x^{n+1} + k = \\ &= \frac{x^{n+1}}{n+1} \left(\operatorname{Ln}(x) - \frac{1}{n+1} \right) + k \end{aligned}$$

$$100. \int \operatorname{arc sen} x dx \quad \text{Sol: } x \operatorname{arc sen} x + \sqrt{1-x^2} + k$$

Hacemos el siguiente cambio: $\left\{ \begin{array}{l} u = \operatorname{arc sen} x \\ dv = dx \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} du = \frac{1}{\sqrt{1-x^2}} \cdot dx \\ v = x \end{array} \right.$

Sustituyendo en la fórmula de integración por partes obtenemos:

$$\begin{aligned} \int \operatorname{arc sen} x \cdot dx &= x \cdot \operatorname{arc sen} x - \int x \cdot \frac{1}{\sqrt{1-x^2}} \cdot dx = x \cdot \operatorname{arc sen} x - \int x \cdot (1-x^2)^{-\frac{1}{2}} \cdot dx = \\ &= x \cdot \operatorname{arc sen} x + \frac{1}{2} \int -2x \cdot (1-x^2)^{-\frac{1}{2}} \cdot dx = x \cdot \operatorname{arc sen} x + \frac{1}{2} \cdot \frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + k = \\ &= x \cdot \operatorname{arc sen} x + \sqrt{1-x^2} + k \end{aligned}$$

$$101. \int \sqrt{1-x^2} dx \quad \text{Sol: } \frac{1}{2} (\operatorname{arc sen} x + x \sqrt{1-x^2}) + k$$

$$\begin{aligned} \int \sqrt{1-x^2} dx &= \int \frac{1-x^2}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{-x^2}{\sqrt{1-x^2}} dx = \\ &= \operatorname{arc sen} x + \int \frac{-x^2}{\sqrt{1-x^2}} dx = \end{aligned}$$

La integral que nos queda la realizaremos por partes:

$$\int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx = \int x \cdot \frac{-x}{\sqrt{1-x^2}} \cdot dx = \left\{ \begin{array}{l} u = x \Rightarrow dx \\ dv = \frac{-x}{\sqrt{1-x^2}} dx \Rightarrow v = \sqrt{1-x^2} \end{array} \right\} =$$

$$= x\sqrt{1-x^2} - \int \sqrt{1-x^2} dx$$

Sustituyendo nos queda:

$$\int \sqrt{1-x^2} dx = \arcsen x + \int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx = \arcsen x + x\sqrt{1-x^2} - \int \sqrt{1-x^2} dx$$

y se nos repite la misma integral. Entonces:

$$\int \sqrt{1-x^2} dx = \arcsen x + x\sqrt{1-x^2} - \int \sqrt{1-x^2} dx \Rightarrow$$

$$\Rightarrow 2 \int \sqrt{1-x^2} dx = \arcsen x + x\sqrt{1-x^2} \Rightarrow \int \sqrt{1-x^2} dx = \frac{1}{2} (\arcsen x + x\sqrt{1-x^2}) + k$$

102. $\int x \arcsen x dx$ Sol: $\frac{1}{4} [(2x^2 - 1) \arcsen x + x\sqrt{1-x^2}] + k$

Hacemos el siguiente cambio: $\left\{ \begin{array}{l} u = \arcsen x \\ dv = x dx \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} du = \frac{1}{\sqrt{1-x^2}} \cdot dx \\ v = \frac{x^2}{2} \end{array} \right.$

Sustituyendo en la fórmula de integración por partes obtenemos:

$$\int x \arcsen x \cdot dx = \frac{x^2}{2} \cdot \arcsen x - \int \frac{x^2}{2} \cdot \frac{1}{\sqrt{1-x^2}} \cdot dx = \frac{x^2}{2} \cdot \arcsen x + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx =$$

Por el ejercicio anterior tenemos que :

$$\int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx = \int x \cdot \frac{-x}{\sqrt{1-x^2}} \cdot dx = \left\{ \begin{array}{l} u = x \Rightarrow dx \\ dv = \frac{-x}{\sqrt{1-x^2}} dx \Rightarrow v = \sqrt{1-x^2} \end{array} \right\} =$$

$$= x\sqrt{1-x^2} - \int \sqrt{1-x^2} dx = x\sqrt{1-x^2} - \int \frac{1-x^2}{\sqrt{1-x^2}} dx \Rightarrow$$

$$\int \frac{-x^2}{\sqrt{1-x^2}} dx = x\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{-x^2}{\sqrt{1-x^2}} dx = x\sqrt{1-x^2} - \arcsen x - \int \frac{-x^2}{\sqrt{1-x^2}} dx$$

En consecuencia:

$$\int \frac{-x^2}{\sqrt{1-x^2}} dx = x\sqrt{1-x^2} - \arcsen x - \int \frac{-x^2}{\sqrt{1-x^2}} dx$$

Por tanto:

$$2 \int \frac{-x^2}{\sqrt{1-x^2}} dx = x\sqrt{1-x^2} - \arcsen x \Rightarrow \int \frac{-x^2}{\sqrt{1-x^2}} dx = \frac{1}{2} (x\sqrt{1-x^2} - \arcsen x)$$

Sustituyendo obtenemos:

$$\begin{aligned} \int x \arcsen x \cdot dx &= \frac{x^2}{2} \cdot \arcsen x + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx = \\ &= \frac{x^2}{2} \cdot \arcsen x + \frac{1}{2} \cdot \frac{1}{2} (x\sqrt{1-x^2} - \arcsen x) + k = \\ &= \frac{x^2}{2} \cdot \arcsen x + \frac{1}{4} x\sqrt{1-x^2} - \frac{1}{4} \arcsen x + k = \\ &= \frac{1}{4} [(2x^2 - 1) \arcsen x + x\sqrt{1-x^2}] + k \end{aligned}$$

103. $\int \arctg x dx$ Sol: $x \arctg x - \frac{1}{2} \ln(1+x^2) + k$

$$\begin{aligned} \int \arctg x dx &= \left\{ \begin{array}{l} u = \arctg x \Rightarrow du = \frac{1}{1+x^2} dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x \cdot \arctg x - \int x \cdot \frac{1}{1+x^2} dx = \\ &= x \cdot \arctg x - \int \frac{x}{1+x^2} dx = x \cdot \arctg x - \frac{1}{2} \int \frac{2x}{1+x^2} dx = x \cdot \arctg x - \frac{1}{2} \ln(1+x^2) + k \end{aligned}$$

104. $\int \arctg \sqrt{x} dx$ Sol: $(x+1) \arctg \sqrt{x} - \sqrt{x} + k$

$$\begin{aligned} \int \arctg \sqrt{x} dx &= \left\{ \begin{array}{l} u = \arctg \sqrt{x} \Rightarrow du = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x \arctg \sqrt{x} - \int x \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} dx = \\ &= x \arctg \sqrt{x} - \frac{1}{2} \int \frac{\sqrt{x}}{1+x} dx = \left\{ \begin{array}{l} x = t^2 \Rightarrow \\ \Rightarrow dx = 2t dt \end{array} \right\} = x \arctg \sqrt{x} - \frac{1}{2} \int \frac{t}{1+t^2} \cdot 2t dt = \end{aligned}$$

$$\begin{aligned}
 &= x \operatorname{arctg} \sqrt{x} - \int \frac{t^2}{1+t^2} dt = x \operatorname{arctg} \sqrt{x} - \int \frac{1+t^2-1}{1+t^2} dt = \\
 &= x \operatorname{arctg} \sqrt{x} - \int \frac{1+t^2-1}{1+t^2} dt = x \operatorname{arctg} \sqrt{x} - \int \left(1 - \frac{1}{1+t^2}\right) dt = \\
 &= x \operatorname{arctg} \sqrt{x} - \int dt + \int \frac{1}{1+t^2} dt = x \operatorname{arctg} \sqrt{x} - t + \operatorname{arctg} t + k = \\
 &= x \operatorname{arctg} \sqrt{x} - \sqrt{x} + \operatorname{arctg} \sqrt{x} + k = (x+1) \operatorname{arctg} \sqrt{x} - \sqrt{x} + k
 \end{aligned}$$

105. $\int \operatorname{Ln}(x + \sqrt{1+x^2}) dx$ Sol: $x \operatorname{Ln}(x + \sqrt{1+x^2}) - \sqrt{1+x^2} + k$

Hacemos: $u = \operatorname{Ln}(x + \sqrt{1+x^2})$ y $dv = dx$ con lo cual

$$\begin{aligned}
 du &= \frac{1}{x + \sqrt{1+x^2}} \cdot \left(1 + \frac{2x}{2\sqrt{1+x^2}}\right) dx = \frac{1}{x + \sqrt{1+x^2}} \cdot \left(1 + \frac{x}{\sqrt{1+x^2}}\right) dx \Rightarrow \\
 \Rightarrow du &= \frac{1}{x + \sqrt{1+x^2}} \cdot \left(\frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}}\right) dx = \frac{1}{\sqrt{1+x^2}} dx \quad \text{y} \quad v = x
 \end{aligned}$$

Sustituyendo en la fórmula de integración por partes, obtenemos:

$$\begin{aligned}
 \int \operatorname{Ln}(x + \sqrt{1+x^2}) dx &= x \cdot \operatorname{Ln}(x + \sqrt{1+x^2}) - \int x \cdot \frac{1}{\sqrt{1+x^2}} dx = \\
 &= x \cdot \operatorname{Ln}(x + \sqrt{1+x^2}) - \int \frac{2x}{2\sqrt{1+x^2}} dx = x \cdot \operatorname{Ln}(x + \sqrt{1+x^2}) - \sqrt{1+x^2} + k
 \end{aligned}$$

106. $\int \frac{x \operatorname{arcsen} x}{\sqrt{1-x^2}} dx$ Sol: $x - \sqrt{1-x^2} \operatorname{arcsen} x + k$

$$\int \frac{x \operatorname{arcsen} x}{\sqrt{1-x^2}} dx = \left\{ \begin{array}{l} u = \operatorname{arcsen} x \Rightarrow du = \frac{1}{\sqrt{1-x^2}} dx \\ dv = \frac{x}{\sqrt{1-x^2}} dx \Rightarrow v = -\int \frac{-2x}{2\sqrt{1-x^2}} dx = -\sqrt{1-x^2} \end{array} \right\} =$$

$$\begin{aligned}
 &= -\sqrt{1-x^2} \cdot \operatorname{arcsen} x - \int -\sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} \cdot \operatorname{arcsen} x + \int dx = \\
 &= -\sqrt{1-x^2} \cdot \operatorname{arcsen} x + x + k
 \end{aligned}$$

$$107. \int \frac{2x-1}{(x-1)(x-2)} dx \qquad \text{Sol: } \text{Ln} \left| \frac{(x-2)^3}{x-1} \right| + k$$

Tenemos una integral de tipo racional donde el grado del numerador es menor que el grado del denominador. Vamos a descomponer el integrando en fracciones simples:

$$(x-1)(x-2) = 0 \Rightarrow \begin{cases} x=1 \\ x=2 \end{cases} \text{ (raíces reales simples)}$$

Entonces:

$$\frac{2x-1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2) + B(x-1)}{(x-1)(x-2)}$$

Vamos a calcular los coeficientes indeterminados. Al ser los denominadores iguales, los numeradores también lo serán. Por tanto:

$$2x-1 = A(x-2) + B(x-1) \Rightarrow \begin{cases} x=1 \rightarrow 1 = -A \Rightarrow A = -1 \\ x=2 \rightarrow 3 = B \Rightarrow B = 3 \end{cases}$$

Por tanto,

$$\begin{aligned} \int \frac{2x-1}{(x-1)(x-2)} dx &= \int \left(\frac{-1}{x-1} + \frac{3}{x-2} \right) dx = -\int \frac{1}{x-1} dx + 3 \int \frac{1}{x-2} dx = \\ &= -\text{Ln} (x-1) + 3 \text{ Ln} (x-2) + k = \text{Ln} \left| \frac{(x-2)^3}{x-1} \right| + k \end{aligned}$$

$$108. \int \frac{x dx}{(x+1)(x+3)(x+5)} \qquad \text{Sol: } \frac{1}{8} \text{Ln} \left| \frac{(x+3)^6}{(x+1)(x+5)^5} \right| + k$$

Tenemos una integral de tipo racional donde el grado del numerador es menor que el grado del denominador. Vamos a descomponer el integrando en fracciones simples:

$$(x+1)(x+3)(x+5) = 0 \Rightarrow \begin{cases} x=-1 \\ x=-3 \\ x=-5 \end{cases} \text{ (raíces reales simples)}$$

Entonces:

$$\begin{aligned} \frac{x}{(x+1)(x+3)(x+5)} &= \frac{A}{x+1} + \frac{B}{x+3} + \frac{C}{x+5} = \\ &= \frac{A(x+3)(x+5) + B(x+1)(x+5) + C(x+1)(x+3)}{(x+1)(x+3)(x+5)} \end{aligned}$$

Para calcular los coeficientes indeterminados, al ser los denominadores iguales, los numeradores también lo serán. Por tanto:

$$x = A(x+3)(x+5) + B(x+1)(x+5) + C(x+1)(x+3) \Rightarrow \begin{cases} x = -1 \rightarrow -1 = 8A \Rightarrow A = -\frac{1}{8} \\ x = -3 \rightarrow -3 = -4B \Rightarrow B = \frac{3}{4} \\ x = -5 \rightarrow -5 = 8C \Rightarrow C = -\frac{5}{8} \end{cases}$$

Por tanto,

$$\begin{aligned} \int \frac{x dx}{(x+1)(x+3)(x+5)} &= \int \left(\frac{-\frac{1}{8}}{x+1} + \frac{\frac{3}{4}}{x+3} + \frac{-\frac{5}{8}}{x+5} \right) dx = \\ &= -\frac{1}{8} \int \frac{1}{x+1} dx + \frac{3}{4} \int \frac{1}{x+3} dx - \frac{5}{8} \int \frac{1}{x+5} dx = -\frac{1}{8} \text{Ln}(x+1) + \frac{3}{4} \text{Ln}(x+3) - \frac{5}{8} \text{Ln}(x+5) + k = \\ &= \frac{1}{8} (\text{Ln}(x+1) + 6\text{Ln}(x+3) - 5\text{Ln}(x+5)) + k = \frac{1}{8} \text{Ln} \left| \frac{(x+3)^6}{(x+1)(x+5)^5} \right| + k \end{aligned}$$

109. $\int \frac{x^5 + x^4 - 8}{x^3 - 4x} dx$ Sol: $\frac{x^3}{3} + \frac{x^2}{2} + 4x + \text{Ln} \left| \frac{x^2(x-2)^5}{(x+2)^3} \right| + k$

Al ser el grado del numerador mayor que el grado del denominador, antes de aplicar el método de descomposición en fracciones simples tendremos que dividir. De esta forma obtenemos:

$$\frac{x^5 + x^4 - 8}{x^3 - 4x} = x^2 + x + 4 + \frac{4x^2 + 16x - 8}{x^3 - 4x}$$

En consecuencia:

$$\begin{aligned} \int \frac{x^5 + x^4 - 8}{x^3 - 4x} \cdot dx &= \int (x^2 + x + 4) dx + \int \frac{4x^2 + 16x - 8}{x^3 - 4x} \cdot dx = \\ &= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \int \frac{4x^2 + 16x - 8}{x^3 - 4x} \cdot dx \end{aligned}$$

A la integral que nos queda le aplicamos el método de descomposición en fracciones simples. Calculamos las raíces del denominador:

$$x^3 - 4x = 0 \rightarrow x \cdot (x^2 - 4) = 0 \rightarrow \begin{cases} x = 0 \\ x = \pm 2 \end{cases}$$

Entonces:

$$\frac{4x^2 + 16x - 8}{x^3 - 4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} = \frac{A(x-2)(x+2) + Bx(x+2) + Cx(x-2)}{x(x-2)(x+2)}$$

Como los denominadores son iguales, los numeradores también lo serán; por tanto:

$$4x^2 + 16x - 8 = A(x-2)(x+2) + Bx(x+2) + Cx(x-2)$$

Calculamos los coeficientes indeterminados: le vamos asignando los valores de las raíces

$$\begin{aligned} x = 0 &\rightarrow -8 = -4A \rightarrow A = 2 \\ x = 2 &\rightarrow 40 = 8B \rightarrow B = 5 \\ x = -2 &\rightarrow -24 = 8C \rightarrow C = -3 \end{aligned}$$

Por tanto, la fracción descompuesta en fracciones simples nos queda:

$$\frac{4x^2 + 16x - 8}{x^3 - 4x} = \frac{2}{x} + \frac{5}{x-2} - \frac{3}{x+2}$$

La integral de la función pedida será:

$$\begin{aligned} \int \frac{x^5 + x^4 - 8}{x^3 - 4x} \cdot dx &= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \int \frac{4x^2 + 16x - 8}{x^3 - 4x} \cdot dx = \\ &= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \int \left(\frac{2}{x} + \frac{5}{x-2} - \frac{3}{x+2} \right) \cdot dx \\ &= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \int \frac{2}{x} \cdot dx + \int \frac{5}{x-2} \cdot dx - \int \frac{3}{x+2} \cdot dx = \\ &= \frac{x^3}{3} + \frac{x^2}{2} + 4x + 2 \int \frac{1}{x} \cdot dx + 5 \int \frac{1}{x-2} \cdot dx - 3 \int \frac{1}{x+2} \cdot dx = \\ &= \frac{x^3}{3} + \frac{x^2}{2} + 4x + 2 \cdot \text{Ln} |x| + 5 \cdot \text{Ln} |x-2| - 3 \cdot \text{Ln} |x+2| + k = \\ &= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \text{Ln} \left| \frac{x^2(x-2)^5}{(x+2)^3} \right| + k \end{aligned}$$

$$110. \int \frac{x^4 dx}{(x^2-1)(x+2)} \quad \text{Sol: } \frac{x^2}{2} - 2x + \frac{1}{6} \ln \left| \frac{(x-1)}{(x+1)^3} \right| + \frac{16}{3} \cdot \ln |x+2| + k$$

Como el grado del numerador es mayor que el del denominador, tenemos que dividir, obteniendo:

$$\frac{x^4}{(x^2-1)(x+2)} = x-2 + \frac{5x^2-4}{(x^2-1)(x+2)}$$

Con lo que

$$\int \frac{x^4 \cdot dx}{(x^2-1)(x+2)} = \int (x-2) dx + \int \frac{5x^2-4}{(x^2-1)(x+2)} dx = \frac{x^2}{2} - 2x + \int \frac{5x^2-4}{(x^2-1)(x+2)} dx$$

y tendremos que integrar la función racional que nos queda, donde el grado del numerador es menor que el grado del denominador.

Descomponemos en fracciones simples:

$$\frac{5x^2-4}{(x^2-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2} = \frac{A(x+1)(x+2) + B(x-1)(x+2) + C(x-1)(x+1)}{(x^2-1)(x+2)}$$

Como los denominadores son iguales, también lo serán los numeradores. Entonces:

$$5x^2 - 4 = A(x+1)(x+2) + B(x-1)(x+2) + C(x-1)(x+1)$$

Calculamos los coeficientes indeterminados:

$$x=1 \quad \rightarrow \quad 1=6A \quad \rightarrow \quad A=\frac{1}{6}$$

$$x=-1 \quad \rightarrow \quad 1=-2B \quad \rightarrow \quad B=-\frac{1}{2}$$

$$x=-2 \quad \rightarrow \quad 16=3C \quad \rightarrow \quad C=\frac{16}{3}$$

Entonces:
$$\frac{5x^2-4}{(x^2-1)(x+2)} = \frac{1}{6} + \frac{-1}{2} + \frac{16}{3}$$

Y, por tanto:

$$\begin{aligned}
 \int \frac{x^4 \cdot dx}{(x^2-1)(x+2)} &= \frac{x^2}{2} - 2x + \int \frac{5x^2-4}{(x^2-1)(x+2)} dx = \frac{x^2}{2} - 2x + \int \left(\frac{\frac{1}{6}}{x-1} + \frac{-\frac{1}{2}}{x+1} + \frac{\frac{16}{3}}{x+2} \right) \cdot dx = \\
 &= \frac{x^2}{2} - 2x + \int \frac{\frac{1}{6}}{x-1} \cdot dx + \int \frac{-\frac{1}{2}}{x+1} \cdot dx + \int \frac{\frac{16}{3}}{x+2} \cdot dx = \\
 &= \frac{x^2}{2} - 2x + \frac{1}{6} \int \frac{1}{x-1} \cdot dx - \frac{1}{2} \int \frac{1}{x+1} \cdot dx + \frac{16}{3} \int \frac{1}{x+2} \cdot dx = \\
 &= \frac{x^2}{2} - 2x + \frac{1}{6} \cdot \text{Ln} |x-1| - \frac{1}{2} \cdot \text{Ln} |x+1| + \frac{16}{3} \cdot \text{Ln} |x+2| + k = \\
 &= \frac{x^2}{2} - 2x + \frac{1}{6} \cdot (\text{Ln} |x-1| - 3 \cdot \text{Ln} |x+1|) + \frac{16}{3} \cdot \text{Ln} |x+2| + k = \\
 &= \frac{x^2}{2} - 2x + \frac{1}{6} \cdot \text{Ln} \left| \frac{x-1}{(x+1)^3} \right| + \frac{16}{3} \cdot \text{Ln} |x+2| + k
 \end{aligned}$$

111. $\int \frac{dx}{(x-1)^2(x-2)}$ Sol: $\frac{1}{x-1} + \text{Ln} \left| \frac{x-2}{x-1} \right| + k$

Como el grado del numerador es menor que el grado del denominador aplicamos la descomposición en fracciones simples directamente:

$$\begin{aligned}
 \frac{1}{(x-1)^2(x-2)} &= \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-2)} = \frac{A(x-1)(x-2) + B(x-2) + C(x-1)^2}{(x-1)^2(x-2)} \rightarrow \\
 &\rightarrow 1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2
 \end{aligned}$$

Calculamos los coeficientes:

$$x=1: 1=-B \rightarrow B=-1$$

$$x=2: 1=C$$

$$x=0: 1=2A-2B+C \rightarrow 1=2A+2+1 \rightarrow A=-1$$

Entonces:

$$\int \frac{1}{(x-1)^2(x-2)} \cdot dx = \int \frac{-1}{x-1} \cdot dx + \int \frac{-1}{(x-1)^2} \cdot dx + \int \frac{1}{(x-2)} \cdot dx =$$

$$= -\int \frac{1}{x-1} \cdot dx - \int (x-1)^{-2} \cdot dx + \int \frac{1}{x-2} \cdot dx =$$

$$= -\text{Ln} |x-1| - \frac{(x-1)^{-1}}{-1} + \text{Ln} |x-2| + k = \frac{1}{x-1} + \text{Ln} \left| \frac{x-2}{x-1} \right| + k$$

112. $\int \frac{x-8}{x^3-4x^2+4x} dx$ Sol: $\frac{3}{x-2} + \text{Ln} \frac{(x-2)^2}{x^2} + k$

Igual que en el anterior, aplicamos la descomposición en fracciones simples:

Calculamos las raíces del denominador:

$$x^3 - 4x^2 + 4x = 0 \rightarrow x \cdot (x^2 - 4x + 4) = 0 \rightarrow x \cdot (x-2)^2 = 0 \rightarrow \begin{cases} x=0 \\ x=2 \text{ (doble)} \end{cases}$$

Entonces:

$$\frac{x-8}{x^3-4x^2+4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} = \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2} \Rightarrow$$

$$\Rightarrow x-8 = A(x-2)^2 + Bx(x-2) + Cx \Rightarrow$$

Calculamos los coeficientes:

$$x=0 \rightarrow -8 = 4A \rightarrow A = -2$$

$$x=2 \rightarrow -6 = 2C \rightarrow C = -3$$

$$x=1 \rightarrow -7 = A - B + C \rightarrow B = 7 + A + C = 7 - 2 - 3 = 2 \rightarrow B = 2$$

Entonces:

$$\int \frac{x-8}{x^3-4x^2+4x} dx = \int \left(\frac{-2}{x} + \frac{2}{x-2} + \frac{-3}{(x-2)^2} \right) \cdot dx =$$

$$= -2 \int \frac{1}{x} \cdot dx + 2 \int \frac{1}{x-2} \cdot dx - 3 \int \frac{1}{(x-2)^2} \cdot dx = -2 \text{Ln} |x| + 2 \text{Ln} |x-2| - 3 \int (x-2)^{-2} dx =$$

$$= -2 \text{Ln} |x| + 2 \text{Ln} |x-2| - 3 \cdot \frac{(x-2)^{-1}}{-1} + k = \frac{3}{x-2} + \text{Ln} \frac{(x-2)^2}{x^2} + k$$

113. $\int \frac{3x+2}{x(x+1)^3} dx$ Sol: $\frac{4x+3}{2(x+1)^2} + \text{Ln} \frac{x^2}{(x+1)^2} + k$

114. $\int \frac{3x-2}{x(x+1)^3} dx$ Sol: $\text{Ln} \frac{(x+1)^2}{x^2} - \frac{4x+9}{2(x+1)^2} + k$

Descomponemos el integrando en fracciones simples:

$$\frac{3x-2}{x(x+1)^3} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} = \frac{A(x+1)^3 + Bx(x+1)^2 + Cx(x+1) + Dx}{x \cdot (x+1)^3} \rightarrow$$

$$\rightarrow 3x-2 = A(x+1)^3 + Bx(x+1)^2 + Cx(x+1) + Dx \rightarrow$$

Calculamos los coeficientes:

$$x=0 \rightarrow -2 = A$$

$$x=-1 \rightarrow -5 = -D \rightarrow D = 5$$

$$x=1 \rightarrow 1 = 8A + 4B + 2C + D \rightarrow 1 = -16 + 4B + 2C + 5 \rightarrow 2B + C = 6$$

$$x=-2 \rightarrow -8 = -A - 2B + 2C - 2D \rightarrow -8 = 2 - 2B + 2C - 10 \rightarrow -B + C = 0$$

Resolviendo el sistema resultante, obtenemos:

$$\left. \begin{array}{l} 2B + C = 6 \\ -B + C = 0 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} 2B + B = 6 \rightarrow B = 2 \\ C = B = 2 \end{array} \right.$$

Entonces:

$$\begin{aligned} \int \frac{3x-2}{x(x+1)^3} \cdot dx &= \int \frac{-2}{x} \cdot dx + \int \frac{2}{x+1} \cdot dx + \int \frac{2}{(x+1)^2} \cdot dx + \int \frac{5}{(x+1)^3} \cdot dx = \\ &= -2 \int \frac{1}{x} \cdot dx + 2 \int \frac{1}{x+1} \cdot dx + 2 \int \frac{1}{(x+1)^2} \cdot dx + 5 \int \frac{1}{(x+1)^3} \cdot dx = \\ &= -2 \cdot \text{Ln} |x| + 2 \cdot \text{Ln} |x+1| + 2 \int (x+1)^{-2} \cdot dx + 5 \int (x+1)^{-3} \cdot dx = \\ &= -2 \cdot \text{Ln} |x| + 2 \cdot \text{Ln} |x+1| + 2 \frac{(x+1)^{-1}}{-1} + 5 \frac{(x+1)^{-2}}{-2} + k = \\ &= \text{Ln} \frac{(x+1)^2}{x^2} - \frac{2}{x+1} - \frac{5}{2(x+1)^2} + k = \text{Ln} \frac{(x+1)^2}{x^2} - \frac{4(x+1)+5}{2(x+1)^2} + k = \\ &= \text{Ln} \frac{(x+1)^2}{x^2} - \frac{4x+9}{2(x+1)^2} + k \end{aligned}$$

115. $\int \frac{x^2 dx}{(x+2)^2(x+4)^2}$ Sol: $-\frac{5x+12}{x^2+6x+8} + \text{Ln}\left(\frac{x+4}{x+2}\right)^2 + k$
116. $\int \frac{\sqrt{x}}{\sqrt[4]{x^3+1}} dx$ Sol: $\frac{4}{3}\left(\sqrt[4]{x^3} - \text{Ln}\left(\sqrt[4]{x^3+1}\right)\right) + k$
117. $\int \frac{\sqrt{x^3} - \sqrt[3]{x}}{6\sqrt[4]{x}} dx$ Sol: $\frac{2}{27}\sqrt[4]{x^9} - \frac{2}{13}\sqrt[12]{x^{13}} + k$
118. $\int \frac{\sqrt[6]{x}+1}{\sqrt[6]{x^7} + \sqrt[4]{x^5}} dx$ Sol: $-\frac{6}{\sqrt[6]{x}} + \frac{12}{\sqrt[12]{x}} + 2\text{Ln } x - 24\text{Ln}(\sqrt[12]{x}+1) + k$
119. $\int \frac{\sqrt[7]{x} + \sqrt{x}}{\sqrt[7]{x^8} + \sqrt[14]{x^{15}}} dx$ Sol: $4\left[\sqrt[14]{x} - \frac{1}{2}\sqrt[7]{x} + \frac{1}{3}\sqrt[14]{x^3} - \frac{1}{4}\sqrt[7]{x^2} + \frac{1}{5}\sqrt[14]{x^5}\right] + k$
120. $\int \frac{dx}{\sqrt{1-x} + \sqrt[3]{1-x}}$ Sol: $6\left[\frac{\sqrt{1-x}}{3} - \frac{\sqrt[3]{1-x}}{2} + \sqrt[6]{1-x} - \text{Ln}(1 + \sqrt[6]{1-x})\right] + k$
121. $\int \frac{e^x}{e^{2x} + e^x - 2} dx$ Sol: $\frac{1}{3} \text{Ln} \left| \frac{e^x - 1}{e^x + 2} \right| + k$
122. $\int \frac{dx}{e^x + 1}$ Sol: $x - \text{Ln}(e^x + 1) + k$
123. $\int \frac{e^x}{e^{2x} + 3e^x + 2} dx$ Sol: $\text{Ln} \left| \frac{e^x + 1}{e^x + 2} \right| + k$
124. $\int \text{sen}^3 x dx$ Sol: $\frac{1}{3}\cos^3 x - \cos x + k$
- 125.