# REPASO DERIVADAS

Calcula as derivadas das seguintes funcións, e se se pode simplifica:

$a) y = \ln \sqrt{\frac{1+x}{1-x}}$

$b) y = \frac{x}{x+1} \operatorname{tag} x$

$c) y = \frac{sen^2 x \cos x^2 - arccos\sqrt{x}}{3}$

$d) y = \ln (cosx)^2 - \sqrt{4x^2 - 5x} + 3 \cdot 5^{6-3x^3}$

$e) y = \left(\frac{senx}{x}\right)^x$

a) $y = \ln \sqrt{\frac{1+x}{1-x}}$

Aplico propiedades dos logaritmos:

$$y = \ln \sqrt{\frac{1+x}{1-x}} = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) =$$

$$= \frac{1}{2}\left[\ln(1+x) - \ln(1-x)\right]$$

Derivo agora:

$$y' = \frac{1}{2} \cdot \left[\frac{1}{1+x} - \frac{(-1)}{1-x}\right] = \frac{1}{2}\left[\frac{1}{1+x} + \frac{1}{1-x}\right] \Rightarrow$$

$$y' = \frac{1}{2}\left[\frac{1-x+1+x}{1-x^2}\right] = \frac{1}{1-x^2}$$

b) $y = \underbrace{\frac{x}{x+1}}_{f} \cdot \underbrace{\tan x}_{g}$

$$y' = \frac{1\cdot(x+1) - x\cdot 1}{(x+1)^2} \cdot \tan x + \frac{x}{x+1} \cdot \frac{1}{\cos^2 x}$$

$$= \frac{1}{(x+1)^2} \tan x + \frac{x}{x+1} \cdot \frac{1}{\cos^2 x}$$

c) $y = \frac{\operatorname{sen}^2 x \cos^2 x - \arccos\sqrt{x}}{3} =$

$$= \frac{1}{3}\left(\operatorname{sen}^2 x \cos^2 x - \arccos\sqrt{x}\right)$$

$$y' = \frac{1}{3}\left[2\operatorname{sen}x\cos x\cos^2 x + \operatorname{sen}^2 x\, 2\cos x\,(-\operatorname{sen}x) + \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}\right]$$

$$y' = \frac{1}{3}\left[2\cdot\operatorname{sen}x\cos^3 x - 2\cos x\operatorname{sen}^3 x + \frac{1}{2\sqrt{x}\cdot\sqrt{1-x}}\right]$$

$$= \frac{1}{3}\left[\underbrace{(\cos^2 x - \operatorname{sen}^2 x)}_{\cos 2x}\underbrace{2\operatorname{sen}x\cos x}_{\operatorname{sen}2x} + \frac{1}{2\sqrt{x}\sqrt{1-x}}\right] =$$

$$= \frac{1}{3}\left[\cos 2x\cdot\operatorname{sen}2x + \frac{1}{2\sqrt{x(1-x)}}\right]$$

a) $y = \underbrace{\ln(\cos x)^2} - \sqrt{4x^2-5x} + 3 \cdot 5^{6-3x^3}$

$y = \overbrace{2\ln(\cos x)} - \sqrt{4x^2-5x} + 3 \cdot 5^{6-3x^3}$

$y' = 2\frac{(-\operatorname{sen} x)}{\cos x} - \frac{8x-5}{2\sqrt{4x^2-5x}} + 3 \cdot 5^{6-3x^3} \cdot \ln 5 \cdot (-9x^2)$

$y' = \frac{-2\operatorname{sen} x}{\cos x} - \frac{8x-5}{2\sqrt{4x^2-5x}} - 27x^2 \ln 5 \cdot 5^{6-3x^3}$

e) $y = \left(\frac{\operatorname{sen} x}{x}\right)^x$

Derivación logarítmica

$\ln y = x \ln\left(\frac{\operatorname{sen} x}{x}\right)$

$\frac{y'}{y} = 1 \ln\left(\frac{\operatorname{sen} x}{x}\right) + \cancel{x} \cdot \frac{1}{\frac{\operatorname{sen} x}{\cancel{x}}} \cdot \frac{\cos x \cdot x - \operatorname{sen} x}{\cancel{x^2}}$

$y' = \left(\frac{\operatorname{sen} x}{x}\right)^x \cdot \left[\ln\left(\frac{\operatorname{sen} x}{x}\right) + \frac{x\cos x - \operatorname{sen} x}{\operatorname{sen} x}\right]$