EJERCICIOS

1) Extrae factores fuera de la raíz:

$a)\ \sqrt{2^3\cdot 3\cdot 5^2} = 2\cdot 5\sqrt{2\cdot 3} = 10\sqrt{6}$

$b)\ \sqrt{120} = \sqrt{2^3\cdot 3\cdot 5} = 2\cdot\sqrt{30}$

$c)\ \sqrt[3]{144} = \sqrt[3]{2^4\cdot 3^2} = 2\sqrt[3]{2\cdot 3^2} = 2\sqrt[3]{18}$

$d)\ \sqrt[4]{64\cdot a^3\cdot b^4} = \sqrt[4]{2^6\cdot a^3\cdot b^4} = 2b\sqrt[4]{2^2 a^3} = 2b\sqrt[4]{4a^3}$

$e)\ \sqrt{72\cdot a^5\cdot b^3\cdot c} = \sqrt{2^3\cdot 3^2 a^5\cdot b^3\cdot c} = 2\cdot 3a^2\cdot b\sqrt{2ac}$

$f)\ \sqrt{45\cdot x\cdot y^6} = \sqrt{3^2\cdot 5xy^6} = 3y^3\sqrt{5x}$

2) Efectúa:

$a)\ \sqrt{2}\cdot\sqrt{5}\cdot\sqrt{3} = \sqrt{2\cdot 5\cdot 3} = \sqrt{30}$

$b)\ \sqrt[3]{7}\cdot\sqrt[3]{5} = \sqrt[3]{35}$

$c)\ \sqrt[a]{b}\cdot\sqrt[a]{x} = \sqrt[a]{bx}$

$d)\ \left(\sqrt{2}+\sqrt{3}\right)^2 = \left(\sqrt{2}\right)^2 + 2\cdot\sqrt{2}\cdot\sqrt{3} + \left(\sqrt{3}\right)^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$

$e)\ \sqrt{5}\left(\sqrt{3}-\sqrt{2}\right)^2 = \sqrt{5}\left[\left(\sqrt{3}\right)^2 - 2\sqrt{3}\cdot\sqrt{2} + \left(\sqrt{2}\right)^2\right] = \sqrt{5}\left(3 - 2\sqrt{6} + 2\right) = 5\sqrt{5} - 2\sqrt{30}$

$f)\ \left(\sqrt{5}-2\sqrt{3}\right)\left(3\sqrt{5}+\sqrt{2}\right) = 3\cdot\sqrt{5}\cdot\sqrt{5} + \sqrt{5}\cdot\sqrt{2} - 2\cdot 3\cdot\sqrt{3}\cdot\sqrt{5} - 2\cdot\sqrt{3}\cdot\sqrt{2} =$

$= 3\cdot 5 + \sqrt{10} - 6\sqrt{15} - 2\sqrt{6} = 15 + \sqrt{10} - 6\sqrt{15} - 2\sqrt{6}$

$g)\ \sqrt{2}\left(2\sqrt{2}-3\sqrt{3}\right)\left(\sqrt{5}-\sqrt{2}\right) = \sqrt{2}\left(2\sqrt{10} - 2\cdot 2 - 2\sqrt{15} + 3\sqrt{6}\right) = 2\sqrt{20} - 4\sqrt{2} - 3\sqrt{30} + 3\sqrt{18} =$

$= 2\sqrt{4\cdot 5} - 4\sqrt{2} - 3\sqrt{30} + 3\sqrt{9\cdot 2} = 4\sqrt{5} - 4\sqrt{2} - 3\sqrt{30} + 9\sqrt{2}$

$h)\ \sqrt{3}\left(\sqrt{3}-2\sqrt{2}\right)^2 = \sqrt{3}\left[\left(\sqrt{3}\right)^2 - 2\cdot\sqrt{3}\cdot 2\sqrt{2} + \left(2\sqrt{2}\right)^2\right] = \sqrt{3}\left(3 - 4\sqrt{6} + 4\cdot 2\right) = 11\sqrt{3} - 4\sqrt{18} =$

$= 11\sqrt{3} - 4\sqrt{9\cdot 2} = 11\sqrt{3} - 12\sqrt{2}$

$i)\ \left(\frac{3}{2\sqrt{3}} - \frac{2\sqrt{2}}{5}\right)\cdot\left(\frac{3}{2\sqrt{3}} + \frac{2\sqrt{2}}{5}\right) = \left(\frac{3}{2\sqrt{3}}\right)^2 - \left(\frac{2\sqrt{2}}{5}\right)^2 = \frac{9}{4\cdot 3} - \frac{4\cdot 2}{25} = \frac{3}{4} - \frac{8}{25} = \frac{75-32}{100} = \frac{43}{100}$

3) Extrae factores fuera de la raíz:

$a)\ \sqrt[5]{\frac{a^7\cdot b^{12}}{c^6}} = \sqrt[5]{\frac{a^5\cdot a^2\cdot\left(b^2\right)^5\cdot b^2}{c^5\cdot c}} = \frac{a\cdot b^2}{c}\sqrt[5]{\frac{a^2\cdot b^2}{c}}$

$b)\ \sqrt[3]{a^5\sqrt{a^6}} = \sqrt[3]{a^5\cdot a^3} = \sqrt[3]{a^8} = a^2\cdot\sqrt[3]{a^2}$

$$c)\quad \sqrt{\frac{81a^8}{32b^3}} = \sqrt{\frac{81\left(a^4\right)^2}{16\cdot 2b^2\cdot b}} = \frac{9a^4}{4b}\sqrt{\frac{1}{2b}}$$

$$d)\quad \sqrt[5]{\frac{a^9\sqrt[3]{a^{18}}}{64b^{15}}} = \sqrt[5]{\frac{a^9\cdot a^6}{2^5\cdot 2\left(b^3\right)^5}} = \sqrt[5]{\frac{a^{15}}{2^5\cdot 2\left(b^3\right)^5}} = \frac{a^3}{2b^3}\sqrt[5]{\frac{1}{2}}$$

$$e)\quad \sqrt[3]{\frac{125\sqrt{a^7}}{48b^8}} = \sqrt[3]{\frac{5^3\cdot a^3\sqrt{a}}{2^4\cdot 3\cdot b^6\cdot b^2}} = \frac{5a}{2b^2}\sqrt[3]{\frac{\sqrt{a}}{6b^2}}$$

4) Introduce dentro de la raíz y simplifica:

$$a)\quad 7\sqrt{a} = \sqrt{49a}$$

$$b)\quad 2a\sqrt{3a} = \sqrt{(2a)^2\cdot 3a} = \sqrt{4a^2\cdot 3a} = \sqrt{12a^3}$$

$$c)\quad x\sqrt{\frac{1}{x}} = \sqrt{\frac{x^2}{x}} = \sqrt{x}$$

$$d)\quad x^3y\sqrt{\frac{x}{y}} = \sqrt{\left(x^3y\right)^2\frac{x}{y}} = \sqrt{\frac{x^6\cdot y^2\cdot x}{y}} = \sqrt{x^7y}$$

$$e)\quad \frac{1}{3}\sqrt[4]{\frac{27}{2}} = \sqrt[4]{\frac{27}{3^4\cdot 2}} = \sqrt[4]{\frac{3^3}{3^4\cdot 2}} = \sqrt[4]{\frac{1}{6}}$$

$$f)\quad \frac{3}{2}\sqrt{\frac{2}{3}} = \sqrt{\frac{9\cdot 2}{4\cdot 3}} = \sqrt{\frac{3}{2}}$$

$$g)\quad \frac{2}{3}\sqrt[3]{\frac{81}{4}} = \sqrt[3]{\frac{2^3\cdot 3^4}{3^3\cdot 2^2}} = \sqrt[3]{2\cdot 3} = \sqrt[3]{6}$$

$$h)\quad \left(1+\frac{1}{2}\right)\sqrt[3]{\frac{4}{81}} = \frac{3}{2}\sqrt[3]{\frac{2^2}{3^4}} = \sqrt[3]{\frac{3^3\cdot 2^2}{2^3\cdot 3^4}} = \sqrt[3]{\frac{1}{6}}$$

$$i)\quad \frac{a-b}{a+b}\sqrt{\frac{a+b}{a-b}} = \sqrt{\frac{(a-b)^2(a+b)}{(a+b)^2(a-b)}} = \sqrt{\frac{a-b}{a+b}}$$

$$j)\quad (1-a)\sqrt{a-a^2} = \sqrt{(1-a)^2\left(a-a^2\right)} = \sqrt{(1-a)^2a(1-a)} = \sqrt{a(1-a)^3}$$

$$k)\quad (x+y)\sqrt{\frac{x-y}{x+y}} = \sqrt{\frac{(x+y)^2\cdot(x-y)}{x+y}} = \sqrt{(x+y)(x-y)} = \sqrt{x^2-y^2}$$

5) Efectúa las siguientes sumas de radicales:

$$a)\quad \sqrt{18}-\sqrt{8}-\sqrt{2} = \sqrt{9\cdot 2}-\sqrt{4\cdot 2}-\sqrt{2} = 3\sqrt{2}-2\sqrt{2}-\sqrt{2} = (3-2-1)\sqrt{2} = 0$$

$$b)\quad \sqrt{27}+\sqrt{12}+\sqrt{3} = \sqrt{9\cdot 3}+\sqrt{4\cdot 3}+\sqrt{3} = 3\sqrt{3}+2\sqrt{3}+\sqrt{3} = 6\sqrt{3}$$

$$c)\quad \sqrt{18}+\sqrt{32}-\sqrt{48}-\sqrt{12} = \sqrt{9\cdot 2}+\sqrt{16\cdot 2}-\sqrt{16\cdot 3}-\sqrt{4\cdot 3} = 3\sqrt{2}+4\sqrt{2}-4\sqrt{3}-2\sqrt{3} =$$

$$= (3+4)\sqrt{2}-(4+2)\sqrt{3} = 7\sqrt{2}-6\sqrt{3}$$

$$d)\quad \sqrt[3]{24}+\sqrt[4]{32}-\sqrt[3]{81}+\sqrt[4]{162} = \sqrt[3]{8\cdot 3}+\sqrt[4]{16\cdot 2}-\sqrt[3]{27\cdot 3}+\sqrt[4]{81\cdot 2} = 2\sqrt[3]{3}+2\sqrt[4]{2}-3\sqrt[3]{3}+3\sqrt[4]{2} =$$

$$= (2-3)\sqrt[3]{3}+(2+3)\sqrt[4]{2} = -\sqrt[3]{3}+5\sqrt[4]{2}$$

6) Efectúa las siguientes sumas de radicales. Expresa el resultado simplificado y racionalizado.

$$a)\ \frac{1}{2}\sqrt{12}-\frac{2}{9}\sqrt{75}+\frac{1}{6}\sqrt{108}-\frac{2}{5}\sqrt{300}=\frac{1}{2}\sqrt{4\cdot3}-\frac{2}{9}\sqrt{25\cdot3}+\frac{1}{6}\sqrt{36\cdot3}-\frac{2}{5}\sqrt{100\cdot3}=$$

$$=\frac{2}{2}\sqrt{3}-\frac{2\cdot5}{9}\sqrt{3}+\frac{6}{6}\sqrt{3}-\frac{2\cdot10}{5}\sqrt{3}=\sqrt{3}-\frac{10}{9}\sqrt{3}+\sqrt{3}-4\sqrt{3}=-2\sqrt{3}-\frac{10}{9}\sqrt{3}=\frac{-18-10}{9}\sqrt{3}=-\frac{28}{9}\sqrt{3}$$

$$b)\ \sqrt{18a}-\frac{4}{a}\sqrt{\frac{a^3}{32}}+\sqrt{\frac{a}{8}}-2\sqrt{\frac{a}{72}}=\sqrt{9\cdot2a}-\frac{4\cancel{a}}{\cancel{a}}\sqrt{\frac{a}{16\cdot2}}+\sqrt{\frac{a}{4\cdot2}}-2\sqrt{\frac{a}{36\cdot2}}=$$

$$=3\sqrt{2a}-\frac{4}{4}\sqrt{\frac{a}{2}}+\frac{1}{2}\sqrt{\frac{a}{2}}-\frac{2}{6}\sqrt{\frac{a}{2}}=3\sqrt{2a}-\sqrt{\frac{a\cdot2}{2\cdot2}}+\frac{1}{2}\sqrt{\frac{a\cdot2}{2\cdot2}}-\frac{1}{3}\sqrt{\frac{a\cdot2}{2\cdot2}}=$$

$$=3\sqrt{2a}-\frac{1}{2}\sqrt{2a}+\frac{1}{4}\sqrt{2a}-\frac{1}{6}\sqrt{2a}=\left(3-\frac{1}{2}+\frac{1}{4}-\frac{1}{6}\right)\sqrt{2a}=\frac{36-6+3-2}{12}\sqrt{2a}=\frac{31}{12}\sqrt{2a}$$

$$c)\ \frac{1}{2}\sqrt{\frac{4}{125}}-\frac{1}{5}\sqrt{500}+\sqrt{1-\frac{16}{25}}-\frac{4}{3}\sqrt{45}+\frac{2}{7}\sqrt{147}=$$

$$=\frac{2}{2}\sqrt{\frac{1}{25\cdot5}}-\frac{1}{5}\sqrt{100\cdot5}+\sqrt{\frac{25-16}{25}}-\frac{4}{3}\sqrt{9\cdot5}+\frac{2}{7}\sqrt{49\cdot3}=\frac{1}{5}\sqrt{\frac{1}{5}}-\frac{10}{5}\sqrt{5}+\sqrt{\frac{9}{25}}-\frac{4\cdot3}{3}\sqrt{5}+\frac{2\cdot7}{7}\sqrt{3}=$$

$$=\frac{1}{5}\sqrt{\frac{5}{25}}-2\sqrt{5}+\frac{3}{5}-4\sqrt{5}+2\sqrt{3}=\frac{1}{25}\sqrt{5}-6\sqrt{5}+\frac{3}{5}+2\sqrt{3}=\frac{1-150}{25}\sqrt{5}+\frac{3}{5}+2\sqrt{3}=$$

$$=-\frac{149}{25}\sqrt{5}+\frac{3}{5}+2\sqrt{3}$$

$$d)\ \frac{1}{b}\sqrt{\frac{ab}{3}}-a\sqrt{\frac{48}{ab}}+\sqrt{\frac{27a}{b}}-\frac{a}{b}\sqrt{\frac{b}{12a}}=\frac{1}{b}\sqrt{\frac{3ab}{3^2}}-a\sqrt{\frac{16\cdot3ab}{(ab)^2}}+\sqrt{\frac{9\cdot3ab}{b^2}}-\frac{a}{b}\sqrt{\frac{b\cdot3a}{4\cdot3a\cdot3a}}=$$

$$=\frac{1}{3b}\sqrt{3ab}-\frac{4a}{ab}\sqrt{3ab}+\frac{3}{b}\sqrt{3ab}-\frac{a}{2b\cdot3a}\sqrt{3a}=\left(\frac{1}{3b}-\frac{4}{b}+\frac{3}{b}-\frac{1}{6b}\right)\sqrt{3a}=\frac{2-24+18-1}{6b}\sqrt{3a}=$$

$$=-\frac{5}{6b}\sqrt{3a}$$

$$e)\ 3\sqrt{27x}-\frac{5}{2}\sqrt{\frac{x}{3}}+5\sqrt{\frac{3x}{81}}-\frac{1}{4}\sqrt{48x}=3\sqrt{9\cdot3x}-\frac{5}{2}\sqrt{\frac{3x}{3^2}}+\frac{5}{9}\sqrt{3x}-\frac{1}{4}\sqrt{16\cdot3x}=$$

$$=9\sqrt{3x}-\frac{5}{6}\sqrt{3x}+\frac{5}{9}\sqrt{3x}-\frac{4}{4}\sqrt{3x}=\left(8-\frac{5}{6}+\frac{5}{9}\right)\sqrt{3x}=\frac{144-15+10}{18}\sqrt{3x}=\frac{139}{18}\sqrt{3x}$$

$$f)\ 3\sqrt{24x}-\sqrt{\frac{x}{6}}+5\sqrt{\frac{6x}{100}}-\sqrt{54x}=3\sqrt{4\cdot6x}-\sqrt{\frac{6x}{6^2}}+\frac{5}{10}\sqrt{6x}-\sqrt{9\cdot6x}=$$

$$=6\sqrt{6x}-\frac{1}{6}\sqrt{6x}+\frac{1}{2}\sqrt{6x}-3\sqrt{6x}=\left(3-\frac{1}{6}+\frac{1}{2}\right)\sqrt{6x}=\frac{18-1+3}{6}\sqrt{6x}=\frac{20}{6}\sqrt{6x}=\frac{10}{3}\sqrt{6x}$$

$$g)\ \sqrt{\frac{3a}{2}}+\sqrt{\frac{2a}{3}}-\sqrt{6a}+\sqrt{\frac{a}{6}}=\sqrt{\frac{3a\cdot2}{2^2}}+\sqrt{\frac{2a\cdot3}{3^2}}-\sqrt{6a}+\sqrt{\frac{6a}{6^2}}=\frac{1}{2}\sqrt{6a}+\frac{1}{3}\sqrt{6a}-\sqrt{6a}+\frac{1}{6}\sqrt{6a}=$$

$$=\left(\frac{1}{2}+\frac{1}{3}-1+\frac{1}{6}\right)\sqrt{6a}=\frac{3+2-6+1}{6}\sqrt{6a}=\frac{0}{6}\sqrt{6a}=0$$

$h)\ \frac{2}{3}\sqrt{24}-\sqrt{\frac{432}{2}}-\frac{2}{5}\sqrt{50}+\frac{12}{\sqrt{6}}=\frac{2}{3}\sqrt{4\cdot 6}-\sqrt{216}-\frac{2}{5}\sqrt{25\cdot 2}+\frac{12\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}}=$

$=\frac{4}{3}\sqrt{6}-\sqrt{36\cdot 6}-\frac{2\cdot 5}{5}\sqrt{2}+\frac{12}{6}\sqrt{6}=\frac{4}{3}\sqrt{6}-6\sqrt{6}-2\sqrt{2}+2\sqrt{6}=\left(\frac{4}{3}-4\right)\sqrt{6}-2\sqrt{2}=$

$=\frac{4-12}{3}\sqrt{6}-2\sqrt{2}=-\frac{8}{3}\sqrt{6}-2\sqrt{2}$

$i)\ 2\sqrt{80}+\frac{14}{5}\sqrt{1+\frac{1}{49}}-\sqrt{8}-\frac{9}{4}\sqrt{1-\frac{1}{81}}=2\sqrt{16\cdot 5}+\frac{14}{5}\sqrt{\frac{49+1}{49}}-\sqrt{4\cdot 2}-\frac{9}{4}\sqrt{\frac{81-1}{81}}=$

$=8\sqrt{5}+\frac{14}{5\cdot 7}\sqrt{50}-2\sqrt{2}-\frac{9}{4\cdot 9}\sqrt{80}=8\sqrt{5}+\frac{2}{5}\sqrt{25\cdot 2}-2\sqrt{2}-\frac{1}{4}\sqrt{16\cdot 5}=$

$=8\sqrt{5}+\frac{2\cdot 5}{5}\sqrt{2}-2\sqrt{2}-\frac{4}{4}\sqrt{5}=7\sqrt{5}$

$j)\ \sqrt{4a-8b}-\sqrt{9a-18b}+2\sqrt{16a-32b}=\sqrt{4(a-2b)}-\sqrt{9(a-2b)}+2\sqrt{16(a-2b)}=$

$=2\sqrt{a-2b}-3\sqrt{a-2b}+8\sqrt{a-2b}=7\sqrt{a-2b}$

$k)\ \sqrt{\frac{2a}{b}}-\sqrt{\frac{2b}{a}}+\sqrt{\frac{a}{2b}}+\sqrt{\frac{2}{ab}}-\sqrt{\frac{ab}{2}}=\sqrt{\frac{2a\cdot b}{b^2}}-\sqrt{\frac{2b\cdot a}{a^2}}+\sqrt{\frac{a\cdot 2b}{(2b)^2}}+\sqrt{\frac{2ab}{(ab)^2}}-\sqrt{\frac{2ab}{2^2}}=$

$=\frac{1}{b}\sqrt{2ab}-\frac{1}{a}\sqrt{2ab}+\frac{1}{2b}\sqrt{2ab}+\frac{1}{ab}\sqrt{2ab}-\frac{1}{2}\sqrt{2ab}=\left(\frac{1}{b}-\frac{1}{a}+\frac{1}{2b}+\frac{1}{ab}-\frac{1}{2}\right)\sqrt{2ab}=$

$=\frac{2a-2b+a+2-ab}{2ab}\sqrt{2ab}=\frac{3a-2b+a+2-ab}{2ab}\sqrt{2ab}$

$l)\ \frac{cd}{a}\sqrt{\frac{a^6}{cd}}-\frac{b^2d}{a}\sqrt{\frac{4a^4c}{b^2d}}+\frac{d^2}{c}\sqrt{\frac{b^4c^3}{d^3}}=\frac{cd\cdot a^3}{a}\sqrt{\frac{1}{cd}}-\frac{b^2d\cdot 2a^2}{ab}\sqrt{\frac{c}{d}}+\frac{d^2b^2c}{cd}\sqrt{\frac{c}{d}}=$

$=a^2cd\sqrt{\frac{cd}{(cd)^2}}-2abd\sqrt{\frac{cd}{d^2}}+b^2d\sqrt{\frac{cd}{d^2}}=a^2\sqrt{cd}-2ab\sqrt{cd}+b^2\sqrt{cd}=\left(a^2-2ab+b^2\right)\sqrt{cd}=$

$=(a-b)^2\sqrt{cd}$

7) Racionaliza y simplifica

$a)\ \frac{5}{\sqrt{5}}=\frac{5\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}}=\frac{5\sqrt{5}}{5}=\sqrt{5}$

$b)\ \frac{2\sqrt{6}}{\sqrt{2}}=\frac{2\sqrt{6}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}=\frac{2\sqrt{12}}{2}=\sqrt{4\cdot 3}=2\sqrt{3}$; $b)\ \frac{2\sqrt{6}}{\sqrt{2}}=2\sqrt{\frac{6}{2}}=2\sqrt{3}$

$c)\ \sqrt{\frac{2}{3}}=\sqrt{\frac{2\cdot 3}{3^2}}=\frac{1}{3}\sqrt{6}$; $c)\ \sqrt{\frac{2}{3}}=\frac{\sqrt{2}}{\sqrt{3}}=\frac{\sqrt{2}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}=\frac{\sqrt{6}}{3}$

$d)\ \sqrt{\frac{5}{2}}=\sqrt{\frac{5\cdot 2}{2^2}}=\frac{1}{2}\sqrt{10}$

$e)\ \sqrt{\dfrac{a}{b}}=\sqrt{\dfrac{ab}{b^2}}=\dfrac{1}{b}\sqrt{ab}$

$f)\ \sqrt[3]{\dfrac{3}{2}}=\sqrt[3]{\dfrac{3\cdot 2^2}{2\cdot 2^2}}=\sqrt[3]{\dfrac{12}{2^3}}=\dfrac{1}{2}\sqrt[3]{12}$

$g)\ \dfrac{2}{3\sqrt{3}}=\dfrac{2\sqrt{3}}{3\sqrt{3}\cdot\sqrt{3}}=\dfrac{2\sqrt{3}}{3\cdot 3}=\dfrac{2\sqrt{3}}{9}$

$h)\ \dfrac{3y}{x\sqrt{y}}=\dfrac{3y\sqrt{y}}{x\sqrt{y}\cdot\sqrt{y}}=\dfrac{3y\sqrt{y}}{xy}=\dfrac{3\sqrt{y}}{x}$

$i)\ \dfrac{x\sqrt{y}}{y\sqrt{x}}=\dfrac{x\sqrt{y}}{y\sqrt{x}\cdot\sqrt{x}}=\dfrac{x\sqrt{y}}{yx}=\dfrac{\sqrt{y}}{y}$

$j)\ \dfrac{2\sqrt{12}}{5\sqrt{3}}=\dfrac{2}{5}\sqrt{\dfrac{12}{3}}=\dfrac{2}{5}\sqrt{4}=\dfrac{4}{5}$

$k)\ \dfrac{3}{\sqrt{2-x}}=\dfrac{3\sqrt{2-x}}{\sqrt{2-x}\cdot\sqrt{2-x}}=\dfrac{3\sqrt{2-x}}{2-x}$

$l)\ \dfrac{\sqrt{2-x}}{\sqrt{2+x}}=\dfrac{\sqrt{2-x}\cdot\sqrt{2+x}}{\sqrt{2+x}\cdot\sqrt{2+x}}=\dfrac{\sqrt{(2-x)\cdot(2+x)}}{\sqrt{(2+x)^2}}=\dfrac{\sqrt{2^2-x^2}}{2+x}=\dfrac{\sqrt{4-x^2}}{2+x}$

$m)\ \dfrac{3xy^2}{\sqrt[3]{x^2y}}=\dfrac{3xy^2\cdot\sqrt[3]{xy^2}}{\sqrt[3]{x^2y}\cdot\sqrt[3]{xy^2}}=\dfrac{3xy^2\cdot\sqrt[3]{xy^2}}{\sqrt[3]{x^3y^3}}=\dfrac{3xy^2\cdot\sqrt[3]{xy^2}}{xy}=3y\sqrt[3]{xy^2}$

$n)\ \dfrac{a}{\sqrt{a}}=\dfrac{a\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}}=\dfrac{a\sqrt{a}}{a}=\sqrt{a}$

$ñ)\ \dfrac{2+x}{\sqrt{2+x}}=\dfrac{(2+x)\cdot\sqrt{2+x}}{\sqrt{2+x}\cdot\sqrt{2+x}}=\dfrac{(2+x)\cdot\sqrt{2+x}}{2+x}=\sqrt{2+x}$

$o)\dfrac{\sqrt{b}}{\sqrt{ab}}=\dfrac{\sqrt{b}\cdot\sqrt{ab}}{\sqrt{ab}\cdot\sqrt{ab}}=\dfrac{\sqrt{ab^2}}{ab}=\dfrac{b\sqrt{a}}{ab}=\dfrac{\sqrt{a}}{a}\quad ;\ o)\dfrac{\sqrt{b}}{\sqrt{ab}}=\sqrt{\dfrac{b}{ab}}=\dfrac{1}{\sqrt{a}}=\dfrac{\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}}=\dfrac{\sqrt{a}}{a}$

$p)\ \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}}=\dfrac{\left(\sqrt{x}-\sqrt{y}\right)\sqrt{x}}{\sqrt{x}\cdot\sqrt{x}}=\dfrac{x-\sqrt{xy}}{x}$

$q)\ \dfrac{x}{1-\sqrt{x}}=\dfrac{x\left(1+\sqrt{x}\right)}{\left(1-\sqrt{x}\right)\left(1+\sqrt{x}\right)}=\dfrac{x+x\sqrt{x}}{1^2-\left(\sqrt{x}\right)^2}=\dfrac{x+x\sqrt{x}}{1-x}$

8) Racionaliza y simplifica

$$a)\ \frac{6(3-y)}{\sqrt[3]{(3-y)^2}}=\frac{6(3-y)\sqrt[3]{3-y}}{\sqrt[3]{(3-y)^2}\cdot\sqrt[3]{3-y}}=\frac{6(3-y)\sqrt[3]{3-y}}{\sqrt[3]{(3-y)^3}}=\frac{6(3-y)\sqrt[3]{3-y}}{3-y}=6\sqrt[3]{3-y}$$

$$b)\ \frac{5}{\sqrt[3]{a^2-1}}=\frac{5\sqrt[3]{\left(a^2-1\right)^2}}{\sqrt[3]{a^2-1}\cdot\sqrt[3]{\left(a^2-1\right)^2}}=\frac{5\sqrt[3]{\left(a^2-1\right)^2}}{\sqrt[3]{\left(a^2-1\right)^3}}=\frac{5\sqrt[3]{\left(a^2-1\right)^2}}{a^2-1}$$

$$c)\ \frac{1}{\sqrt[3]{a^2b^2}}=\frac{\sqrt[3]{ab}}{\sqrt[3]{a^2b^2}\cdot\sqrt[3]{ab}}=\frac{\sqrt[3]{ab}}{ab}$$

$$d)\ \frac{1}{\sqrt[3]{a^2+b^2}}=\frac{\sqrt[3]{\left(a^2+b^2\right)^2}}{\sqrt[3]{a^2+b^2}\cdot\sqrt[3]{\left(a^2+b^2\right)^2}}=\frac{\sqrt[3]{\left(a^2+b^2\right)^2}}{\sqrt[3]{\left(a^2+b^2\right)^3}}=\frac{\sqrt[3]{\left(a^2+b^2\right)^2}}{a^2+b^2}$$

$$e)\ \frac{1}{\sqrt[4]{a^2-b^3+x}}=\frac{\sqrt[4]{\left(a^2-b^3+x\right)^3}}{\sqrt[4]{a^2-b^3+x}\cdot\sqrt[4]{\left(a^2-b^3+x\right)^3}}=\frac{\sqrt[4]{\left(a^2-b^3+x\right)^3}}{\sqrt[4]{\left(a^2-b^3+x\right)^4}}=\frac{\sqrt[4]{\left(a^2-b^3+x\right)^3}}{a^2-b^3+x}$$

$$f)\ \frac{x^2p^3}{\sqrt[6]{z^2x^4p}}=\frac{x^2p^3\sqrt[6]{z^4x^2p^5}}{\sqrt[6]{z^2x^4p}\cdot\sqrt[6]{z^4x^2p^5}}=\frac{x^2p^3\sqrt[6]{z^4x^2p^5}}{\sqrt[6]{z^6x^6p^6}}=\frac{x^2p^3\sqrt[6]{z^4x^2p^5}}{zxp}=\frac{xp^2\sqrt[6]{z^4x^2p^5}}{z}$$

$$g)\ \frac{\sqrt{a}-a}{a\sqrt{a}}=\frac{\left(\sqrt{a}-a\right)\sqrt{a}}{a\sqrt{a}\sqrt{a}}=\frac{a-a\sqrt{a}}{a^2}=\frac{a\left(1-\sqrt{a}\right)}{a^2}=\frac{1-\sqrt{a}}{a}$$

$$h)\ \frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}}=\frac{\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)}=\frac{a-\sqrt{ab}}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}=\frac{a-\sqrt{ab}}{a-b}$$

$$i)\ \frac{a^2-b^2}{\sqrt[3]{a+b}}=\frac{\left(a^2-b^2\right)\cdot\sqrt[3]{(a+b)^2}}{\sqrt[3]{a+b}\cdot\sqrt[3]{(a+b)^2}}=\frac{\left(a^2-b^2\right)\cdot\sqrt[3]{(a+b)^2}}{\sqrt[3]{(a+b)^3}}=\frac{(a+b)(a-b)\sqrt[3]{(a+b)^2}}{a+b}=(a-b)\sqrt[3]{(a+b)^2}$$

$$j)\ \frac{\sqrt{p-h}}{\sqrt{p+h}}=\frac{\sqrt{p-h}\cdot\sqrt{p+h}}{\sqrt{p+h}\cdot\sqrt{p+h}}=\frac{\sqrt{p^2-h^2}}{\sqrt{(p+h)^2}}=\frac{\sqrt{p^2-h^2}}{p+h}$$

$$k)\ \frac{\sqrt{p}-\sqrt{h}}{\sqrt{p}+\sqrt{h}}=\frac{\left(\sqrt{p}-\sqrt{h}\right)\cdot\left(\sqrt{p}-\sqrt{h}\right)}{\left(\sqrt{p}+\sqrt{h}\right)\cdot\left(\sqrt{p}-\sqrt{h}\right)}=\frac{\left(\sqrt{p}-\sqrt{h}\right)^2}{\left(\sqrt{p}\right)^2-\left(\sqrt{h}\right)^2}=\frac{\left(\sqrt{p}\right)^2-2\sqrt{ph}+\left(\sqrt{h}\right)^2}{p-h}=\frac{p-2\sqrt{ph}+h}{p-h}$$

$$l)\ \frac{b\sqrt{a}+a\sqrt{b}}{\sqrt{ab}}=\frac{\left(b\sqrt{a}+a\sqrt{b}\right)\sqrt{ab}}{\sqrt{ab}\cdot\sqrt{ab}}=\frac{b\sqrt{a^2b}+a\sqrt{ab^2}}{ab}=\frac{ba\sqrt{b}+ab\sqrt{a}}{ab}=\frac{ab\left(\sqrt{b}+\sqrt{a}\right)}{ab}=\sqrt{b}+\sqrt{a}$$

$$m)\ \frac{b\sqrt{a}+a\sqrt{b}}{a\sqrt{b}-b\sqrt{a}}=\frac{\left(b\sqrt{a}+a\sqrt{b}\right)\left(a\sqrt{b}+b\sqrt{a}\right)}{\left(a\sqrt{b}-b\sqrt{a}\right)\left(a\sqrt{b}+b\sqrt{a}\right)}=\frac{ba\sqrt{ab}+b^2\cdot a+a^2\cdot b+ab\sqrt{ba}}{\left(a\sqrt{b}\right)^2-\left(b\sqrt{a}\right)^2}=$$

$$=\frac{2ab\sqrt{ab}+b^2\cdot a+a^2\cdot b}{a^2b-b^2a}=\frac{ab\left(2\sqrt{ab}+b+a\right)}{ab\left(a-b\right)}=\frac{2\sqrt{ab}+b+a}{a-b}$$

9) Racionaliza y simplifica

$$a)\ \frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}}=\frac{\sqrt{2}\left(\sqrt{2}-\sqrt{3}\right)}{\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)}=\frac{2-\sqrt{6}}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}=\frac{2-\sqrt{6}}{2-3}=-2+\sqrt{6}$$

$$b)\ \frac{\sqrt{6}}{2\sqrt{2}-\sqrt{6}}=\frac{\sqrt{6}\left(2\sqrt{2}+\sqrt{6}\right)}{\left(2\sqrt{2}-\sqrt{6}\right)\left(2\sqrt{2}+\sqrt{6}\right)}=\frac{2\sqrt{12}+6}{\left(2\sqrt{2}\right)^2-\left(\sqrt{6}\right)^2}=\frac{2\sqrt{4\cdot 3}+6}{4\cdot 2-6}=\frac{4\sqrt{3}+6}{2}=2\sqrt{3}+3$$

$$c)\ \frac{1-\sqrt{5}}{\sqrt{5}-\sqrt{2}}=\frac{\left(1-\sqrt{5}\right)\left(\sqrt{5}+\sqrt{2}\right)}{\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)}=\frac{\sqrt{5}+\sqrt{2}-5-\sqrt{10}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\sqrt{5}+\sqrt{2}-5-\sqrt{10}}{5-2}=\frac{\sqrt{5}+\sqrt{2}-5-\sqrt{10}}{3}$$

$$d)\ \frac{\sqrt{3}-\sqrt{6}}{2\sqrt{3}-1}=\frac{\left(\sqrt{3}-\sqrt{6}\right)\left(2\sqrt{3}+1\right)}{\left(2\sqrt{3}-1\right)\left(2\sqrt{3}+1\right)}=\frac{2\cdot 3+\sqrt{3}-2\sqrt{18}-\sqrt{6}}{\left(2\sqrt{3}\right)^2-1^2}=\frac{6+\sqrt{3}-6\sqrt{2}-\sqrt{6}}{4\cdot 3-1}=$$

$$=\frac{6+\sqrt{3}-6\sqrt{2}-\sqrt{6}}{11}$$

$$e)\ \frac{\sqrt{10}-\sqrt{2}}{\sqrt{5}-2\sqrt{2}}=\frac{\left(\sqrt{10}-\sqrt{2}\right)\left(\sqrt{5}+2\sqrt{2}\right)}{\left(\sqrt{5}-2\sqrt{2}\right)\left(\sqrt{5}+2\sqrt{2}\right)}=\frac{\sqrt{50}+2\sqrt{20}-\sqrt{10}-2\cdot 2}{\left(\sqrt{5}\right)^2-\left(2\sqrt{2}\right)^2}=\frac{\sqrt{25\cdot 2}+2\sqrt{4\cdot 5}-\sqrt{10}-4}{5-4\cdot 2}=$$

$$=\frac{5\sqrt{2}+4\sqrt{5}-\sqrt{10}-4}{-3}$$

$$f)\ \frac{2\sqrt{3}-3\sqrt{2}}{2\sqrt{2}+3\sqrt{3}}=\frac{\left(2\sqrt{3}-3\sqrt{2}\right)\left(2\sqrt{2}-3\sqrt{3}\right)}{\left(2\sqrt{2}+3\sqrt{3}\right)\left(2\sqrt{2}-3\sqrt{3}\right)}=\frac{4\sqrt{6}-6\cdot 3-6\cdot 2+9\sqrt{6}}{\left(2\sqrt{2}\right)^2-\left(3\sqrt{3}\right)^2}=\frac{13\sqrt{6}-30}{4\cdot 2-9\cdot 3}=\frac{13\sqrt{6}-30}{-19}$$