Integrales propuestas

1) $\int (x^2 + 3x)\,dx$

2) $\int (6x^2 + 3x + 3)\,dx$

3) $\int (x^3 - 4x + 2)\,dx$

4) $\int (x^5 + 4x)\,dx$

5) $\int (x - 1)^3\,dx$

6) $\int (4x^3 - 7x^2 + 5x + 1)\,dx$

7) $\int \sqrt[5]{x^2}\,dx$

8) $\int \sqrt[3]{x}\,dx$

9) $\int \frac{dx}{\sqrt[7]{x^6}}$

10) $\int \frac{dx}{\sqrt[7]{x^5}}$

11) $\int \frac{2}{x}\,dx$

12) $\int (x - 1)(x^2 + x + 1)\,dx$

13) $\int (x^2 + 4x)(x^2 - 1)\,dx$

14) $\int (x^3 - 3x^4 + 5x^2 - 1)\,dx$

15) $\int (x^7 - 3x)\,dx$

16) $\int (4x^2 - 5x + 7)\,dx$

17) $\int (x - \sin x)\,dx$

18) $\int (x + \cos x)\,dx$

19) $\int (x - 2\tan x)\,dx$

20) $\int (\sin x + e^x)\,dx$

21) $\int (\cos x - 5e^x)\,dx$

22) $\int (e^x - e^{-x})\,dx$

23) $\int (e^x + 3e^{-x})\,dx$

24) $\int \frac{3}{x - 5}\,dx$

25) $\int \frac{1}{2x - 7}\,dx$

26) $\int \frac{3}{1 + x^2}\,dx$

27) $\int \frac{2}{1 + 4x^2}\,d$

28) $\int (2^x - x^2)\,dx$

29) $\int e^{x+2}\,dx$

30) $\int \sqrt{x\sqrt{x}}\,dx$

31) $\int \frac{1}{3 - x}\,dx$

32) $\int \frac{2x}{\sqrt{1 - x^2}}\,dx$

33) $\int \frac{\cos x}{\sin x}\,dx$

34) $\int \tan x\,dx$

35) $\int \frac{1}{1 + x}$

36) $\int \frac{e^x + 1}{e^x + x}\,dx$

37) $\int \frac{2x}{1 + x^4}\,dx$

38) $\int \frac{1}{x - 4}\,dx$

39) $\int \cos(-x)\,dx$

40) $\int \sin(2x + 1)\,dx$

41) $\int \sin(2 - 3x)\,dx$

42) $\int \cos\left(\frac{2x + 1}{3}\right)dx$

43) $\int \frac{-3x}{2 - 6x^2}\,dx$

44) $\int \frac{5}{\cos^2 x}\,dx$

45) $\int \frac{3}{\sqrt{1 - x^2}}\,dx$

46) $\int \frac{2}{\sqrt{1 - 4x^2}}\,dx$

47) $\int \frac{2x-5}{x^2-5x+6}dx$

48) $\int \sin x \cos^4 x dx$

49) $\int \sin\left(\frac{x}{2}\right)dx$

50) $\int \frac{-3x^3}{1+x^4}dx$

51) $\int \frac{x^2}{1+x^3}dx$

52) $\int \frac{1}{(x+2)^3}dx$

53) $\int \cot g\, xdx$

54) $\int e^{7x}dx$

55) $\int e^{-2x+3}dx$

56) $\int x \cdot e^{x^2}\, dx$

57) $\int x^2 \cdot e^{x^3} dx$

58) $\int \cos 3x\, dx$

59) $\int x \cdot \sin\left(x^2+4\right)dx$

60) $\int \frac{2x}{1+x^4}dx$

61) $\int \frac{2x^3}{1+x^4}dx$

62) $\int \frac{e^x}{1+e^{2x}}dx$

63) $\int \frac{dx}{\sqrt{9-x^2}}$

64) $\int \frac{dx}{1+4x^2}$

65) $\int \frac{xdx}{\sqrt{1-4x^2}}$

66) $\int \frac{1}{\cos^2 3x}dx$

67) $\int \frac{1+\cos x}{x+\sin x}dx$

68) $\int \frac{5e^x}{1+e^{2x}}dx$

69) $\int 5x \cdot e^{3x^2+3}dx$

70) $\int \arcsin x\, dx$

71) $\int \arccos x\, dx$

72) $\int \arctan x\, dx$

73) $\int x \cdot \sin x\, dx$

74) $\int x \cdot \cos x\, dx$

75) $\int x^2 \cdot \cos x\, dx$

76) $\int x^2 \cdot \sin x\, dx$

77) $\int x \cdot e^x dx$

78) $\int x^2 \cdot e^x dx$

79) $\int x^3 \cdot e^x dx$

80) $\int e^x \sin x\, dx$

81) $\int e^x \cos x\, dx$

82) $\int \ln x\, dx$

83) $\int x \cdot \ln x\, dx$

84) $\int (x^2-7x+12)dx$

85) $\int (x^3-12x^2+14x-5)dx$

86) $\int \left(x^5-7+\frac{4}{x^2}\right)dx$

87) $\int \left(6x^2-\frac{6}{x}+\frac{8}{x^3}\right)dx$

88) $\int \frac{x^3-7x^2+4}{x}dx$

89) $\int \frac{x^3-5x+3}{x}dx$

90) $\int (x^2-5)^2 dx$

91) $\int (x+4)^3 dx$

92) $\int \frac{x^3-5x+3}{x-2}dx$

93) $\int \frac{\sqrt{3x^3}}{\sqrt[3]{x^2}}dx$

94) $\int \frac{\sqrt{3x^3}}{\sqrt[3]{5x^2}}dx$

95) $\int \frac{\sqrt{3x}-5}{\sqrt[3]{2x}}dx$

96) $\int \frac{(x^2-1)^2}{\sqrt{x}}dx$

97) $\int (2x+3)^3\sqrt{x}\,dx$

98) $\int (\sqrt[3]{x^2}-3\sqrt{x}+12)\,dx$

99) $\int \frac{3x+2}{\sqrt{x}}dx$

100) $\int 5x\sqrt{x^2+7}\ dx$

101) $\int x^3\cdot\sqrt[3]{x-12}dx$

102) $\int x(2x^2-7)^{99}\ dx$

103) $\int \frac{4x\,dx}{\sqrt[3]{8-x^2}}dx$

104) $\int (3x^2-1)^{240}\,x\,dx$

105) $\int (2-7x)^{2/3}\,dx$

106) $\int \frac{dx}{(3x+1)^5}$

107) $\int x\cdot\sqrt{2x^2+17}\ dx$

108) $\int \frac{x+1}{\sqrt[3]{x^2+2x+12}}dx$

109) $\int \sqrt{x^2-2x+3}\cdot(x-1)dx$

110) $\int \cos 5x\,dx$

111) $\int \sin 2x\cos 2x\,dx$

112) $\int e^{\sin x}\ \cos x\,dx$

113) $\int e^{-5x}dx$

114) $\int \frac{e^{-2x}\cdot e^{2x}}{2}dx$

115) $\int \sin^2\frac{x}{2}\cdot\cos\frac{x}{2}\ dx$

116) $\int \tan x\cdot\sec^2\ x\ dx$

117) $\int \sin^2\ 4x\ dx$

118) $\int \cos^3\ 3x\,dx$

119) $\int \cos^5\ x\ dx$

120) $\int \frac{x^2-4}{x-3}\ dx$

121) $\int (2^x+x^2)dx$

122) $\int \frac{x^2}{x^3+2}dx$

123) $\int x^2\cdot\sin x^3dx$

124) $\int \frac{-3x}{1+x^4}dx$

125) $\int \frac{2x}{4+x^4}dx$

126) $\int x^3dx$

127) $\int \frac{x^4}{3}dx$

128) $\int \frac{x^5}{6}dx$

129) $\int (x^2+3)dx$

130) $\int \left(x^3+2x-\frac{1}{x}\right)dx$

131) $\int \frac{x^4-x^2+1}{x}dx$

132) $\int \frac{dx}{x^2}$

133) $\int \frac{1}{x^4}dx$

134) $\int \frac{x^4-2x+3}{x^7}dx$

135) $\int \frac{x^5}{1+x}\ dx$

136) $\int \frac{4\sqrt[3]{x}}{3}\ dx$

137) $\int \frac{dx}{\sqrt[5]{x}}$

138) $\int \left(\frac{8}{3}\sqrt[3]{x^5}+6\sqrt{x}\right)dx$

139) $\int \sqrt[3]{x}\cdot\left(\sqrt{x}+3\right)dx$

140) $\int (x-\sin x+2\cos x)dx$

141) $\int \left(e^x-\frac{1}{x}\right)dx$

142) $\int \frac{dx}{4x-2}$

143) $\int \frac{dx}{2-x}$

144) $\int \frac{x}{1-x^2}\,dx$

145) $\int \frac{3dx}{(x+1)^4}$

146) $\int \frac{x^3}{1+x^4}\,dx$

147) $\int \frac{\sin 2x}{1+\sin^2 x}\,dx$

148) $\int \frac{x+2}{\sqrt{x^2+4x+2}}\,dx$

149) $\int e^x \sqrt{1-e^x}\,dx$

150) $\int \frac{\ln x}{x}\,dx$

151) $\int x\sqrt{x^2+5}dx$

152) $\int \sin 7x\,dx$

153) $\int 8x\cos x^2\,dx$

154) $\int \frac{\cos x}{1+\sin^2 x}\,dx$

155) $\int \frac{dx}{\cos^2 x\sqrt{1-\tan^2 x}}$

156) $\int 4x^2 \cdot e^{x^3}\,dx$

157) $\int \frac{3x^2}{1+x^6}\,dx$

158) $\int 3^x\,dx$

159) $\int \frac{dx}{x^2+4}$

160) $\int e^{5x}\,dx$

161) $\int (e^x+e^{-x})^2\,dx$

162) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

163) $\int \frac{\sin x}{e^{\cos x}}\,dx$

164) $\int x^2 \ln x\,dx$

165) $\int x\arctan x\,dx$

166) $\int \frac{x+2}{x^2+x-6}\,dx$

167) $\int \frac{dx}{x^2+5x+6}$

168) $\int \frac{x^3+1}{x^2+x-2}\,dx$

169) $\int \frac{1}{x^2+3x}\,dx$

170) $\int \frac{1}{\sin x\cos x}\,dx$

171) $\int \frac{2x-3}{x+2}\,dx$

172) $\int \frac{dx}{x^2-4}$

173) $\int \frac{x-1}{x^2+x-6}\,dx$

174) $\int \frac{2}{x^2+5x+6}\,dx$

175) $\int \frac{x+1}{x\cdot(x-1)^2}\,dx$

176) $\int \frac{dx}{x^2+2x}$

177) $\int \frac{x^2+1}{x^2+x-6}\,dx$

178) $\int \frac{(x^3-1)}{x^2+x}\,dx$

179) $\int \frac{x^2+1}{x^2-1}\,dx$

180) $\int \frac{1}{x^2\cdot(x+1)}\,dx$

181) $\int \frac{1}{x^2-9}\,dx$

182) $\int \frac{x}{(x-1)^2\cdot(x+1)}\,dx$

183) $\int \frac{6}{x\cdot(x-1)\cdot(x+2)}\,dx$

184) $\int \frac{3}{x^3-1}\,dx$

185) $\int \frac{5x^2-2x+25}{x^3-6x^2+25x}\,dx$

186) $\int (1-x)^2 dx$

187) $\int (1-x^2)^2 dx$

188) $\int 2x\sqrt{x^2+3}\, dx$

189) $\int x\,(x^2+3)\, dx$

190) $\int (x-2)(x^2-4x+1)^3 dx$

191) $\int \sqrt{2x-1}\, dx$

192) $\int x\sqrt{x^2-2}\, dx$

193) $\int e^{2x} dx$

194) $\int (x+1)\cdot e^{x^2+2x-1} dx$

195) $\int \frac{4x}{2x^2+1} dx$

196) $\int \frac{x^2}{x^2+3} dx$

197) $\int \frac{x+1}{x^2+2x} dx$

198) $\int \frac{5x}{1-x^2} dx$

199) $\int \frac{4x^3}{x^4+2} dx$

200) $\int \frac{4x^3}{(x^4+2)^2} dx$

201) $\int \frac{4x^3}{\sqrt[3]{x^4+2}} dx$

202) $\int \frac{x-1}{\sqrt{x^2-2x}} dx$

203) $\int 3^{x/2} dx$

204) $\int e^{x+1} dx$

205) $\int \left(\frac{1}{2}\right)^{4x} dx$

206) $\int (e^{-3x}+e^{x-2})\, dx$

207) $\int 7^{x^2+1}\cdot 2x\cdot dx$

208) $\int 5e^{x/2+2} dx$

209) $\int \frac{3^{5x-1}}{7} dx$

210) $\int \frac{x}{e^{x^2}}\,dx$

211) $\int \operatorname{sen}(2x)\,dx$

212) $\int \cos(x+1)\,dx$

213) $\int \frac{\operatorname{sen}\left(\frac{x}{2}\right)}{2}\,dx$

214) $\int \operatorname{sen}(-x)\,dx$

215) $\int \frac{1}{\cos^2(x+1)}\,dx$

216) $\int -3\cdot\operatorname{sen}(2x+1)\,dx$

217) $\int (x+1)\cdot\cos(x^2+2x)\,dx$

218) $\int \frac{x}{\cos^2(x^2-3)}\,dx$

219) $\int \frac{1}{\sqrt{1-25x^2}}\,dx$

220) $\int \frac{1}{\sqrt{1-(2x-3)^2}}\,dx$

221) $\int \frac{x}{1+(x-3)^2}\,dx$

222) $\int \frac{x}{1+9x^4}\,dx$

223) $\int (x^2+x)\cdot e^{-2x+1}\,dx$

224) $\int x^2\cdot\cos(3x)\,dx$

225) $\int 2x^2\cdot\ln x\,dx$

226) $\int x^2\cdot 2^x\,dx$

227) $\int \frac{2}{x^2-1}\,dx$

228) $\int \frac{-3}{x^2+x-2}\,dx$

229) $\int \frac{2x+1}{x^4-5x^2+4}\,dx$

230) $\int \frac{7x-2}{x^3-2x^2-x+2}\,dx$

231) $\int \frac{x^2}{(x-1)^3}\,dx$

232) $\int -\frac{3x-2}{(2-x)^2}\,dx$

233) $\int \frac{-2x^2+1}{x^3+6x^2+12x+8}\,dx$

234) $\int \frac{x-2}{x^4}\,dx$

235) $\int \frac{4x^2-2x}{(x+2)(x-3)^2}\,dx$

236) $\int \frac{-x^2+7x}{x^3-x^2-x+1}\,dx$

237) $\int \frac{2}{x^2+1}\,dx$

238) $\int -\frac{3x-2}{2+x^2}\,dx$

239) $\int \frac{-2x^2+1}{x^3-x^2+3x-3}\,dx$

240) $\int \frac{x-2}{x^2(x^2+1)}\,dx$

241) $\int \frac{2x^4}{(x-1)^3}\,dx$

242) $\int -\frac{3x^3-2}{(2-x)^2}\,dx$

243) $\int \frac{-2x^5+1}{x^4-2x^2+1}\,dx$

244) $\int \frac{x^6-1}{x^2(x^2+1)(x-1)}\,dx$

245) $\int x\cdot 2^{x^2-3}\,dx$

246) $\int \frac{\ln^3 x}{2x}\,dx$

247) $\int x\cdot \ln(1+x^2)\,dx$

248) $\int \frac{1}{e^x+e^{-x}}\,dx$

249) $\int \frac{x^2+2}{\sqrt{x^3+6x}}\,dx$

250) $\int \frac{x}{1+x^4}\,dx$

251) $\displaystyle\int \frac{\operatorname{arctg}(x)}{1+x^2}\,dx$

252) $\displaystyle\int \frac{dx}{(\operatorname{arcsen} x)^5 \cdot \sqrt{1+x^2}}$

253) $\displaystyle\int \operatorname{sen}^5 x \cdot \cos^2 x\,dx$

254) $\displaystyle\int \sqrt{4-x^2}\,dx$

255) $\displaystyle\int \operatorname{sen}^2 x\,dx$

256) $\displaystyle\int \frac{\sqrt{2-x^2}}{4}\,dx$

257) $\displaystyle\int (2x-3)\,dx$

258) $\displaystyle\int (3x^2+4x-2)\,dx$

259) $\displaystyle\int \left(\frac{3}{4}x^3-3x^2+6x-1\right)dx$

260) $\displaystyle\int \frac{7}{\operatorname{sen}^2(3x)}\,dx$

261) $\displaystyle\int 3\cdot \sec^2\left(\frac{1}{5}x\right)dx$

262) $\displaystyle\int \frac{2x+\sqrt{x}}{x^2}\,dx$

263) $\displaystyle\int \frac{1-\operatorname{sen} x}{2x+2\cos x}\,dx$

264) $\displaystyle\int \ln\left(\frac{x+1}{x-1}\right)^x dx$

265) $\displaystyle\int x\cdot \ln^2 x\,dx$

266) $\displaystyle\int x\cdot \operatorname{arctg}(x+1)\,dx$

267) $\displaystyle\int \frac{x\cdot \operatorname{arcsen}(x)}{\sqrt{1-x^2}}\,dx$

268) $\displaystyle\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$

269) $\displaystyle\int \frac{x^4}{\sqrt{1-x^{10}}}\,dx$

270) $\displaystyle\int e^x\cdot \sqrt{(e^x+1)^3}\,dx$

271) $\displaystyle\int \cos^3 x\cdot \operatorname{sen}^5 x\,dx$

272) $\displaystyle\int \operatorname{sen}^3 x\cdot \cos^{15} x\,dx$

273) $\int \frac{\operatorname{sen}^3 x}{\cos^2 x}\,dx$

274) $\int \frac{1}{\cos^3 x\,\operatorname{sen} x}\,dx$

275) $\int tg^3 x \cdot \sec^3 x\,dx$

276) $\int \frac{\cos x}{\operatorname{sen} x + \cos x}\,dx$

277) $\int \frac{\cos x}{2\operatorname{sen} x \cos^2 x + \operatorname{sen}^3 x}\,dx$

278) $\int \frac{1+\sqrt{x+1}}{1+\sqrt[3]{x+1}}\,dx$

279) $\int \frac{1+x+\sqrt{x+1}}{(x+1)\sqrt[3]{x+1}}\,dx$

280) $\int \frac{1}{(1-e^x)^2}\,dx$

281) $\int \frac{1+\sqrt{e^x}}{(1-\sqrt[4]{e^x})^2}\,dx$

282) $\int \frac{1}{e^x+e^{-x}+1}\,dx$

283) $\int \frac{e^{3x}}{e^{2x}-3e^x+2}\,dx$

284) $\int (1-x^2)^{-3/2}\,dx$

285) $\int \left(1-(2x+1)^2\right)^{-1/2}dx$

286) $\int \frac{2^{3x}}{2^x-4}\,dx$

287) $\int \frac{\operatorname{sen}(2x)+\cos x}{\cos x}\,dx$

288) $\int \frac{7}{x\sqrt[3]{\ln x}}\,dx$

289) $\int \frac{(\ln x)^2+x}{x}\,dx$

290) $\int \frac{1}{\sqrt{x+2}+\sqrt{x-2}}\,dx$

291) $\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$

292) $\int \frac{x-1}{\sqrt{2x}-\sqrt{x+1}}\,dx$

293) $\int e^{2x}\cdot \operatorname{sen}(e^x)\,dx$

1) $\int (x^2+3x)\,dx = \dfrac{x^3}{3} + \dfrac{3x^2}{2} + C$

2) $\int (6x^2+3x+3)\,dx = \dfrac{6x^3}{3} + \dfrac{3x^2}{2} + 3x + C = 2x^3 + \dfrac{3x^2}{2} + 3x + C$

3) $\int (x^3-4x+2)\,dx = \dfrac{x^4}{4} - \dfrac{4x^2}{2} + 2x + C = \dfrac{x^4}{4} - 2x^2 + 2x + C$

4) $\int (x^5+4x)\,dx = \dfrac{x^6}{6} + \dfrac{4x^2}{2} + C = \dfrac{x^6}{6} + 2x^2 + C$

5) $\int (x-1)^3\,dx = \dfrac{(x-1)^4}{4} + C$

6) $\int (4x^3-7x^2+5x+1)\,dx = \dfrac{4x^4}{4} - \dfrac{7x^3}{3} + \dfrac{5x^2}{2} + x + C = x^4 - \dfrac{7x^3}{3} + \dfrac{5x^2}{2} + x + C$

7) $\int \sqrt[5]{x^2}\,dx = \int x^{2/5}\,dx = \dfrac{x^{7/5}}{7/5} + C = \dfrac{5}{7}\sqrt[5]{x^7} + C$

8) $\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx = \dfrac{x^{4/3}}{4/3} + C = \dfrac{3}{4}\sqrt[3]{x^4} + C$

9) $\int \dfrac{dx}{\sqrt[7]{x^6}} = \int x^{-6/7}\,dx = \dfrac{x^{1/7}}{1/7} = 7\sqrt[7]{x} + C$

10) $\int \dfrac{dx}{\sqrt[7]{x^5}} = \int x^{-5/7}\,dx = \dfrac{x^{2/7}}{2/7} + C = \dfrac{7}{2}\cdot\sqrt[7]{x^2} + C$

11) $\int \frac{2}{x}\,dx = 2\int \frac{1}{x}\,dx = 2\ln|x| + C$

12) $\int (x-1)(x^2+x+1)\,dx = \int (x^3+\cancel{x^2}+\cancel{x}-\cancel{x^2}-\cancel{x}-1)\,dx = \int (x^3-1)\,dx =$

$= \frac{x^4}{4} - x + C$

13) $\int (x^2+4x)(x^2-1)\,dx = \int (x^4-x^2+4x^3-4x)\,dx =$

$= \frac{x^5}{5} - \frac{x^3}{3} + \frac{4x^4}{4} - \frac{4x^2}{2} + C = \frac{x^5}{5} - \frac{x^3}{3} + x^4 - 2x^2 + C$

14) $\int (x^3-3x^4+5x^2-1)\,dx = \frac{x^4}{4} - \frac{3x^5}{5} + \frac{5x^3}{3} - x + C$

15) $\int (x^7-3x)\,dx = \frac{x^8}{8} - \frac{3x^2}{2} + C$

16) $\int (4x^2-5x+7)\,dx = \frac{4x^3}{3} - \frac{5x^2}{2} + 7x + C$

17) $\int (x - \operatorname{sen} x)\,dx = \frac{x^2}{2} + \cos x + C$

18) $\int (x + \cos x)\,dx = \frac{x^2}{2} + \operatorname{sen} x + C$

19) $\int (x - 2\operatorname{tg} x)\,dx = \frac{x^2}{2} + 2\int -\frac{\operatorname{sen} x}{\cos x}\,dx = \frac{x^2}{2} + 2\ln|\cos x| + C$

20) $\int (\operatorname{sen} x + e^x)\,dx = -\cos x + e^x + C$

21) $\int (\cos x - 5e^x)\,dx = \operatorname{sen} x - 5e^x + C$

22) $\int (e^x - e^{-x})\,dx = \int e^x\,dx + \int -e^{-x}\,dx = e^x + e^{-x} + C$

23) $\int (e^x + 3e^{-x})\,dx = e^x - 3e^{-x} + C$

24) $\int \frac{3}{x-5}\,dx = 3\int \frac{1}{x-5}\,dx = 3\ln|x-5| + C$

25) $\int \frac{1}{2x-7}\,dx = \frac{1}{2}\int \frac{2}{2x-7}\,dx = \frac{1}{2}\ln|2x-7| + C$

26) $\int \frac{3}{1+x^2}\,dx = 3\int \frac{1}{1+x^2}\,dx = 3\operatorname{arctg} x + C$

27) $\int \frac{2}{1+4x^2}\,dx = \int \frac{2}{1+(2x)^2}\,dx = \operatorname{arctg}(2x) + C$

28) $\int (2^x - x^2)\,dx = \frac{2^x}{\ln 2} - \frac{x^3}{3} + C$

29) $\int e^{x+2}\,dx = e^{x+2} + C$

30) $\int \sqrt{x\sqrt{x}}\,dx = \int \sqrt{x}\cdot\sqrt{\sqrt{x}}\,dx = \int x^{1/2}\cdot x^{1/4}\,dx = \int x^{3/4}\,dx =$

$= \frac{x^{7/4}}{7/4} = \frac{4}{7}\cdot\sqrt[4]{x^7} + C$

31) $\int \frac{1}{3-x}dx = -\int \frac{-1}{3-x}dx = -\ln|3-x|+C$

32) $\int \frac{2x}{\sqrt{1-x^2}}dx = -\int -2x\cdot(1-x^2)^{-1/2}dx = -\frac{(1-x^2)^{1/2}}{1/2}+C =$

$= -2\sqrt{1-x^2}+C$

33) $\int \frac{\cos x}{\operatorname{sen} x}dx = \ln|\operatorname{sen} x|+C$

34) $\int \operatorname{tg} x\, dx = -\int \frac{-\operatorname{sen} x}{\cos x}dx = -\ln|\cos x|+C$

35) $\int \frac{1}{1+x}dx = \ln|1+x|+C$

36) $\int \frac{e^x+1}{e^x+x}dx = \ln|e^x+x|+C$

37) $\int \frac{2x}{1+x^4}dx = \int \frac{2x}{1+(x^2)^2}dx = \operatorname{arctg}(x^2)+C$

38) $\int \frac{1}{x-4}dx = \ln|x-4|+C$

39) $\int \cos(-x)dx = -\int -\cos(-x)dx = -\operatorname{sen}(-x)+C$

40) $\int \operatorname{sen}(2x+1)dx = \frac{1}{2}\int 2\operatorname{sen}(2x+1)dx = -\frac{1}{2}\cos(2x+1)+C$

41) $\int \operatorname{sen}(2-3x)\,dx = -\frac{1}{3}\int -3\operatorname{sen}(2-3x)\,dx = \frac{1}{3}\cos(2-3x) + C$

42) $\int \cos\left(\frac{2x+1}{3}\right)dx = \frac{3}{2}\int \frac{2}{3}\cos\left(\frac{2x+1}{3}\right)dx = \frac{3}{2}\operatorname{sen}\left(\frac{2x+1}{3}\right) + C$

43) $\int \frac{-3x}{2-6x^2}\,dx = \frac{1}{4}\int \frac{4\cdot(-3x)}{2-6x^2}\,dx = \frac{1}{4}\ln|2-6x^2| + C$

44) $\int \frac{5}{\cos^2 x}\,dx = 5\int \frac{1}{\cos^2 x}\,dx = 5\operatorname{tg} x + C$

45) $\int \frac{3}{\sqrt{1-x^2}}\,dx = 3\int \frac{1}{\sqrt{1-x^2}}\,dx = 3\operatorname{arcsen}(x) + C$

46) $\int \frac{2}{\sqrt{1-4x^2}}\,dx = \int \frac{2\,dx}{\sqrt{1-(2x)^2}} = \operatorname{arcsen}(2x) + C$

47) $\int \frac{2x-5}{x^2-5x+6}\,dx = \ln|x^2-5x+6| + C$

48) $\int \operatorname{sen} x\cdot(\cos x)^4\,dx = -\int -\operatorname{sen} x\cdot(\cos x)^4\,dx = -\frac{(\cos x)^5}{5} + C$

49) $\int \operatorname{sen}\left(\frac{x}{2}\right)dx = 2\int \frac{1}{2}\operatorname{sen}\left(\frac{x}{2}\right)dx = -2\cos\left(\frac{x}{2}\right) + C$

50) $\int \frac{-3x^3}{1+x^4}\,dx = \frac{-3}{4}\int \frac{4x^3}{1+x^4}\,dx = -\frac{3}{4}\ln|1+x^4| + C$

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51) $\int \frac{x^2}{1+x^3}\,dx = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx = \frac{1}{3}\ln|1+x^3| + C$

52) $\int \frac{1}{(x+2)^3}\,dx = \int (x+2)^{-3}\,dx = \frac{(x+2)^{-2}}{-2} = \frac{-1}{2(x+2)^2} + C$

53) $\int \operatorname{cotg} x\,dx = \int \frac{\cos x}{\operatorname{sen} x}\,dx = \ln|\operatorname{sen} x| + C$

54) $\int e^{7x}\,dx = \frac{1}{7}\int 7\,e^{7x}\,dx = \frac{1}{7}e^{7x} + C$

55) $\int e^{-2x+3}\,dx = -\frac{1}{2}\int -2\,e^{-2x+3}\,dx = -\frac{1}{2}e^{-2x+3} + C$

56) $\int x\cdot e^{x^2}\,dx = \frac{1}{2}\int 2x\,e^{x^2}\,dx = \frac{1}{2}e^{x^2} + C$

57) $\int x^2\cdot e^{x^3}\,dx = \frac{1}{3}\int 3x^2 e^{x^3}\,dx = \frac{1}{3}e^{x^3} + C$

58) $\int \cos(3x)\,dx = \frac{1}{3}\int 3\cos(3x)\,dx = \frac{1}{3}\operatorname{sen}(3x) + C$

59) $\int x\operatorname{sen}(x^2+4)\,dx = \frac{1}{2}\int 2x\operatorname{sen}(x^2+4)\,dx = -\frac{1}{2}\cos(x^2+4) + C$

60) $\int \frac{2x}{1+x^4}\,dx = \int \frac{2x}{1+(x^2)^2}\,dx = \operatorname{arctg}(x^2) + C$

61) $\int \frac{2x^3}{1+x^4}\,dx = \frac{1}{2}\int \frac{2\cdot 2x^3}{1+x^4}\,dx = \frac{1}{2}\ln|1+x^4| + C$

62) $\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx = \text{arctg}\,(e^x) + C$

63) $\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{dx}{\sqrt{\frac{9}{9}(9-x^2)}} = \frac{1}{3}\int \frac{dx}{\sqrt{1-\frac{x^2}{9}}} = \frac{1}{3}\int \frac{dx}{\sqrt{1-\left(\frac{x}{3}\right)^2}} =$

$= \int \frac{1/3\,dx}{\sqrt{1-\left(\frac{x}{3}\right)^2}} = \text{arcsen}\left(\frac{x}{3}\right) + C$

64) $\int \frac{dx}{1+4x^2} = \frac{1}{2}\int \frac{2\,dx}{1+(2x)^2} = \frac{1}{2}\,\text{arctg}\,(2x) + C$

65) $\int \frac{x\,dx}{\sqrt{1-4x^2}} = -\frac{1}{8}\int -8x \cdot (1-4x^2)^{-1/2}\,dx = -\frac{1}{8}\cdot\frac{(1-4x^2)^{1/2}}{1/2} + C =$

$= -\frac{1}{4}\cdot\sqrt{1-4x^2} + C$

66) $\int \frac{1}{\cos^2(3x)}\,dx = \frac{1}{3}\int \frac{3}{\cos^2(3x)}\,dx = \frac{1}{3}\,\text{tg}\,(3x) + C$

67) $\int \frac{1+\cos x}{x+\text{sen}\,x}\,dx = \ln|x+\text{sen}\,x| + C$

68) $\displaystyle\int \frac{5e^x}{1+e^{2x}}dx = 5\int \frac{e^x}{1+(e^x)^2}dx = 5\,\text{arctg}(e^x) + C$

69) $\displaystyle\int 5x\cdot e^{3x^2+3}dx = \frac{5}{6}\int 6x\cdot e^{3x^2+3}dx = \frac{5}{6}e^{3x^2+3} + C$

70) $\displaystyle\int \underbrace{\text{arcsen}\,x}_{u}\,\underbrace{dx}_{dv} = x\cdot\text{arcsen}\,x - \int \frac{x}{\sqrt{1-x^2}}dx = (*)$

$u = \text{arcsen}\,x \qquad du = \dfrac{1}{\sqrt{1-x^2}}dx$

$dv = dx \qquad v = x$

$(*) = x\,\text{arcsen}\,x + \dfrac{1}{2}\displaystyle\int -2x\,(1-x^2)^{-1/2}dx =$

$= x\,\text{arcsen}\,x + \dfrac{1}{\cancel{2}}\dfrac{(1-x^2)^{1/2}}{\cancel{1/2}} = x\,\text{arcsen}\,x + \sqrt{1-x^2} + C$

71) $\displaystyle\int \underbrace{\text{arccos}\,x}_{u}\,\underbrace{dx}_{dv} = x\,\text{arccos}\,x + \int \frac{x}{\sqrt{1-x^2}}dx = (*)$

$u = \text{arccos}\,x \qquad du = \dfrac{-1}{\sqrt{1-x^2}}dx$

$dv = dx \qquad v = x$

$(*) = x\,\text{arccos}\,x + \dfrac{-1}{2}\displaystyle\int -2x\,(1-x^2)^{-1/2}dx =$

$= x\,\text{arccos}\,x - \dfrac{1}{\cancel{2}}\dfrac{(1-x^2)^{1/2}}{\cancel{1/2}} = x\,\text{arccos}\,x - \sqrt{1-x^2} + C$

72) $\displaystyle\int \underbrace{\operatorname{arctg} x}_{u} \underbrace{dx}_{dv} = x \operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{1+x^2}dx =$

$u = \operatorname{arctg} x \quad du = \frac{1}{1+x^2}dx$

$dv = dx \quad v = x$

$= x \operatorname{arctg} x - \frac{1}{2}\ln|1+x^2| + C$

73) $\displaystyle\int \underbrace{x}_{u} \underbrace{\operatorname{sen} x\, dx}_{dv} = -x\cos x + \int \cos x\, dx = -x\cos x + \operatorname{sen} x + C$

$u = x \quad du = dx$

$dv = \operatorname{sen} x\, dx \quad v = -\cos x$

74) $\displaystyle\int \underbrace{x}_{u} \cdot \underbrace{\cos x\, dx}_{dv} = x \operatorname{sen} x - \int \operatorname{sen} x\, dx = x \operatorname{sen} x + \cos x + C$

$u = x \quad du = 1dx$

$dv = \cos x\, dx \quad v = \operatorname{sen} x$

75) $\displaystyle\int \underbrace{x^2}_{u} \cdot \underbrace{\cos x\, dx}_{dv} = x^2 \operatorname{sen} x - \int \underbrace{2x}_{u} \underbrace{\operatorname{sen} x\, dx}_{dv} = (*)$

$u = x^2 \quad du = 2x\,dx$ || $u = 2x \quad du = 2dx$

$dv = \cos x\, dx \quad v = \operatorname{sen} x$ || $dv = \operatorname{sen} x\, dx \quad v = -\cos x$

$(*) = x^2 \operatorname{sen} x - \left[-2x\cos x + \int 2\cos x\, dx\right] =$

$= x^2 \operatorname{sen} x + 2x\cos x - 2\operatorname{sen} x + C$

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76) $\displaystyle\int \underbrace{x^2}_{u} \cdot \underbrace{\operatorname{sen} x\,dx}_{dv} = -x^2\cos x + \int \underbrace{2x}_{u}\,\underbrace{\cos x\,dx}_{dv} = ⊛$

$u = x^2 \quad du = 2x\,dx \quad \| \quad u = 2x \quad du = 2\,dx$

$dv = \operatorname{sen} x\,dx \quad v = -\cos x \quad \| \quad dv = \cos x\,dx \quad v = \operatorname{sen} x$

$⊛ = -x^2\cos x + 2x\operatorname{sen} x - \displaystyle\int 2\operatorname{sen} x\,dx =$

$= -x^2\cos x + 2x\operatorname{sen} x + 2\cos x + C$

77) $\displaystyle\int \underbrace{x}_{u} \cdot \underbrace{e^x dx}_{dv} = xe^x - \int 1 e^x dx = xe^x - e^x + C$

$u = x \quad du = 1\,dx$

$dv = e^x dx \quad v = e^x$

78) $\displaystyle\int \underbrace{x^2}_{u} \cdot \underbrace{e^x dx}_{dv} = x^2 \cdot e^x - \int \underbrace{2x}_{u}\,\underbrace{e^x dx}_{dv} = ⊛$

$u = x^2 \quad du = 2x\,dx \quad \| \quad u = 2x \quad du = 2\,dx$

$dv = e^x dx \quad v = e^x \quad \| \quad dv = e^x dx \quad v = e^x$

$⊛ = x^2 \cdot e^x - \left[2xe^x - \displaystyle\int 2e^x dx\right] = x^2e^x - 2xe^x + 2e^x + C$

79) $\displaystyle\int \underbrace{x^3}_{u} \cdot \underbrace{e^x dx}_{dv} = x^3 \cdot e^x - \int \underbrace{3x^2}_{u} \cdot \underbrace{e^x dv}_{dv} = x^3 \cdot e^x - \left[3x^2e^x - \int \underbrace{6x}_{u}\,\underbrace{e^x dx}_{dv}\right] = ⊛$

$u = x^3 \quad du = 3x^2 dx \quad \| \quad u = 3x^2 \quad du = 6x\,dx \quad \| \quad u = 6x \quad du = 6\,dx$

$dv = e^x dx \quad v = e^x \quad \| \quad dv = e^x dx \quad v = e^x \quad \| \quad dv = e^x dx \quad v = e^x$

$⊛ = x^3 \cdot e^x - 3x^2e^x + 6xe^x - \displaystyle\int 6e^x dx = x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

80) $\underbrace{\int \underbrace{e^x}_{u} \underbrace{\operatorname{sen} x\, dx}_{dv}}_{=I} = -e^x \cos x + \int \underbrace{e^x}_{u} \underbrace{\cos dx}_{dv} = ⊛$

$u = e^x \quad du = e^x dx \quad \| \quad u = e^x \quad du = e^x dx$

$dv = \operatorname{sen} x\, dx \quad v = -\cos x \quad \| \quad dv = \cos x\, dx \quad v = \operatorname{sen} x$

$⊛ = -e^x \cos x + e^x \operatorname{sen} x - \underbrace{\int e^x \operatorname{sen} x\, dx}_{=I}$

$\Rightarrow I = -e^x \cos x + e^x \operatorname{sen} x - I \Rightarrow$

$\Rightarrow 2I = -e^x \cos x + e^x \operatorname{sen} x$

$$\Rightarrow I = \frac{-e^x \cos x + e^x \operatorname{sen} x}{2} + C$$

81) $\underbrace{\int \underbrace{e^x}_{u} \underbrace{\cos x\, dx}_{dv}}_{=I} = e^x \operatorname{sen} x - \int \underbrace{e^x}_{u} \underbrace{\operatorname{sen} x\, dx}_{dv} = ⊛$

$u = e^x \quad du = e^x dx \quad \| \quad u = e^x \quad du = e^x dx$

$dv = \cos x\, dx \quad v = \operatorname{sen} x \quad \| \quad dv = \operatorname{sen} x \quad v = -\cos x$

$⊛ = e^x \operatorname{sen} x - \left[-e^x \cos x + \int e^x \cos dx\right] = e^x \operatorname{sen} x + e^x \cos x - \underbrace{\int e^x \cos x\, dx}_{=I}$

$\Rightarrow I = e^x \operatorname{sen} x + e^x \cos x - I \Rightarrow$

$\Rightarrow 2I = e^x \operatorname{sen} x + e^x \cos x$

$$\Rightarrow I = \frac{e^x \operatorname{sen} x + e^x \cos x}{2} + C$$

82) $$\int \underbrace{\ln x}_{u}\,\underbrace{dx}_{dv} = x\ln x - \int 1\,dx = x\ln x - x + C$$

$u = \ln x \qquad du = \frac{1}{x}dx$

$dv = dx \qquad v = x$

83) $$\int \underbrace{x\,dx}_{dv}\,\underbrace{\ln x}_{u} = \frac{x^2}{2}\ln x - \int \frac{x}{2}dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

$u = \ln x \qquad du = \frac{1}{x}dx$

$dv = x\,dx \qquad v = \frac{x^2}{2}$

84) $$\int (x^2 - 7x + 12)dx = \frac{x^3}{3} - \frac{7x^2}{2} + 12x + C$$

85) $$\int (x^3 - 12x^2 + 14x - 5)dx = \frac{x^4}{4} - \frac{12x^3}{3} + \frac{14x^2}{2} - 5x =$$

$$= \frac{x^4}{4} - 4x^3 + 7x^2 - 5x + C$$

86) $$\int \left(x^5 - 7 + \frac{4}{x^2}\right)dx = \frac{x^6}{6} - 7x + 4\int x^{-2}dx = \frac{x^6}{6} - 7x + \frac{4x^{-1}}{-1} =$$

$$= \frac{x^6}{6} - 7x - \frac{4}{x} + C$$

87) $$\int \left(6x^2 - \frac{6}{x} + \frac{8}{x^3}\right)dx = \frac{6x^3}{3} - 6\ln|x| + 8\frac{x^{-2}}{-2} + C =$$

$$= 2x^3 - 6\ln|x| - \frac{4}{x^2} + C$$

88) $\int \frac{x^3-7x^2+4}{x}\,dx = \int\left(x^2-7x+\frac{4}{x}\right)dx = \frac{x^3}{3}-\frac{7x^2}{2}+4\ln|x|+C$

89) $\int \frac{x^3-5x+3}{x}\,dx = \int\left(x^2-5+\frac{3}{x}\right)dx = \frac{x^3}{3}-5x+3\ln|x|+C$

90) $\int (x^2-5)^2\,dx = \int (x^4-10x^2+25)\,dx = \frac{x^5}{5}-\frac{10x^3}{3}+25x+C$

91) $\int (x+4)^3\,dx = \frac{(x+4)^4}{4}+C$

92) $\int \frac{x^3-5x+3}{x-2}\,dx = \int (x^2+2x-1)\,dx + \int \frac{1}{x-2}\,dx =$

$$
\begin{array}{rrrr|l}
\cancel{x^3} & +0x^2 & -5x & +3 & x-2 \\
-\cancel{x^3} & +2x^2 & & & x^2+2x-1 \\
\hline
 & \cancel{2x^2} & -5x & & \\
 & -\cancel{2x^2} & +4x & & \\
\hline
 & & -\cancel{x} & +3 & \\
 & & +\cancel{x} & -2 & \\
\hline
 & & & 1 &
\end{array}
$$

$= \frac{x^3}{3}+\frac{\cancel{2}x^2}{\cancel{2}}-x+\ln|x-2|+C$

93) $\int \frac{\sqrt{3x^3}}{\sqrt[3]{x^2}}\,dx = \int \frac{\sqrt{3}\cdot\sqrt{x^3}}{\sqrt[3]{x^2}}\,dx = \sqrt{3}\int\frac{x^{3/2}}{x^{2/3}}\,dx = \sqrt{3}\int x^{5/6}\,dx =$

$= \sqrt{3}\cdot\frac{x^{11/6}}{11/6}+C = \frac{6\sqrt{3}}{11}\cdot\sqrt[6]{x^{11}}+C$

94) $\displaystyle\int \frac{\sqrt{3x^3}}{\sqrt[3]{5x^2}}\,dx = \int \frac{\sqrt{3}\,\sqrt{x^3}}{\sqrt[3]{5}\cdot\sqrt[3]{x^2}}\,dx = \frac{\sqrt{3}}{\sqrt[3]{5}}\cdot\int \frac{x^{3/2}}{x^{2/3}}\,dx =$

$\displaystyle= \frac{\sqrt{3}}{\sqrt[3]{5}}\cdot\int x^{5/6}\,dx = \frac{\sqrt{3}}{\sqrt[3]{5}}\cdot\frac{x^{11/6}}{11/6} + C = \frac{6\sqrt{3}}{\sqrt[3]{5}}\cdot\sqrt[6]{x^{11}} + C$

95) $\displaystyle\int \frac{\sqrt{3x}-5}{\sqrt[3]{2x}}\,dx = \int \frac{\sqrt{3x}}{\sqrt[3]{2x}}\,dx - \int \frac{5}{\sqrt[3]{2x}}\,dx =$

$\displaystyle= \int \frac{\sqrt{3}\cdot\sqrt{x}}{\sqrt[3]{2}\cdot\sqrt[3]{x}}\,dx - 5\int \frac{1}{\sqrt[3]{2}\cdot\sqrt[3]{x}}\,dx =$

$\displaystyle= \frac{\sqrt{3}}{\sqrt[3]{2}}\int \frac{x^{1/2}}{x^{1/3}}\,dx - \frac{5}{\sqrt[3]{2}}\int x^{-1/3}\,dx = \frac{\sqrt{3}}{\sqrt[3]{2}}\int x^{1/6}\,dx - \frac{5}{\sqrt[3]{2}}\int x^{-1/3}dx$

$\displaystyle= \frac{\sqrt{3}}{\sqrt[3]{2}}\,\frac{x^{7/6}}{7/6} - \frac{5}{\sqrt[3]{2}}\,\frac{x^{2/3}}{2/3} + C = \frac{6\sqrt{3}}{7\sqrt[3]{2}}\cdot\sqrt[6]{x^7} - \frac{15}{2\sqrt[3]{2}}\cdot\sqrt[3]{x^2} + C$

96) $\displaystyle\int \frac{(x^2-1)^2}{\sqrt{x}}\,dx = \int \frac{x^4-2x^2+1}{\sqrt{x}}\,dx = \int \frac{x^4}{x^{1/2}}\,dx - 2\int\frac{x^2}{x^{1/2}}\,dx + \int x^{-1/2}d$

$\displaystyle= \int x^{7/2}\,dx - 2\int x^{3/2}\,dx + \int x^{-1/2}\,dx =$

$\displaystyle= \frac{x^{9/2}}{9/2} - 2\cdot\frac{x^{5/2}}{5/2} + \frac{x^{1/2}}{1/2} + C = \frac{2}{9}\sqrt{x^9} - \frac{4}{5}\sqrt{x^5} + 2\sqrt{x} + C$

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97) $\int (2x+3)^3 \cdot \sqrt{x}\, dx = \int (8x^3+36x^2+54x+27)\cdot \sqrt{x}\, dx = \circledast$

$$\begin{matrix} & & 1 & & \\ & 1 & 2 & 1 & \\ 1 & 3 & & 3 & 1 \end{matrix} \qquad (2x+3)^3 = (2x)^3 + 3\cdot(2x)^2\cdot 3 + 3\cdot(2x)\cdot 3^2 + 3^3 =$$

$$= 8x^3+36x^2+54x+27$$

$\circledast = 8\int x^3\cdot x^{1/2} dx + 36\int x^2\cdot x^{1/2} dx + 54\int x\cdot x^{1/2} + 27\int x^{1/2} dx =$

$= 8\cdot \int x^{7/2} dx + 36\int x^{5/2} dx + 54\int x^{3/2} dx + 27\int x^{1/2} dx =$

$= 8\cdot \dfrac{x^{9/2}}{9/2} + 36\,\dfrac{x^{7/2}}{7/2} + 54\,\dfrac{x^{5/2}}{5/2} + 27\,\dfrac{x^{3/2}}{3/2} + C =$

$= \dfrac{16}{9}\sqrt{x^9} + \dfrac{72}{7}\sqrt{x^7} + \dfrac{108}{5}\sqrt{x^5} + 18\sqrt{x^3} + C$

98) $\int (\sqrt[3]{x^2} - 3\sqrt{x} + 12)\, dx = \int x^{2/3} dx - 3\int x^{1/2} dx + \int 12\, dx =$

$= \dfrac{x^{5/3}}{5/3} - 3\,\dfrac{x^{3/2}}{3/2} + 12x + C = \dfrac{3}{5}\sqrt[3]{x^5} - 2\sqrt{x^3} + 12x + C$

99) $\int \dfrac{3x+2}{\sqrt{x}}\, dx = \int \dfrac{3x}{x^{1/2}}\, dx + \int \dfrac{2}{x^{1/2}}\, dx = 3\int x^{1/2} dx + 2\int x^{-1/2} dx$

$= \dfrac{3x^{3/2}}{3/2} + 2\,\dfrac{x^{1/2}}{1/2} + C = 2\sqrt{x^3} + 4\sqrt{x} + C$

100) $\int 5x\sqrt{x^2+7}\, dx = \dfrac{5}{2}\int 2x\,(x^2+7)^{1/2} dx = \dfrac{5}{\cancel{2}}\,\dfrac{(x^2+7)^{3/2}}{3/\cancel{2}} + C =$

$= \dfrac{5}{3}\sqrt{(x^2+7)^3} + C$

101) $\int x^3 \sqrt[3]{x-12}\, dx = \int (t^3+12)^3 \cdot \sqrt[3]{t^3} \cdot 3t^2 dt = ⊛$

$x - 12 = t^3 \longrightarrow x = t^3 + 12$

$dx = 3t^2 dt$

$⊛ = \int 3t^2 \cdot t \cdot (t^3+12)^3 dt = \int 3t^3 \cdot (t^3+12)^3 dt = ⊛$

$$\begin{matrix} & & 1 & & \\ & 1 & 2 & 1 & \\ 1 & 3 & 3 & 1 \end{matrix} \qquad (t^3+12)^3 = (t^3)^3 + 3\cdot(t^3)^2\cdot 12 + 3\cdot(t^3)\cdot 12^2 + 12^3 =$$

$$= t^9 + 36t^6 + 432t^3 + 1728$$

$⊛ = \int 3t^3 \cdot (t^9 + 36t^6 + 432t^3 + 1728) dt =$

$= \int (3t^{12} + 108t^9 + 1296t^6 + 5184t^3) dt =$

$= \frac{3t^{13}}{13} + \frac{108t^{10}}{10} + \frac{1296t^7}{7} + \frac{5184t^4}{4} + C' =$

$= \frac{3}{13}\sqrt[3]{(x-12)^{13}} + \frac{108}{10}\sqrt[3]{(x-12)^{10}} + \frac{1296}{7}\sqrt[3]{(x-12)^7} + 1296\sqrt[3]{(x-12)^4} + C$

102) $\int x(2x^2-7)^{99} dx = \frac{1}{4}\int 4x(2x^2-7)^{99} dx = \frac{1}{4}\frac{(2x^2-7)^{100}}{100} + C$

103) $\int \frac{4x\, dx}{\sqrt[3]{8-x^2}} = \frac{4}{-2}\int -2x(8-x^2)^{-1/3} dx = -2\frac{(8-x^2)^{2/3}}{2/3} + C =$

$= -3\sqrt[3]{(8-x^2)^2} + C$

104) $\int x(3x^2-1)^{240}dx = \frac{1}{6}\int 6x(3x^2-1)^{240}dx =$

$= \frac{1}{6}\frac{(3x^2-1)^{241}}{241} + C$

105) $\int (2-7x)^{2/3}dx = -\frac{1}{7}\int -7(2-7x)^{2/3}dx = -\frac{1}{7}\frac{(2-7x)^{5/3}}{5/3} + C =$

$= \frac{-3}{35}\sqrt[3]{(2-7x)^5} + C$

106) $\int \frac{dx}{(3x+1)^5} = \frac{1}{3}\int 3(3x+1)^{-5}dx = \frac{1}{3}\frac{(3x+1)^{-4}}{-4} + C =$

$= \frac{-1}{12}\cdot\frac{1}{(3x+)^4} + C$

107) $\int x\cdot\sqrt{2x^2+17}\,dx = \frac{1}{4}\int 4x(2x^2+17)^{1/2}dx = \frac{1}{4}\frac{(2x^2+17)^{3/2}}{3/2} + C =$

$= \frac{1}{6}\sqrt{(2x^2+17)^3} + C$

108) $\int \frac{x+1}{\sqrt[3]{x^2+2x+12}}dx = \frac{1}{2}\int 2(x+1)\cdot(x^2+2x+12)^{-1/3}dx =$

$= \frac{1}{2}\frac{(x^2+2x+12)^{2/3}}{2/3} + C = \frac{3}{4}\cdot\sqrt[3]{(x^2+2x+12)^2} + C$

109) $\int \sqrt{x^2-2x+3}\cdot(x-1)dx = \frac{1}{2}\int 2(x-1)\cdot(x^2-2x+3)^{1/2}dx =$

$= \frac{1}{\cancel{2}}\frac{(x^2-2x+3)^{3/2}}{3/\cancel{2}} + C = \frac{1}{3}\sqrt{(x^2-2x+3)^3} + C$

110) $\int \cos(5x)\,dx = \frac{1}{5}\int 5\cos(5x)\,dx = \frac{1}{5}\,\text{sen}(5x) + C$

111) $\int \text{sen}(2x)\cdot\cos(2x)\,dx = \frac{1}{2}\int 2\cos(2x)\cdot\text{sen}(2x)\,dx =$

$= \frac{1}{2}\cdot\frac{(\text{sen}\,2x)^2}{2} + C$

112) $\int e^{\text{sen}x}\cdot\cos x\,dx = e^{\text{sen}x} + C$

113) $\int e^{-5x}\,dx = -\frac{1}{5}\int -5\,e^{-5x}\,dx = -\frac{1}{5}e^{-5x} + C$

114) $\int \frac{e^{-2x}\cdot e^{2x}}{2}\,dx = \int \frac{e^0}{2}\,dx = \int \frac{1}{2}\,dx = \frac{1}{2}x + C$

115) $\int \text{sen}^2\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)dx = 2\int \frac{1}{2}\cos\left(\frac{x}{2}\right)\cdot\left[\text{sen}\left(\frac{x}{2}\right)\right]^2 dx =$

$= 2\cdot\frac{(\text{sen}(x/2))^3}{3} + C$

116) $\int \text{tg}\,x\cdot\sec^2 x\,dx = \int \underbrace{\sec(x)}_{F(x)}\cdot\underbrace{\sec(x)\cdot\text{tg}(x)\,dx}_{F'(x)} = \frac{\sec^2(x)}{2} + C$

117) $\int \text{sen}^2(4x)\,dx = \int \underbrace{\text{sen}(4x)}_{u}\,\underbrace{\text{sen}(4x)\,dx}_{dv} = (*)$

$u = \text{sen}(4x) \qquad du = 4\cos(4x)\,dx$

$dv = \text{sen}(4x) \qquad v = -\frac{1}{4}\cos(4x)$

$$\circledast = -\frac{1}{4}\cos(4x)\,\text{sen}(4x) + \int \cos^2(4x)\,dx =$$

$$= -\frac{1}{4}\cos(4x)\,\text{sen}(4x) + \int (1-\text{sen}^2(4x))\,dx =$$

$$= -\frac{1}{4}\cos(4x)\,\text{sen}(4x) + x - \underbrace{\int \text{sen}^2(4x)\,dx}_{=I}$$

$$\Rightarrow I = -\frac{1}{4}\cos(4x)\,\text{sen}(4x) + x - I \Rightarrow$$

$$\Rightarrow 2I = -\frac{1}{4}\cos(4x)\,\text{sen}(4x) + x$$

$$\Rightarrow I = \frac{-1/4\cos(4x)\,\text{sen}(4x) + x}{2} + C$$

118) $\int \cos^3(3x)\,dx = \int \cos(3x)\cdot\cos^2(3x)\,dx =$

$$= \int \cos(3x)\cdot(1-\text{sen}^2(3x))\,dx = \frac{1}{3}\int 3\cos(3x)\,dx - \frac{1}{3}\int 3\cos(3x)\,\text{sen}^2(3x)\,dx$$

$$= \frac{1}{3}\,\text{sen}(3x) - \frac{1}{3}\,\frac{\text{sen}^3(3x)}{3} + C$$

119) $\int \cos^5 x\,dx = \int \cos^3 x\cdot\cos^2 x\,dx = \int \cos^3 x\cdot(1-\text{sen}^2 x)\,dx =$

$$= \underbrace{\int \cos^3 x\,dx}_{I_1} - \underbrace{\int \cos^3 x\cdot\text{sen}^2 x\,dx}_{I_2}$$

$$I_1 \longrightarrow \int \cos^3 x\, dx = \int \cos x \cdot \cos^2 x\, dx = \int \cos x (1-\operatorname{sen}^2 x)\, dx =$$

$$= \int \cos x\, dx - \int \cos x \operatorname{sen}^2 x\, dx = \operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} + C_1$$

$$I_2 \longrightarrow \int \cos^3 x \cdot \operatorname{sen}^2 x\, dx = \int \cos x \cdot \cos^2 x \operatorname{sen}^2 x\, dx =$$

$$= \int \cos x\,(1-\operatorname{sen}^2 x) \operatorname{sen}^2 x\, dx = \int \cos x \operatorname{sen}^2 x\, dx - \int \cos x \operatorname{sen}^4 x\, dx =$$

$$= \frac{\operatorname{sen}^3 x}{3} - \frac{\operatorname{sen}^5 x}{5} + C_2$$

$$\Rightarrow I = I_1 - I_2 = \operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} - \left(\frac{\operatorname{sen}^3 x}{3} - \frac{\operatorname{sen}^5 x}{5} \right) + C =$$

$$= \operatorname{sen} x - \frac{2\operatorname{sen}^3 x}{3} + \frac{\operatorname{sen}^5 x}{5} + C$$

120) $$\int \frac{x^2-4}{x-3}\, dx = \int (x+3)\, dx + \int \frac{5}{x-3}\, dx =$$

$$\begin{array}{rl|l} \cancel{x^2} + 0x - 4 & & x-3 \\ \cline{3-3} -\cancel{x^2} + 3x & & x+3 \\ \hline 3\cancel{x} - 4 & & \\ -3\cancel{x} + 9 & & \\ \hline 5 & & \end{array}$$

$$= \frac{x^2}{2} + 3x + 5\ln|x-3| + C$$

121) $\int (2^x + x^2)\,dx = \dfrac{2^x}{\ln 2} - \dfrac{x^3}{3} + C$

122) $\int \dfrac{x^2}{x^3+2}\,dx = \dfrac{1}{3}\int \dfrac{3x^2}{x^3+2}\,dx = \dfrac{1}{3}\ln|x^3+2| + C$

123) $\int x^2 \cdot \operatorname{sen} x^3\,dx = \dfrac{1}{3}\int 3x^2 \operatorname{sen} x^3\,dx = -\dfrac{1}{3}\cos(x^3) + C$

124) $\int \dfrac{-3x}{1+x^4}\,dx = -\dfrac{3}{2}\int \dfrac{2x}{1+(x^2)^2}\,dx = -\dfrac{3}{2}\operatorname{arctg}(x^2) + C$

125) $\int \dfrac{2x}{4+x^4}\,dx = 2\int \dfrac{1x\,dx}{4\,(4+x^4)} = \dfrac{1}{2}\int \dfrac{x\,dx}{1+\left(\frac{x^2}{2}\right)^2} =$

$= \dfrac{1}{2}\operatorname{arctg}\left(\dfrac{x^2}{2}\right) + C$

126) $\int x^3\,dx = \dfrac{x^4}{4} + C$

127) $\int \dfrac{x^4}{3}\,dx = \dfrac{x^5}{15} + C$

128) $\int \dfrac{x^5}{6}\,dx = \dfrac{x^6}{36} + C$

129) $\int (x^2+3)\,dx = \dfrac{x^3}{3} + 3x + C$

130) $\int (x^3 + 2x - 1/x)\,dx = \dfrac{x^4}{4} + x^2 - \ln|x| + C$

131) $\int \frac{x^4-x^2+1}{x}\,dx = \int\left(x^3 - x + \frac{1}{x}\right)dx = \frac{x^4}{4} - \frac{x^2}{2} + \ln|x| + C$

132) $\int \frac{dx}{x^2} = \int x^{-2}dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$

133) $\int \frac{1}{x^4}\,dx = \int x^{-4}dx = \frac{x^{-3}}{-3} + C = \frac{-1}{3x^3} + C$

134) $\int \frac{x^4-2x+3}{x^7}\,dx = \int\left(x^{-3} - 2x^{-6} + 3x^{-7}\right)dx =$

$= \frac{x^{-2}}{-2} - 2\frac{x^{-5}}{-5} + 3\frac{x^{-6}}{-6} = \frac{-1}{x^2} + \frac{2}{5x^5} - \frac{1}{2x^6} + C$

135) $\int \frac{x^5}{1+x}\,dx = \int (x^4-x^3+x^2-x+1)dx + \int \frac{-1}{x+1}dx = \circledast$

$$\begin{array}{l|l}
\cancel{x^5}+0x^4+0x^3+0x^2+0x+0 & x+1 \\ \hline
-\cancel{x^5}-x^4 & x^4-x^3+x^2-x+1 \\
\quad -\cancel{x^4}+0x^3 & \\
\quad +\cancel{x^4}+x^3 & \\
\qquad \cancel{x^3}+0x^2 & \\
\qquad -\cancel{x^3}-x^2 & \\
\qquad\quad -\cancel{x^2}+0x & \\
\qquad\quad +\cancel{x^2}+x & \\
\qquad\qquad \cancel{x}+0 & \\
\qquad\qquad -\cancel{x}-1 & \\
\qquad\qquad\quad -1 &
\end{array}$$

$\circledast = \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x - \ln|x+1| + C$

136) $\int \frac{4\sqrt[3]{x}}{3}\,dx = \frac{4}{3}\int x^{1/3}dx = \frac{\cancel{4}}{\cancel{3}}\cdot\frac{x^{4/3}}{\cancel{4/3}} + C = \sqrt[3]{x^4} + C$

137) $\int \frac{dx}{\sqrt[5]{x}} = \int x^{-1/5} dx = \frac{x^{4/5}}{4/5} + C = \frac{5}{4} \sqrt[5]{x^4} + C$

138) $\int \left(\frac{8}{3}\sqrt[3]{x^5} + 6\sqrt{x}\right) dx = \frac{8}{3}\int x^{5/3} dx + 6\int x^{1/2} dx =$

$= \cancel{\frac{8}{3}} \frac{x^{8/3}}{\cancel{8/3}} + 6 \cdot \frac{x^{3/2}}{3/2} + C = \sqrt[3]{x^8} + 4\sqrt{x^3} + C$

139) $\int \sqrt[3]{x} \cdot (\sqrt{x} + 3)\, dx = \int \sqrt[3]{x} \cdot \sqrt{x}\, dx + 3\int \sqrt[3]{x}\, dx =$

$= \int x^{1/3} \cdot x^{1/2} dx + 3\int x^{1/3} dx = \int x^{5/6} dx + 3\int x^{1/3} dx =$

$= \frac{x^{11/6}}{11/6} + 3\frac{x^{4/3}}{4/3} + C = \frac{6}{11}\sqrt[6]{x^{11}} + \frac{9}{4}\sqrt[3]{x^4} + C$

140) $\int (x - \operatorname{sen} x + 2\cos x)\, dx = \frac{x^2}{2} + \cos x + 2\operatorname{sen} x + C$

141) $\int \left(e^x - \frac{1}{x}\right) dx = e^x - \ln|x| + C$

142) $\int \frac{dx}{4x-2} = \frac{1}{4}\int \frac{4\, dx}{4x-2} = \frac{1}{4} \ln|4x-2| + C$

143) $\int \frac{dx}{2-x} = -\int \frac{-dx}{2-x} = -\ln|2-x| + C$

144) $\int \frac{x}{1-x^2} dx = -\frac{1}{2}\int \frac{-2x\, dx}{1-x^2} = -\frac{1}{2} \ln|1-x^2| + C$

145) $\int \frac{3\, dx}{(x+1)^4} = 3\int (x+1)^{-4} dx = 3\frac{(x+1)^{-3}}{-3} + C = \frac{-1}{(x+1)^3} + C$

146) $\int \frac{x^3}{1+x^4}dx = \frac{1}{4}\int \frac{4x^3}{1+x^4}dx = \frac{1}{4}\ln|1+x^4| + C$

147) $\int \frac{\operatorname{sen}(2x)}{1+\operatorname{sen}^2 x}dx = \int \frac{2\operatorname{sen}x\cos x}{1+\operatorname{sen}^2 x}dx = \ln|1+\operatorname{sen}^2 x| + C$

$\operatorname{sen}(2x) = 2\operatorname{sen}x \cdot \cos x$

148) $\int \frac{x+2}{\sqrt{x^2+4x+2}}dx = \frac{1}{2}\int 2(x+2)(x^2+4x+2)^{-1/2}dx =$

$= \frac{1}{\cancel{2}}\frac{(x^2+4x+2)^{1/2}}{\cancel{1/2}} + C = \sqrt{x^2+4x+2} + C$

149) $\int e^x(\sqrt{1-e^x})dx = -\int -e^x(1-e^x)^{1/2}dx = -\frac{(1-e^x)^{3/2}}{3/2} + C =$

$= -\frac{2}{3}\sqrt{(1-e^x)^3} + C$

150) $\int \frac{\ln x}{x}dx = \int \frac{1}{x}\cdot \ln x\, dx = \frac{(\ln x)^2}{2} + C$

151) $\int x\sqrt{x^2+5}\,dx = \frac{1}{2}\int 2x(x^2+5)^{1/2}dx = \frac{1}{\cancel{2}}\frac{(x^2+5)^{3/2}}{3/\cancel{2}} + C =$

$= \frac{1}{3}\sqrt{(x^2+5)^3} + C$

152) $\int \operatorname{sen}(7x)dx = \frac{1}{7}\int 7\operatorname{sen}(7x)dx = -\frac{1}{7}\cos(7x) + C$

153) $\int 8x \cos x^2 \, dx = \frac{8}{2} \int 2x \cos x^2 \, dx = 4 \operatorname{sen}(x^2) + C$

154) $\int \frac{\cos x}{1+\operatorname{sen}^2 x} \, dx = \operatorname{arctg}(\operatorname{sen} x) + C$

155) $\int \frac{dx}{\cos^2 x \sqrt{1-\tan^2 x}} = \int \frac{{}^{1}/_{\cos^2 x} \, dx}{\sqrt{1-(\operatorname{tg} x)^2}} = \operatorname{arcsen}(\operatorname{tg} x) + C$

156) $\int 4x^2 \cdot e^{x^3} dx = \frac{4}{3} \int 3x^2 \cdot e^{x^3} dx = \frac{4}{3} e^{x^3} + C$

157) $\int \frac{3x^2}{1+x^6} \, dx = \int \frac{3x^2}{1+(x^3)^2} \, dx = \operatorname{arctg}(x^3) + C$

158) $\int 3^x dx = \frac{3^x}{\ln 3} + C$

159) $\int \frac{dx}{4+x^2} dx = \int \frac{dx}{\frac{4}{4}(4+x^2)} = \frac{1}{4} \cdot 2 \int \frac{\frac{1}{2} dx}{1+\left(\frac{x}{2}\right)^2} = \frac{1}{2} \operatorname{arctg}\left(\frac{x}{2}\right) + C$

160) $\int e^{5x} dx = \frac{1}{5} \int 5 e^{5x} dx = \frac{1}{5} e^{5x} + C$

161) $\int (e^x + e^{-x})^2 dx = \int (e^{2x} + 2e^x \cdot e^{-x} + e^{-2x}) dx = \int (e^{2x} + 2 + e^{-2x}) dx$

$= \frac{1}{2} \int 2e^{2x} dx + \int 2 \, dx + \frac{-1}{2} \int -2e^{-2x} dx = \frac{1}{2} e^{2x} + 2x - \frac{1}{2} e^{-2x} + C$

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162) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx = e^{\sqrt{x}} + C$

163) $\int \frac{\operatorname{sen} x}{e^{\cos x}}\,dx = \int \operatorname{sen} x \cdot e^{-\cos x}\,dx = e^{-\cos x} + C$

164) $\int \underbrace{x^2 \cdot \underbrace{\ln x}_{u}\, dx}_{dv} = \frac{x^3}{3} \cdot \ln x - \int \frac{\cancel{x^3}^{2}}{3} \cdot \frac{1}{\cancel{x}}\,dx = \circledast$

$u = \ln x \qquad du = \frac{1}{x}\,dx$

$dv = x^2\,dx \qquad v = \frac{x^3}{3}$

$\circledast = \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$

165) $\int \underbrace{x\, \underbrace{\operatorname{arctg} x}_{u}\, dx}_{dv} = \frac{x^2}{2}\operatorname{arctg} x - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2}\,dx = \circledast$

$u = \operatorname{arctg} x \qquad du = \frac{1}{1+x^2}\,dx$

$dv = x\,dx \qquad v = \frac{x^2}{2}$

$\circledast = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx = ☺$

La hacemos a parte!!

$\int \frac{x^2}{1+x^2}\,dx = \int 1\,dx + \int \frac{-1}{1+x^2}\,dx = x - \operatorname{arctg} x$

$$\begin{array}{r|l} x^2 & x^2+1 \\ \hline -x^2-1 & 1 \\ \hline -1 & \end{array}$$

$☺ = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2} \cdot [x - \operatorname{arctg} x] + C$

166) $\int \frac{x+2}{x^2+x-6}\,dx = (*)$

$x^2+x-6=0 \Rightarrow x = \frac{-1 \pm \sqrt{1+24}}{2}$ $\begin{cases} x = 2 \to \text{Real Sencilla} \\ x = -3 \to \text{Real Sencilla} \end{cases}$

$\frac{x+2}{\cancel{x^2+x-6}} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{A(x+3)+B(x-2)}{\cancel{(x-2)(x+3)}}$

$\Rightarrow x+2 = A(x+3)+B(x-2)$

Si $x=2 \to 4 = 5A \to A = 4/5$

Si $x=-3 \to -1 = -5B \to B = 1/5$

$(*) = \int \frac{4/5}{x-2}\,dx + \int \frac{1/5}{x+3}\,dx = \frac{4}{5}\ln|x-2| + \frac{1}{5}\ln|x+3| + C$

167) $\int \frac{dx}{x^2+5x+6} = (*)$

$x^2+5x+6=0 \Rightarrow x = \frac{-5 \pm \sqrt{25-24}}{2}$ $\begin{cases} x = -2 \to \text{Real Sencilla} \\ x = -3 \to \text{Real Sencilla.} \end{cases}$

$\frac{1}{\cancel{x^2+5x+6}} = \frac{A}{x+2} + \frac{B}{x+3} = \frac{A(x+3)+B(x+2)}{\cancel{(x+2)(x+3)}}$

$\Rightarrow 1 = A(x+3)+B(x+2)$

Si $x=-2 \to 1 = A$

Si $x=-3 \to 1 = -B \Rightarrow B = -1$

$$(*) = \int \frac{1}{x+2}\,dx + \int \frac{-1}{x+3}\,dx = \ln|x+2| - \ln|x+3| + C$$

168) $\displaystyle\int \frac{x^3+1}{x^2+x-2}\,dx = \int (x-1)\,dx + \int \frac{3x-1}{x^2+x-2}\,dx =$ 

$\Downarrow$

La hacemos a parte

$$\begin{array}{rl|l} & x^3+0x^2+0x+1 & x^2+x-2 \\ \cline{3-3} & -x^3-x^2+2x & x-1 \\ \hline & -x^2+2x+1 & \\ & +x^2+x-2 & \\ \hline & 3x-1 & \end{array}$$

$$x^2+x-2=0$$

$$x = \frac{-1 \pm \sqrt{1+8}}{2} \begin{cases} \to x = 1 \\ \to x = -2 \end{cases}$$

$$\frac{3x-1}{\cancel{x^2+x-2}} = \frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2)+B(x-1)}{\cancel{(x-1)(x+2)}}$$

$$\Rightarrow 3x-1 = A(x+2)+B(x-1)$$

Si $x = 1 \rightarrow 2 = 3A \Rightarrow A = 2/3$

Si $x = -2 \rightarrow -7 = -3B \Rightarrow B = 7/3$

$$\int \frac{3x-1}{x^2+x-2}\,dx = \int \frac{2/3}{x-1}\,dx + \int \frac{7/3}{x+2}\,dx = \frac{2}{3}\ln|x-1| + \frac{7}{3}\ln|x+2|$$

 $\displaystyle = \frac{x^2}{2} - x + \frac{2}{3}\ln|x-1| + \frac{7}{3}\ln|x+2| + C$

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169) $\displaystyle\int \frac{1}{x^2+3x}\,dx = \circledast$

$$x^2+3x=0 \Rightarrow x(x+3)=0 \begin{cases} \to x=0 \\ \to x=-3 \end{cases}$$

$$\frac{1}{\cancel{x^2+3x}} = \frac{A}{x} + \frac{B}{x+3} = \frac{A(x+3)+Bx}{\cancel{x(x+3)}}$$

$$\Rightarrow 1 = A(x+3)+Bx$$

$$\text{Si } x=-3 \longrightarrow 1 = -3B \Rightarrow B = -1/3$$

$$\text{Si } x=0 \longrightarrow 1 = 3A \Rightarrow A = 1/3$$

$$\circledast = \int \frac{1/3}{x}\,dx + \int \frac{-1/3}{x+3}\,dx = \frac{1}{3}\ln|x| - \frac{1}{3}\ln|x+3| + C$$

170) $\displaystyle\int \frac{1}{\operatorname{sen}x\cos x}\,dx = \int \frac{\operatorname{sen}^2x+\cos^2x}{\operatorname{sen}x\cdot\cos x}\,dx = \int \frac{\operatorname{sen}^{\cancel{2}}x}{\cancel{\operatorname{sen}x}\cdot\cos x}\,dx + \int \frac{\cos^{\cancel{2}}x}{\operatorname{sen}x\,\cancel{\cos x}}\,dx$

$$= -\int \frac{-\operatorname{sen}x}{\cos x}\,dx + \int \frac{\cos x}{\operatorname{sen}x}\,dx = -\ln|\cos x| + \ln|\operatorname{sen}x| + C$$

$$= \ln|\operatorname{tg}x| + C$$

171) $\int \frac{2x-3}{x+2}\,dx = \int 2\,dx + \int \frac{-7}{x+2}\,dx =$

$$\begin{array}{r|l} 2x - 3 & x+2 \\ -2x - 4 & 2 \\ \hline -7 & \end{array}$$

$= 2x - 7\cdot \ln|x+2| + C$

172) $\int \frac{dx}{x^2-4} = ⊛$

$x^2-4=0 \Rightarrow x^2=4$

$x=2 \rightarrow$ Real Sencilla

$x=-2 \rightarrow$ Real Sencilla

$\frac{1}{\cancel{x^2-4}} = \frac{A}{x+2} + \frac{B}{x-2} = \frac{A(x-2)+B(x+2)}{\cancel{(x+2)(x-2)}}$

$\Rightarrow 1 = A(x-2)+B(x+2)$

Si $x=2 \rightarrow 1 = 4B \rightarrow B = 1/4$

Si $x=-2 \rightarrow 1 = -4A \rightarrow A = -1/4$

$⊛ = \int \frac{-1/4}{x+2}\,dx + \int \frac{1/4}{x-2}\,dx = -\frac{1}{4}\ln|x+2| + \frac{1}{4}\ln|x-2| + C$

173) $\int \frac{x-1}{x^2+x-6}\,dx = ⊛$

$x^2+x-6=0 \Rightarrow x = \frac{-1\pm\sqrt{1+24}}{2}$

$x=2 \rightarrow$ Real Sencilla

$x=-3 \rightarrow$ Real Sencilla.

$$\frac{x-1}{\cancel{x^2+x-6}} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{A(x+3)+B(x-2)}{\cancel{(x-2)(x+3)}}$$

$$\Rightarrow x-1 = A(x+3)+B(x-2)$$

Si $x=2 \longrightarrow 1 = 5A \Rightarrow A = 1/5$

Si $x=-3 \longrightarrow -4 = -5B \Rightarrow B = 4/5$

$$\circledast = \int \frac{1/5}{x-2}\,dx + \int \frac{4/5}{x+3}\,dx = \frac{1}{5}\ln|x-2| + \frac{4}{5}\ln|x+3| + C'$$

174) $\int \frac{2}{x^2+5x+6}\,dx = \circledast$

$$x^2+5x+6=0 \Rightarrow x = \frac{-5\pm\sqrt{25-24}}{2} \begin{cases} x=-2 \text{ Real Sencilla} \\ x=-3 \text{ Real Sencilla} \end{cases}$$

$$\frac{2}{\cancel{x^2+5x+6}} = \frac{A}{x+2} + \frac{B}{x+3} = \frac{A(x+3)+B(x+2)}{\cancel{(x+2)(x+3)}}$$

$$\Rightarrow 2 = A(x+3)+B(x+2)$$

Si $x=-2 \longrightarrow 2 = A$

Si $x=-3 \longrightarrow 2 = -B \Rightarrow B=-2$

$$\circledast = \int \frac{2}{x+2}\,dx + \int \frac{-2}{x+3}\,dx = 2\ln|x+2| - 2\ln|x+3| + C'$$

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175) $\displaystyle\int \frac{x+1}{x\cdot(x-1)^2}\,dx = (*)$

$x\cdot(x-1)^2 = 0$ (ya esta factorizado!) $\begin{cases} x=0 \\ x=1 \text{ (Doble)} \end{cases}$

$$\frac{x+1}{\cancel{x(x-1)^2}} = \frac{A}{x} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)} = \frac{A(x-1)^2 + Bx + C\,x(x-1)}{\cancel{x(x-1)^2}}$$

$$\Rightarrow x+1 = A(x-1)^2 + Bx + Cx(x-1)$$

Si $x=0 \rightarrow 1 = A$

Si $x=1 \rightarrow 2 = B$

Si $x=2 \rightarrow 3 = A + 2B + 2C \rightarrow C = -1$

$$(*) = \int \frac{1}{x}\,dx + \int \frac{2}{(x-1)^2}\,dx + \int \frac{-1}{x-1}\,dx =$$

$$= \ln|x| + 2\int (x-1)^{-2}\,dx - \ln|x-1| =$$

$$= \ln|x| + 2\,\frac{(x-1)^{-1}}{-1} - \ln|x-1| + C_1 =$$

$$= \ln|x| - \frac{2}{(x-1)} - \ln|x-1| + C_1$$

176) $\displaystyle\int \frac{dx}{x^2+2x} = (*)$

$x^2+2x=0 \Rightarrow x(x+2)=0 \begin{cases} x=0 \\ x+2=0 \rightarrow x=-2 \end{cases}$

$$\frac{1}{\cancel{x^2+2x}} = \frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2)+Bx}{\cancel{x(x+2)}}$$

$$\Longrightarrow 1 = A(x+2) + Bx$$

$$\text{Si } x=0 \longrightarrow 1 = 2A \Rightarrow A = 1/2$$

$$\text{Si } x=-2 \longrightarrow 1 = -2B \Rightarrow B = -1/2$$

$$\circledast = \int \frac{1/2}{x}\,dx + \int \frac{-1/2}{x+2}\,dx = \frac{1}{2}\ln|x| - \frac{1}{2}\ln|x+2| + C$$

177) $\displaystyle\int \frac{x^2+1}{x^2+x-6}\,dx = \int 1\,dx + \int \frac{-x+7}{x^2+x-6}\,dx =$ 

$\Downarrow$

La hacemos a parte

$$\begin{array}{l|l} x^2+0x+1 & x^2+x-6 \\ \hline -x^2-x+6 & 1 \\ \hline -x+7 & \end{array}$$

$$x^2+x-6=0$$

$$x = \frac{-1 \pm \sqrt{1+24}}{2} \begin{cases} x = 2 \\ x = -3 \end{cases}$$

$$\frac{-x+7}{\cancel{x^2+x-6}} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{A(x+3)+B(x-2)}{\cancel{(x-2)(x+3)}}$$

$$\Longrightarrow -x+7 = A(x+3) + B(x-2)$$

$$\text{Si } x=2 \longrightarrow 5 = 5A \Rightarrow A = 1$$

$$\text{Si } x=-3 \longrightarrow 10 = -5B \Rightarrow B = -2$$

$$\int \frac{-x+7}{x^2+x-6}\,dx = \int \frac{1}{x-2}\,dx + \int \frac{-2}{x+3}\,dx = \ln|x-2| - 2\ln|x+3|$$

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$$\Rightarrow \; = x + \ln|x-2| - 2\ln|x+3| + C'$$

178) $$\int \frac{x^3-1}{x^2+x}\,dx = \int (x-1)\,dx + \int \frac{x-1}{x^2+x}\,dx =$$

$$\begin{array}{l|l} \cancel{x^3} + 0x^2 + 0x - 1 & x^2 + x \\ \hline -\cancel{x^3} - x^2 & x - 1 \\ \hline -\cancel{x^2} + 0x & \\ +\cancel{x^2} + x & \\ \hline x - 1 & \end{array}$$

La hacemos a parte

$$x^2 + x = 0 \rightarrow x(x+1) = 0$$

$$x = 0 \qquad x = -1$$

$$\frac{x-1}{\cancel{x^2+x}} = \frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1)+Bx}{\cancel{x(x+1)}}$$

$$\Rightarrow x - 1 = A(x+1) + Bx$$

Si $x = 0 \rightarrow -1 = A$

Si $x = -1 \rightarrow -2 = -B \Rightarrow B = 2$

$$\int \frac{x-1}{x^2+x}\,dx = \int \frac{-1}{x}\,dx + \int \frac{2}{x+1}\,dx = -\ln|x| + 2\ln|x+1|$$

$$\Rightarrow \; = \frac{x^2}{2} - x - \ln|x| + 2\ln|x+1| + C'$$

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179) $\displaystyle\int \frac{x^2+1}{x^2-1}\,dx = \int 1\,dx + \int \frac{2}{x^2-1}\,dx =$

$$\begin{array}{r|l} x^2 + 0x + 1 & x^2-1 \\ \cline{2-2} -x^2 \quad\;\; +1 & 1 \\ \hline 2 & \end{array}$$

La hacemos a parte

$x^2-1=0 \rightarrow x=1$, $x=-1$

$$\frac{2}{\cancel{x^2-1}} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(x-1)}{\cancel{(x-1)(x+1)}}$$

$$\Rightarrow 2 = A(x+1)+B(x-1)$$

Si $x=1 \longrightarrow 2 = 2A \Rightarrow A=1$

Si $x=-1 \longrightarrow 2 = -2B \Rightarrow B=-1$

$$\int \frac{2}{x^2-1}\,dx = \int \frac{1}{x-1}\,dx + \int \frac{-1}{x+1}\,dx = \ln|x-1| - \ln|x+1|$$

$$\Rightarrow \; = x + \ln|x-1| - \ln|x+1| + C$$

180) $\displaystyle\int \frac{1}{x^2(x+1)}\,dx = (*)$

$x^2\cdot(x+1)=0$ (Ya está factorizado!) $\rightarrow$ $x=0$ (Doble), $x+1=0 \rightarrow x=-1$

$$\frac{1}{\cancel{x^2(x+1)}} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+1} = \frac{A(x+1)+Bx(x+1)+Cx^2}{\cancel{x^2(x+1)}}$$

$\Rightarrow 1 = A(x+1) + Bx(x+1) + Cx^2$

Si $x = 0 \rightarrow 1 = A$

Si $x = -1 \Rightarrow 1 = C$

Si $x = 1 \Rightarrow 1 = 2A + 2B + C \Rightarrow B = -1$

$$* = \int \frac{1}{x^2}dx + \int \frac{-1}{x}dx + \int \frac{1}{x+1}dx =$$

$$= \int x^{-2}dx - \ln|x| + \ln|x+1| + C_1 =$$

$$= \frac{x^{-1}}{-1} - \ln|x| + \ln|x+1| + C_1 = \frac{-1}{x} - \ln|x| + \ln|x+1| + C_1$$

181) $\int \frac{1}{x^2-9}dx = *$

$$x^2 - 9 = 0 \; ; \; x^2 = 9 \begin{cases} x = +3 \\ x = -3 \end{cases}$$

$$\frac{1}{\cancel{x^2-9}} = \frac{A}{x+3} + \frac{B}{x-3} = \frac{A(x-3) + B(x+3)}{\cancel{(x+3)(x-3)}}$$

$\Rightarrow 1 = A(x-3) + B(x+3)$

Si $x = 3 \rightarrow 1 = 6B \Rightarrow B = 1/6$

Si $x = -3 \rightarrow 1 = -6A \Rightarrow A = -1/6$

$$* = \int \frac{-1/6}{x+3}dx + \int \frac{1/6}{x-3}dx = \frac{-1}{6}\ln|x+3| + \frac{1}{6}\ln|x-3| + C_1$$

182) $\int \frac{x}{(x-1)^2 \cdot (x+1)} dx = ⊛$

$(x-1)^2(x+1) = 0$ (ya está factorizado!) $\to$ $x = 1$ (Doble), $x = -1$

$$\frac{x}{\cancel{(x-1)^2(x+1)}} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)} = \frac{A(x-1)(x+1) + B(x+1) + C(x-1)^2}{\cancel{(x-1)^2(x+1)}}$$

$$\Rightarrow \quad x = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

Si $x = 1 \to 1 = 2B \Rightarrow B = 1/2$

Si $x = -1 \to -1 = 4C \Rightarrow C = -1/4$

Si $x = 0 \to 0 = -A + B + C \Rightarrow A = B + C = 1/4$

$$⊛ = \int \frac{1/4}{x-1} dx + \int \frac{1/2}{(x-1)^2} dx + \int \frac{-1/4}{x+1} dx =$$

$$= \frac{1}{4} \ln|x-1| + \frac{1}{2} \int (x-1)^{-2} dx - \frac{1}{4} \ln|x+1| =$$

$$= \frac{1}{4} \ln|x-1| + \frac{1}{2} \cdot \frac{(x-1)^{-1}}{-1} - \frac{1}{4} \ln|x+1| + C' =$$

$$= \frac{1}{4} \ln|x-1| - \frac{1}{2(x-1)} - \frac{1}{4} \ln|x+1| + C'$$

183) $\displaystyle\int \frac{6}{x\cdot(x-1)\cdot(x+2)}\,dx = \circledast$

$x\cdot(x-1)(x+2)=0$ (ya está factorizado) $\to x=0,\ x=1,\ x=-2$

$$\frac{6}{\cancel{x(x-1)(x+2)}} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2} = \frac{A(x-1)(x+2)+Bx(x+2)+Cx(x-1)}{\cancel{x(x-1)(x+2)}}$$

$$\Rightarrow 6 = A(x-1)(x+2) + Bx(x+2) + Cx(x-1)$$

Si $x=0 \to 6=-2A \Rightarrow A=-3$

Si $x=1 \to 6 = 3B \Rightarrow B=2$

Si $x=-2 \to 6=6C \Rightarrow C=1$

$$\circledast = \int \frac{-3}{x}\,dx + \int \frac{2}{x-1}\,dx + \int \frac{1}{x+2}\,dx = -3\ln|x| + 2\ln|x-1| + \ln|x+2| + C$$

184) $\displaystyle\int \frac{3}{x^3-1}\,dx = \circledast$

$x^3-1=0 \to 1$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \lfloor 0 \end{array}$$

solución real

$\to x^2+x+1=0$

$$x = \frac{-1\pm\sqrt{1-4}}{2} \Rightarrow$$

$$x = \frac{-1\pm i\sqrt{3}}{2} = \underbrace{-\frac{1}{2}}_{\alpha} \pm i\cdot \underbrace{\frac{\sqrt{3}}{2}}_{\beta}$$

Soluciones Complejas

$$\Rightarrow x^2+x+1 = (x-\alpha)^2+\beta^2 = (x+1/2)^2+\frac{3}{4}$$

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$$\frac{3}{\cancel{x^3-1}} = \frac{A}{x-1} + \frac{Mx+N}{x^2+x+1} = \frac{A(x^2+x+1)+Mx(x-1)+N(x-1)}{\cancel{(x-1)(x^2+x+1)}}$$

$$\Rightarrow 3 = A(x^2+x+1)+Mx(x-1)+N(x-1)$$

Si $x=1 \rightarrow 3 = 3A \Rightarrow A=1$

Si $x=0 \rightarrow 3 = A-N \Rightarrow N=-2$

Si $x=2 \rightarrow 3 = 7A+2M+N \rightarrow M=-1$

$$(*) = \int \frac{1}{x-1}\,dx + \int \frac{-x-2}{x^2+x+1}\,dx = \int \frac{1}{x-1}\,dx - \int \frac{x+2}{x^2+x+1}\,dx =$$

La hacemos a parte

$$\int \frac{x+2}{x^2+x+1}\,dx = \frac{1}{2}\int \frac{2x+1}{x^2+x+1}\,dx + \int \frac{2-1\cdot\frac{1}{2}}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx =$$

$$= \frac{1}{2}\ln|x^2+x+1| + \int \frac{\frac{3}{2}\,dx}{\frac{3}{4}\cdot\frac{4}{3}\left[\frac{3}{4}+\left(x+\frac{1}{2}\right)^2\right]} =$$

$$= \frac{1}{2}\ln|x^2+x+1| + \frac{4}{\cancel{3}}\int \frac{\frac{\cancel{3}}{2}\,dx}{1+\left(\frac{2}{\sqrt{3}}(x+\frac{1}{2})\right)^2} = \frac{1}{2}\ln|x^2+x+1| + 2\cdot\frac{\sqrt{3}}{2}\int \frac{1\cdot\frac{2}{\sqrt{3}}\,dx}{1+\left[\frac{2}{\sqrt{3}}(x+\frac{1}{2})\right]^2}$$

$$= \frac{1}{2}\ln|x^2+x+1| + \sqrt{3}\cdot \mathrm{arctg}\left[\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right]$$

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$\Rightarrow$  $= \ln|x-1| - \frac{1}{2}\ln|x^2+x+1| - \sqrt{3}\cdot \operatorname{arctg}\left[\frac{2}{\sqrt{3}}(x+1/2)\right] + C'$

185) $\displaystyle\int \frac{5x^2-2x+25}{x^3-6x^2+25x}\,dx = (*)$

$x^3-6x^2+25x = 0 \Rightarrow x(x^2-6x+25)=0 \begin{cases} x=0 \\ x^2-6x+25=0 \end{cases}$

$x^2-6x+25=0 \rightarrow x = \frac{6\pm\sqrt{36-100}}{2} = \frac{6\pm 8i}{2} = 3\pm 4i$

$\frac{5x^2-2x+25}{\cancel{x^3-6x^2+25x}} = \frac{A}{x} + \frac{Mx+N}{x^2-6x+25} = \frac{A(x^2-6x+25)+Mx^2+Nx}{\cancel{x(x^2-6x+25)}}$

$\Rightarrow 5x^2-2x+25 = A(x^2-6x+25)+Mx^2+Nx$

Si $x=0 \longrightarrow 25 = 25A \Rightarrow A=1$

Si $x=1 \longrightarrow 28 = 20A+M+N \longrightarrow 8 = M+N$

Si $x=-1 \longrightarrow 32 = 32A+M-N \longrightarrow 0 = M-N$

$8 = 2M \Rightarrow M=4$

$\Downarrow$

$N=4$

$(*) = \displaystyle\int \frac{1}{x}\,dx + \boxed{\int \frac{4x+4}{x^2-6x+25}\,dx} =$ 

La hacemos a parte!

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$$\int \frac{4x+4}{x^2-6x+25}\,dx = \int \frac{4x}{x^2-6x+25}\,dx + \int \frac{4\,dx}{(x-3)^2+16} =$$

$$= \frac{4}{2}\int \frac{2x-6}{x^2-6x+25}\,dx + \int \frac{4+6\cdot 2}{(x-3)^2+16}\,dx =$$

$$= 2\int \frac{2x-6}{x^2-6x+25}\,dx + 16\cdot\int \frac{1\quad dx}{\frac{16}{16}\left[16+(x-3)^2\right]} =$$

$$= 2\ln|x^2-6x+25| + 4\cdot\int \frac{1\cdot 1/4\,dx}{1+\left[\frac{x-3}{4}\right]^2} =$$

$$= 2\ln|x^2-6x+25| + 4\cdot \operatorname{arctg}\left(\frac{x-3}{4}\right)$$

$\Rightarrow$  $= \ln x + 2\ln|x^2-6x+25| + 4\operatorname{arctg}\left(\frac{x-3}{4}\right) + C'$

186) $\int (1-x)^2 dx = (-1)\cdot\int (-1)\cdot(1-x)^2 dx = -\frac{(1-x)^3}{3} + C$

187) $\int (1-x^2)^2 dx = \int (1-2x^2+x^4)\,dx = x - 2\cdot\frac{x^3}{3} + \frac{x^5}{5} + C$

188) $\int 2x\sqrt{x^2+3}\,dx = \int 2x\cdot(x^2+3)^{1/2}dx = \frac{(x^2+3)^{3/2}}{3/2} = \frac{2}{3}\sqrt{(x^2+3)^3} + C$

189) $\int x\,(x^2+3)\,dx = \frac{1}{2}\int 2x\,(x^2+3)\,dx = \frac{1}{2}\cdot\frac{(x^2+3)^2}{2} = \frac{(x^2+3)^2}{4} + C$

190) $\int (x-2)(x^2-4x+1)^3 dx = \frac{1}{2}\int 2(x-2)(x^2-4x+1)^3 dx = \frac{1}{2}\cdot\frac{(x^2-4x+1)^4}{4} + C$

191) $\int \sqrt{2x-1}\,dx = \int (2x-1)^{1/2}dx = \frac{1}{2}\int 2\cdot(2x-1)^{1/2}dx = \frac{1}{\cancel{2}}\,\frac{(2x-1)^{3/2}}{3/\cancel{2}} =$

$= \frac{1}{3}\sqrt{(2x-1)^3} + C$

192) $\int x\sqrt{x^2-2}\,dx = \int x\cdot(x^2-2)^{1/2}dx = \frac{1}{2}\int 2x\cdot(x^2-2)^{1/2}dx =$

$= \frac{1}{\cancel{2}}\cdot\frac{(x^2-2)^{3/2}}{3/\cancel{2}} = \frac{1}{3}\sqrt{(x^2-2)^3} + C$

193) $\int e^{2x}dx = \frac{1}{2}\int 2e^{2x}dx = \frac{1}{2}\cdot e^{2x} + C$

194) $\int (x+1)\cdot e^{x^2+2x-1}\cdot dx = \frac{1}{2}\int 2(x+1)\cdot e^{x^2+2x-1}dx = \frac{1}{2}e^{x^2+2x-1} + C$

195) $\int \frac{4x}{2x^2+1}\,dx = \ln|2x^2+1| + C$

196) $\int \frac{x^2}{x^3+3}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3+3}\,dx = \frac{1}{3}\ln|x^3+3| + C$

197) $\int \frac{x+1}{x^2+2x}\,dx = \frac{1}{2}\int \frac{2(x+1)}{x^2+2x}\,dx = \frac{1}{2}\ln|x^2+2| + C$

198) $\int \frac{5x}{1-x^2}\,dx = \frac{-5}{2}\int \frac{-2x}{1-x^2}\,dx = \frac{-5}{2}\cdot \ln|1-x^2| + C$

199) $\int \frac{4x^3}{x^4+2}\,dx = \ln|x^4+2| + C$

200) $\int \frac{4x^3}{(x^4+2)^2}\,dx = \int 4x^3\cdot(x^4+2)^{-2}\,dx = \frac{(x^4+2)^{-1}}{-1} = \frac{-1}{x^4+2} + C$

201) $\int \frac{4x^3}{\sqrt[3]{x^4+2}}\,dx = \int 4x^3\cdot(x^4+2)^{-1/3}\,dx = \frac{(x^4+2)^{2/3}}{2/3} = \frac{3\sqrt[3]{(x^4+2)^2}}{2} + C$

202) $\int \frac{x-1}{\sqrt{x^2-2x}}\,dx = \int (x-1)\cdot(x^2-2x)^{-1/2}\,dx = \frac{1}{2}\int 2(x-1)\cdot(x^2-2x)^{-1/2}\,dx =$

$= \frac{\cancel{1}}{\cancel{2}}\cdot\frac{(x^2-2x)^{1/2}}{\cancel{1/2}} = \sqrt{(x^2-2x)} + C$

203) $\int 3^{x/2} dx = 2 \cdot \int \frac{1}{2} \cdot 3^{x/2} dx = \frac{2 \cdot 3^{x/2}}{\ln 3} + C$

204) $\int e^{x+1} dx = e^{x+1} + C$

205) $\int \left(\frac{1}{2}\right)^{4x} dx = \frac{1}{4} \int 4 \cdot \left(\frac{1}{2}\right)^{4x} dx = \frac{1}{4} \cdot \left(\frac{1}{2}\right)^{4x} \cdot \frac{1}{\ln(1/2)} =$

$= \frac{-1}{\ln 2 \cdot 2^{4x+2}} + C$

206) $\int (e^{-3x} + e^{x-2}) dx = \int e^{-3x} dx + \int e^{x-2} dx =$

$= -\frac{1}{3} \cdot \int -3 \cdot e^{-3x} dx + \int e^{x-2} dx = -\frac{1}{3} e^{-3x} + e^{x-2} + C$

207) $\int 7^{x^2+1} \cdot 2x \cdot dx = \frac{7^{x^2+1}}{\ln 7} + C$

208) $\int 5 \cdot e^{\frac{x}{2}+2} dx = 5 \cdot 2 \int \frac{1}{2} e^{x/2+2} dx = 10 \cdot e^{x/2+2} + C$

209) $\int \frac{3^{5x-1}}{7} dx = \frac{1}{7} \cdot \frac{1}{5} \int 5 \cdot 3^{5x-1} dx = \frac{1}{35} \cdot \frac{3^{5x-1}}{\ln 3} + C$

210) $\int \frac{x}{e^{x^2}} dx = \int x \cdot e^{-x^2} dx = -\frac{1}{2} \int -2x \cdot e^{-x^2} dx = -\frac{1}{2} \cdot e^{-x^2} + C$

211) $\int \operatorname{sen}(2x)\,dx = \frac{1}{2}\int 2\operatorname{sen}(2x)\,dx = -\frac{1}{2}\cos(2x) + C$

212) $\int \cos(x+1)\,dx = \operatorname{sen}(x+1) + C$

213) $\int \frac{\operatorname{sen}\left(\frac{x}{2}\right)}{2}\,dx = -\cos\left(\frac{x}{2}\right) + C$

214) $\int \operatorname{sen}(-x)\,dx = -1\cdot\int -1\operatorname{sen}(-x)\,dx = +\cos(-x) + C$

215) $\int \frac{1}{\cos^2(x+1)}\,dx = \operatorname{tg}(x+1) + C$

216) $\int -3\cdot\operatorname{sen}(2x+1)\,dx = -3\cdot\frac{1}{2}\int 2\operatorname{sen}(2x+1)\,dx = +\frac{3}{2}\cos(2x+1)+C$

217) $\int (x+1)\cos(x^2+2x)\,dx = \frac{1}{2}\int 2(x+1)\cdot\cos(x^2+2x)\,dx = \frac{1}{2}\operatorname{sen}(x^2+2x) + C$

218) $\int \frac{x}{\cos^2(x^2-3)}\,dx = \frac{1}{2}\int \frac{2x\,dx}{\cos^2(x^2-3)} = \frac{1}{2}\operatorname{tg}(x^2-3) + C$

219) $\int \frac{1}{\sqrt{1-25x^2}}\,dx = \frac{1}{5}\int \frac{5}{\sqrt{1-(5x)^2}}\,dx = \frac{1}{5}\cdot\operatorname{arcsen}(5x) + C$

220) $\int \frac{1}{\sqrt{1-(2x-3)^2}}\,dx = \frac{1}{2}\int \frac{2\,dx}{\sqrt{1-(2x-3)^2}} = \frac{1}{2}\operatorname{arcsen}(2x-3) + C$

221) $\int \frac{x}{1+(x-3)^2}dx = \frac{1}{2}\int \frac{2x-6+6}{x^2-6x+10}dx = ⊛$

$1+(x-3)^2 = 1+x^2-6x+9 = x^2-6x+10$

$⊛ = \frac{1}{2}\int \frac{2x-6}{x^2-6x+10}dx + \frac{1}{2}\int \frac{6}{1+(x-3)^2}dx =$

$= \frac{1}{2}\ln|x^2-6x+10| + 3\operatorname{arctg}(x-3) + C'$

222) $\int \frac{x}{1+9x^4}dx = \frac{1}{6}\int \frac{6x}{1+(3x^2)^2}dx = \frac{1}{6}\operatorname{arctg}(3x^2) + C'$

223) $\int \underbrace{(x^2+x)}_{u}\cdot \underbrace{e^{-2x+1}dx}_{dv} = -\frac{1}{2}(x^2+x)\cdot e^{-2x+1} + \int \underbrace{\left(x+\frac{1}{2}\right)}_{u}\cdot \underbrace{e^{-2x+1}dx}_{dv} = ⊛$

$u = x^2+x \quad du = (2x+1)dx$ || $u = x+1/2 \quad du = dx$

$dv = e^{-2x+1}dx \quad v = -\frac{1}{2}e^{-2x+1}$ || $dv = e^{-2x+1}dx \quad v = -\frac{1}{2}e^{-2x+1}$

$⊛ = -\frac{1}{2}(x^2+x)\cdot e^{-2x+1} - \frac{1}{2}(x+1/2)\cdot e^{-2x+1} + \int \frac{1}{2}e^{-2x+1}dx =$

$= -\frac{1}{2}(x^2+x)\cdot e^{-2x+1} - \frac{1}{2}(x+1/2)\cdot e^{-2x+1} + \frac{1}{2}\cdot\frac{-1}{2}\int -2e^{-2x+1}dx =$

$= -\frac{1}{2}(x^2+x)\cdot e^{-2x+1} - \frac{1}{2}(x+1/2)\cdot e^{-2x+1} - \frac{1}{4}e^{-2x+1} =$

$= -\frac{1}{2}e^{-2x+1}\cdot\left[x^2+x+x+\frac{1}{2}+\frac{1}{2}\right] = -\frac{1}{2}e^{-2x+1}(x^2+2x+1) + C'$

224) $\int \underbrace{x^2}_{u} \cdot \underbrace{\cos(3x)\,dx}_{dv} = \frac{1}{3}x^2 \operatorname{sen}(3x) - \int \underbrace{\frac{2}{3}x}_{u} \underbrace{\operatorname{sen}(3x)dx}_{dv} = \circledast$

$u = x^2 \qquad du = 2x\,dx$

$dv = \cos(3x)dx \qquad v = \frac{1}{3}\operatorname{sen}(3x)$

$u = \frac{2}{3}x \qquad du = \frac{2}{3}dx$

$dv = \operatorname{sen}(3x)dx \qquad v = -\frac{1}{3}\cos(3x)$

$\circledast = \frac{1}{3}x^2\operatorname{sen}(3x) - \left[-\frac{2}{9}x\cos(3x) + \int \frac{2}{9}\cos(3x)dx\right] =$

$= \frac{1}{3}x^2\operatorname{sen}(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{9}\cdot\frac{1}{3}\int 3\cos(3x)dx =$

$= \frac{1}{3}x^2\operatorname{sen}(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{27}\operatorname{sen}(3x) + C$

225) $\int \underbrace{2x^2}_{dv} \cdot \underbrace{\ln x}_{u}\, \underbrace{dx}_{dv} = \frac{2x^3}{3}\ln x - \int \frac{2x^{\cancel{3}\,2}}{3}\cdot\frac{1}{\cancel{x}}dx =$

$u = \ln x \qquad du = \frac{1}{x}dx$

$dv = 2x^2dx \qquad v = 2\cdot\frac{x^3}{3}$

$= \frac{2x^3}{3}\ln x - \frac{2}{3}\int x^2dx = \frac{2x^3}{3}\ln x - \frac{2x^3}{9} + C$

226) $\int \underbrace{x^2}_{u} \cdot \underbrace{2^x \cdot dx}_{dv} = \frac{x^2 \cdot 2^x}{\ln 2} - \int \underbrace{\frac{2x}{\ln 2}}_{u} \cdot \underbrace{2^x dx}_{dv} = \circledast$

$u = x^2 \qquad du = 2x\,dx$

$dv = 2^x dx \qquad v = \frac{2^x}{\ln 2}$

$u = \frac{2x}{\ln 2} \qquad du = \frac{2}{\ln 2}dx$

$dv = 2^x dx \qquad v = \frac{2^x}{\ln 2}$

$$\circledast = \frac{x^2 \cdot 2^x}{\ln 2} - \left[\frac{2x \cdot 2^x}{(\ln 2)^2} - \int \frac{2 \cdot 2^x}{(\ln 2)^2} dx \right] =$$

$$= \frac{x^2 \cdot 2^x}{\ln 2} - \frac{2x \cdot 2^x}{(\ln 2)^2} + \frac{2}{(\ln 2)^2} \cdot \int 2^x dx =$$

$$= \frac{x^2 \cdot 2^x}{\ln 2} - \frac{2x \cdot 2^x}{(\ln 2)^2} + \frac{2 \cdot 2^x}{(\ln 2)^3} + C$$

227) $\displaystyle\int \frac{2}{x^2-1} dx = \circledast$

$$x^2 - 1 = 0 \begin{cases} x = 1 \\ x = -1 \end{cases} \Rightarrow \frac{2}{\cancel{x^2-1}} = \frac{A}{(x-1)} + \frac{B}{(x+1)} = \frac{A(x+1)+B(x-1)}{\cancel{(x-1)(x+1)}}$$

$$\Rightarrow 2 = A(x+1) + B(x-1)$$

Si $x = 1 \longrightarrow 2 = 2A \Rightarrow A = 1$

Si $x = -1 \longrightarrow 2 = -2B \Rightarrow B = -1$

$$\circledast = \int \frac{1}{x-1} dx + \int \frac{-1}{x+1} dx = \ln|x-1| - \ln|x+1| + C' =$$

$$= \ln\left(\left|\frac{x-1}{x+1}\right|\right) + C'$$

228) $\int \frac{-3}{x^2+x-2}\,dx = ⊛$

$x^2+x-2=0 \rightarrow x = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2}$ → $x=1$; → $x=-2$

$\frac{-3}{\cancel{x^2+x-2}} = \frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2)+B(x-1)}{\cancel{(x-1)(x+2)}}$

$\Longrightarrow -3 = A(x+2)+B(x-1)$

Si $x=1 \longrightarrow -3 = 3A \Rightarrow A=-1$

Si $x=-2 \longrightarrow -3 = -3B \Rightarrow B=1$

$⊛ = \int \frac{-1}{x-1}\,dx + \int \frac{1}{x+2}\,dx = -1 \cdot \ln|x-1| + \ln|x+2| + C =$

$= \ln\left(\frac{x+2}{x-1}\right) + C$

229) $\int \frac{2x+1}{x^4-5x^2+4}\,dx = ⊛$

$x^4-5x^2+4 = 0 \longrightarrow t^2-5t+4=0$

$\left.\begin{array}{l} x^2 = t \\ x^4 = (x^2)^2 = t^2 \end{array}\right\}$

$t = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2}$ → $t=4$; → $t=1$

$t=4$ → $x=2$; → $x=-2$; $t=1$ → $x=1$; → $x=-1$

$$\frac{2x+1}{x^4-5x^2+4} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x-1} + \frac{D}{x+1} =$$

$$= \frac{A(x+2)(x-1)(x+1) + B(x-2)(x-1)(x+1) + C(x-2)(x+2)(x+1) + D(x-2)(x+2)(x-1)}{(x-2)(x+2)(x-1)(x+1)}$$

$$\Rightarrow 2x+1 = A(x^2-1)(x+2) + B(x^2-1)(x-2) + C(x^2-4)(x+1) + D(x^2-4)(x-1)$$

Si $x=2 \longrightarrow 5 = 12A \Rightarrow A = 5/12$

Si $x=-2 \longrightarrow -3 = -12B \Rightarrow B = 1/4$

Si $x=1 \longrightarrow 3 = -6C \Rightarrow C = -1/2$

Si $x=-1 \longrightarrow -1 = 6D \Rightarrow D = -1/6$

$$\circledast = \int \frac{5/12}{x-2}\,dx + \int \frac{1/4}{x+2}\,dx + \int \frac{-1/2}{x-1}\,dx + \int \frac{-1/6}{x+1}\,dx =$$

$$= \frac{5}{12}\ln|x-2| + \frac{1}{4}\ln|x+2| - \frac{1}{2}\ln|x-1| - \frac{1}{6}\ln|x+1| + C$$

230) $\displaystyle\int \frac{7x-2}{x^3-2x^2-x+2}\,dx = \circledast$

$x^3-2x^2-x+2=0$

$$\begin{array}{r|rrrr} & 1 & -2 & -1 & 2 \\ 1 & & 1 & -1 & -2 \\ \hline & 1 & -1 & -2 & \underline{|0} \end{array}$$

$$x^2 - x - 2 = 0 \; ; \quad x = \frac{1 \pm \sqrt{1 + 4 \cdot 2}}{2} = \frac{1 \pm 3}{2} \begin{cases} \to x = 2 \\ \to x = -1 \end{cases}$$

$$\frac{7x-2}{\cancel{x^3 - 2x^2 - x + 2}} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1} = \frac{A(x-2)(x+1) + B(x-1)(x+1) + C(x-1)(x-2)}{\cancel{(x-1)(x-2)(x+1)}}$$

$$\Rightarrow 7x - 2 = A(x-2)(x+1) + B(x^2-1) + C(x-1)(x-2)$$

Si $x = 1 \longrightarrow 5 = -2A \Rightarrow A = -5/2$

Si $x = 2 \longrightarrow 12 = 3B \Rightarrow B = 4$

Si $x = -1 \longrightarrow -9 = 6C \Rightarrow C = -3/2$

$$(*) = \int \frac{-5/2}{x-1}\,dx + \int \frac{4}{x-2}\,dx + \int \frac{-3/2}{x+1}\,dx =$$

$$= -\frac{5}{2} \ln|x-1| + 4 \ln|x-2| - 3/2 \ln|x+1| + C$$

231) $\displaystyle\int \frac{x^2}{(x-1)^3}\,dx = (*)$

$$\frac{x^2}{\cancel{(x-1)^3}} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} = \frac{A(x-1)^2 + B(x-1) + C}{\cancel{(x-1)^3}}$$

$$\Rightarrow x^2 = A(x-1)^2 + B(x-1) + C$$

Si $x = 1 \longrightarrow 1 = C$

Si $x = 0 \longrightarrow 0 = A - B + 1$

Si $x = 2 \longrightarrow 4 = A + B + 1$

$$\left.\begin{array}{l} A - B = -1 \\ A + B = 3 \end{array}\right\}$$

$2A = 2 \Rightarrow A = 1 \Rightarrow B = 2$

$$(*) = \int \frac{1}{x-1}\,dx + \int \frac{2}{(x-1)^2}\,dx + \int \frac{1}{(x-1)^3}\,dx =$$

$$= \ln|x-1| + 2\cdot\int (x-1)^{-2}dx + \int (x-1)^{-3}dx =$$

$$= \ln|x-1| + 2\cdot\frac{(x-1)^{-1}}{-1} + \frac{(x-1)^{-2}}{-2} =$$

$$= \ln|x-1| - \frac{2}{x-1} - \frac{2}{(x-1)^2} + C$$

232) $\displaystyle\int -\frac{3x-2}{(2-x)^2}\,dx = \int \frac{2-3x}{(2-x)^2}\,dx = (*)$

$$\frac{2-3x}{\cancel{(2-x)^2}} = \frac{A}{2-x} + \frac{B}{(2-x)^2} = \frac{A(2-x)+B}{\cancel{(2-x)^2}}$$

$$\Rightarrow 2 - 3x = A(2-x) + B$$

Si $x = 2 \longrightarrow -4 = B$

Si $x = 1 \longrightarrow -1 = A - 4 \Rightarrow A = 3$

$$(*) = \int \frac{3}{2-x}\,dx + \int \frac{-4}{(2-x)^2}\,dx =$$

$$= 3(-1)\int \frac{-1}{2-x}\,dx + 4\int (-1)\cdot(2-x)^{-2}\,dx =$$

$$= -3\ln|2-x| + 4\cdot\frac{(2-x)^{-1}}{-1} = -3\ln|2-x| - \frac{4}{2-x} + C'$$

233) $\displaystyle\int \frac{-2x^2+1}{x^3+6x^2+12x+8}\,dx = (*)$

$x^3+6x^2+12x+8=0$

	1	6	12	8
-2		-2	-8	-8
	1	4	4	0
-2		-2	-4	
	1	2	0	
-2		-2		
	1	0		

$$\frac{-2x^2+1}{\cancel{x^3+6x^2+12x+8}} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} = \frac{A(x+2)^2+B(x+2)+C}{\cancel{(x+2)^3}}$$

$$\Longrightarrow -2x^2+1 = A(x+2)^2+B(x+2)+C$$

Si $x=-2 \longrightarrow -7 = C$

Si $x=-1 \longrightarrow -1 = A+B-7$

Si $x=0 \longrightarrow 1 = 4A+2B-7$

$$\left.\begin{array}{l} A+B=6 \\ 4A+2B=8 \end{array}\right\}$$

$$\left.\begin{array}{l} -2A-2B=-12 \\ 4A+2B=8 \end{array}\right\} \quad 2A=-4 \Rightarrow A=-2 \qquad B=8$$

$$\circledast = \int \frac{-2}{x+2}\,dx + \int \frac{8}{(x+2)^2}\,dx + \int \frac{-7}{(x+2)^3}\,dx =$$

$$= -2\ln|x+2| + 8\cdot\frac{(x+2)^{-1}}{-1} - 7\cdot\frac{(x+2)^{-2}}{-2} + C' =$$

$$= -2\ln|x+2| - \frac{8}{x+2} + \frac{7}{2(x+2)^2} + C'$$

234) $\int \frac{x-2}{x^4}\,dx = \int \frac{x}{x^4}\,dx - \int \frac{2}{x^4}\,dx = \int x^{-3}\,dx - \int 2x^{-4}\,dx =$

$$= \frac{x^{-2}}{-2} - 2\frac{x^{-3}}{-3} + C' = -\frac{1}{2x^2} + \frac{2}{3x^3} + C'$$

235) $\int \frac{4x^2-2x}{(x+2)(x-3)^2}\,dx = \circledast$

$$\frac{4x^2-2x}{\cancel{(x+2)(x-3)^2}} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2} = \frac{A(x-3)^2+B(x+2)(x-3)+C(x+2)}{\cancel{(x+2)(x-3)^2}}$$

$$\Rightarrow 4x^2-2x = A(x-3)^2 + B(x+2)(x-3) + C(x+2)$$

Si $x=-2 \longrightarrow 20 = 25A \Rightarrow A = 4/5$

Si $x = 3 \longrightarrow 30 = 5C \Rightarrow C = 6$

Si $x = 0 \longrightarrow 0 = \frac{36}{5} - 6B + 12 \Rightarrow B = 16/5$

$$(*) = \int \frac{4/5}{x+2}\,dx + \int \frac{16/5}{x-3}\,dx + \int \frac{6}{(x-3)^2}\,dx =$$

$$= \frac{4}{5}\ln|x+2| + \frac{16}{5}\ln|x-3| + 6\cdot\frac{(x-3)^{-1}}{-1} + C' =$$

$$= \frac{4}{5}\ln|x+2| + \frac{16}{5}\ln|x-3| - \frac{6}{x-3} + C'$$

236) $\displaystyle\int \frac{-x^2+7x}{x^3-x^2-x+1}\,dx = (*)$

$x^3-x^2-x+1=0$

$$\begin{array}{c|cccc} & 1 & -1 & -1 & 1 \\ 1 & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & \boxed{0} \end{array}$$

$$\underbrace{1 \quad 0 \quad -1}_{x^2-1=0} \quad \begin{cases} \to x=1 \\ \to x=-1 \end{cases}$$

$$\frac{-x^2+7x}{\cancel{x^3-x^2-x+1}} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2+B(x+1)(x-1)+C(x+1)}{\cancel{(x+1)(x-1)^2}}$$

$$-x^2+7x = A(x-1)^2+B(x^2-1)+C(x+1)$$

Si $x=1 \longrightarrow 6 = 2C \Rightarrow C=3$

Si $x=-1 \longrightarrow -8 = 4A \Rightarrow A=-2$

Si $x=0 \longrightarrow 0 = -2 - B + 3 \Rightarrow B = 1$

$$(*) = \int \frac{-2}{x+1}\,dx + \int \frac{1}{x-1}\,dx + \int \frac{3}{(x-1)^2}\,dx =$$

$$= -2\ln|x+1| + \ln|x-1| - \frac{3}{x-1} + C'$$

237) $\int \frac{2}{x^2+1}\,dx = 2\operatorname{arctg}(x) + C$

238) $\int -\frac{3x-2}{2+x^2}\,dx = \int \frac{2-3x}{2+x^2}\,dx = \int \frac{2\,dx}{2+x^2} - \int \frac{3x}{2+x^2}\,dx =$

$= 2\int \frac{1\,dx}{2(2+x^2)} - 3\cdot\frac{1}{2}\int \frac{2x\,dx}{2+x^2} = \int \frac{1\,dx}{1+\frac{x^2}{2}} - \frac{3}{2}\ln|2+x^2| =$

$= \sqrt{2}\cdot\int \frac{1/\sqrt{2}}{1+\left(\frac{x}{\sqrt{2}}\right)^2}\,dx - \frac{3}{2}\ln|2+x^2| = \sqrt{2}\cdot\operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right) - \frac{3}{2}\ln|2+x^2| + C$

239) $\int \frac{-2x^2+1}{x^3-x^2+3x-3}\,dx = (*)$

$x^3 - x^2 + 3x - 3 = 0$

	1	-1	3	-3
1		1	0	3
	1	0	3	0

$\underbrace{1 \quad 0 \quad 3}$

$x^2+3 \to$ Irreducible (Imaginaria!!)

$\frac{-2x^2+1}{\cancel{x^3-x^2+3x-3}} = \frac{A}{x-1} + \frac{Mx+N}{x^2+3} = \frac{A(x^2+3)+Mx(x-1)+N(x-1)}{\cancel{(x-1)(x^2+3)}}$

$\Rightarrow -2x^2+1 = A(x^2+3) + Mx(x-1) + N(x-1)$

Si $x=1 \longrightarrow -1 = 4A \Rightarrow A = -1/4$

Si $x=0 \longrightarrow 1 = \frac{-3}{4} - N \Rightarrow N = -7/4$

Si $x=2 \longrightarrow -7 = \frac{-7}{4} + 2M \frac{-7}{4} \Rightarrow M = -7/4$

$$(*) = \int \frac{-1/4}{x-1}\,dx + \int \frac{-7/4\,x - \frac{7}{4}}{x^2+3}\,dx =$$

$$= -\frac{1}{4}\ln|x-1| - \frac{7}{4}\cdot\frac{1}{2}\int \frac{2x}{x^2+3}\,dx - \frac{7}{4}\int \frac{\frac{3}{3}\,1\,dx}{3\,(x^2+3)} =$$

$$= -\frac{1}{4}\ln|x-1| - \frac{7}{8}\ln|x^2+3| - \frac{7}{12}\cdot\sqrt{3}\int \frac{1/\sqrt{3}}{1+\left(\frac{x}{\sqrt{3}}\right)^2}\,dx =$$

$$= -\frac{1}{4}\ln|x-1| - \frac{7}{8}\ln|x^2+3| - \frac{7\sqrt{3}}{12}\cdot \operatorname{arctg}\left(\frac{x}{\sqrt{3}}\right) + C$$

240) $\displaystyle\int \frac{x-2}{x^2(x^2+1)}\,dx = (*)$

$$\frac{x-2}{\cancel{x^2(x^2+1)}} = \frac{A}{x} + \frac{B}{x^2} + \frac{Mx+N}{x^2+1} = \frac{Ax(x^2+1)+B(x^2+1)+Mx^3+Nx^2}{\cancel{x^2(x^2+1)}}$$

$$\Rightarrow x-2 = Ax(x^2+1)+B(x^2+1)+Mx^3+Nx^2$$

Si $x=0 \longrightarrow -2 = B$

Si $x=1 \longrightarrow -1 = 2A-4+M+N$
Si $x=-1 \longrightarrow -3 = -2A-4-M+N$
Si $x=2 \longrightarrow 0 = 10A-10+8M+4N$

$$\left.\begin{array}{l} 2A+M+N = 3 \\ -2A-M+N = 1 \\ 10A+8M+4N = 10 \end{array}\right\}$$

Ecuación 1 + Ecuación 2 $\Rightarrow 2N = 4 \Rightarrow N = 2$

$$\left.\begin{array}{l}\text{Ecuación } 1 \\ \text{Ecuación } 3\end{array}\right\} \Rightarrow \left.\begin{array}{l} 2A+M = 1 \\ 10A+8M = 2 \end{array}\right\} \begin{array}{r} -10A-5M = -5 \\ 10A+8M = 2 \\ \hline 3M = -3 \end{array}$$

$M=-1$

$A=1$

$$\circledast = \int \frac{1}{x}\,dx + \int \frac{-2}{x^2}\,dx + \int \frac{-x+2}{x^2+1}\,dx =$$

$$= \ln|x| - 2\cdot\frac{x^{-1}}{-1} - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx + 2\int \frac{1}{x^2+1}\,dx =$$

$$= \ln|x| + \frac{2}{x} - \frac{1}{2}\ln|x^2+1| + 2\,\mathrm{arctg}(x) + C$$

241) $\displaystyle\int \frac{2x^4}{(x-1)^3}\,dx = ⊛$

$(x-1)^3 = x^3-3x^2+3x-1$

$$
\begin{array}{rl}
2x^4+0x^3+0x^2+0x+0 & \underline{|\ x^3-3x^2+3x-1} \\
-2x^4+6x^3-6x^2+2x & \ 2x+6 \\
\hline
/ \quad 6x^3-6x^2+2x+0 & \\
-6x^3+18x^2-18x+6 & \\
\hline
/ \quad 12x^2-16x+6 &
\end{array}
$$

$⊛ = \displaystyle\int (2x+6)\,dx + \int \frac{12x^2-16x+6}{(x-1)^3}\,dx =$ 

la haremos a parte

$$\frac{12x^2-16x+6}{\cancel{(x-1)^3}} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} = \frac{A(x-1)^2+B(x-1)+C}{\cancel{(x-1)^3}}$$

$$\Rightarrow 12x^2-16x+6 = A(x-1)^2+B(x-1)+C$$

Si $x=1 \rightarrow 2 = C$

$$\left.\begin{array}{l}\text{Si } x=0 \rightarrow 6 = A-B+2 \\ \text{Si } x=2 \rightarrow 22 = A+B+2\end{array}\right\} \Rightarrow \left.\begin{array}{l} A-B=4 \\ A+B=20\end{array}\right\}$$

$2A = 24 \Rightarrow A=12$

$\Downarrow$

$B=8$

$$\int \frac{12x^2-16x+6}{(x-1)^3}\,dx = \int \frac{12}{x-1}\,dx + \int \frac{8}{(x-1)^2}\,dx + \int \frac{2}{(x-1)^3}\,dx =$$

$$= 12\ln|x-1| + 8\cdot\frac{(x-1)^{-1}}{-1} + 2\cdot\frac{(x-1)^{-2}}{-2} + C =$$

$$= 12\ln|x-1| - \frac{8}{x-1} - \frac{1}{(x-1)^2} + C$$

$$= x^2+6x+12\ln|x-1| - \frac{8}{x-1} - \frac{1}{(x-1)^2} + C$$

242) $\displaystyle\int -\frac{3x^3-2}{(2-x)^2}\,dx = \int \frac{2-3x^3}{(2-x)^2}\,dx = (*)$

$$(2-x)^2 = 4-4x+x^2$$

$$\begin{array}{l|l}
-3x^3+0x^2+0x+2 & x^2-4x+4 \\ \hline
+3x^3-12x^2+12x & -3x-12 \\ \hline
\quad -12x^2+12x+2 & \\
\quad +12x^2-48x+48 & \\ \hline
\quad\quad -36x+50 &
\end{array}$$

$$(*) = \int(-3x-12)\,dx + \int \frac{-36x+50}{(2-x)^2}\,dx =$$

La hacemos aparte

$$\frac{-36x+50}{\cancel{(2-x)^2}} = \frac{A}{2-x} + \frac{B}{(2-x)^2} = \frac{A(2-x)+B}{\cancel{(2-x)^2}}$$

$$\Rightarrow -36x+50 = A(2-x)+B$$

Si $x=2 \longrightarrow -22 = B$

Si $x=1 \longrightarrow 14 = A-22 \Rightarrow A = 36$

$$\int \frac{-36x+50}{(2-x)^2}\,dx = \int \frac{36}{2-x}\,dx + \int \frac{-22}{(2-x)^2}\,dx =$$

$$= (-1)\cdot 36 \int \frac{-1}{2-x}\,dx + 22\int(-1)\cdot(2-x)^{-2}\,dx =$$

$$= -36\ln|2-x| + 22\cdot\frac{(2-x)^{-1}}{-1} + C'$$

$$= -3\frac{x^2}{2} - 12x - 36\ln|2-x| - \frac{22}{2-x} + C'$$

243) $\displaystyle\int \frac{-2x^5+1}{x^4-2x^2+1}\,dx = (*)$

$x^4-2x^2+1=0 \longrightarrow t^2-2t+1=0$

$\left.\begin{array}{l} x^2=t \\ x^4=(x^2)^2=t^2 \end{array}\right\}$

$t = \dfrac{2\pm\sqrt{4-4}}{2} = 1 \begin{cases} \rightarrow x = 1 \text{ (Doble)} \\ \rightarrow x = -1 \text{ (Doble)} \end{cases}$

$$\begin{array}{rl}
-\cancel{2x^5}+0x^4+0x^3+0x^2+0x+1 & \underline{|\,x^4-2x^2+1} \\
+\cancel{2x^5}\qquad\quad -4x^3\qquad\quad +2x & -2x \\
\hline
-4x^3+2x+1 &
\end{array}$$

$$(*) = \int (-2x)\,dx + \int \frac{-4x^3+2x+1}{(x-1)^2(x+1)^2}\,dx =$$

La hacemos a parte

$$\frac{-4x^3+2x+1}{\cancel{(x-1)^2(x+1)^2}} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} =$$

$$= \frac{A(x-1)(x+1)^2 + B(x+1)^2 + C(x-1)^2(x+1) + D(x-1)^2}{\cancel{(x-1)^2(x+1)^2}}$$

$$\Longrightarrow -4x^3+2x+1 = A(x-1)(x+1)^2 + B(x+1)^2 + C(x-1)^2(x+1) + D(x-1)^2$$

Si $x = 1 \longrightarrow -1 = 4B \Rightarrow B = -1/4$

Si $x = -1 \longrightarrow 3 = 4D \Rightarrow D = 3/4$

Si $x = 0 \longrightarrow 1 = -A - 1/4 + C + \frac{3}{4}$

Si $x = 2 \longrightarrow -27 = 9A - 9/4 + 3C + \frac{3}{4}$

$$\left.\begin{array}{l} -A + C = 1/2 \\ 9A + 3C = -\frac{51}{2} \end{array}\right\}$$

$$\left.\begin{array}{l} -9A + 9C = 9/2 \\ 9A + 3C = -51/2 \end{array}\right\} 12C = -21 \Rightarrow C = \frac{-21}{12} \Rightarrow A = -9/4$$

$$\int \frac{-4x^3+2x+1}{(x-1)^2\cdot(x+1)^2}\,dx = \int \frac{-9/4}{x-1}\,dx + \int \frac{-1/4}{(x-1)^2}\,dx + \int \frac{-21/12}{(x+1)}\,dx + \int \frac{3/4}{(x+1)^2}\,dx$$

$$= -\frac{9}{4}\ln|x-1| - \frac{1}{4}\frac{(x-1)^{-1}}{-1} - \frac{21}{12}\ln|x+1| + \frac{3}{4}\frac{(x+1)^{-1}}{-1} =$$

$$= -\frac{9}{4}\ln|x-1| + \frac{1}{4(x-1)} - \frac{21}{12}\ln|x+1| - \frac{3}{4(x+1)}$$

$$= -x^2 - \frac{9}{4}\ln|x-1| - \frac{21}{12}\ln|x+1| + \frac{-2x+4}{4(x^2-1)} + C$$

244) $$\int \frac{x^6-1}{x^2(x^2+1)(x-1)}\,dx \overset{\Delta}{=} \int \frac{\cancel{(x-1)}(x^5+x^4+x^3+x^2+x+1)}{x^2(x^2+1)\cancel{(x-1)}}\,dx =$$

$$\begin{array}{c|ccccccc} & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 1 & \boxed{0} \end{array}$$

$$= \int \frac{x^5+x^4+x^3+x^2+x+1}{x^4+x^2}\,dx = (*)$$

$$\begin{array}{l|l} x^5+x^4+x^3+x^2+x+1 & x^4+x^2 \\ \hline -x^5 \qquad -x^3 & x+1 \\ \quad x^4 \qquad\; +x^2+x+1 & \\ -x^4 \qquad -x^2 & \\ \hline \qquad\qquad\quad x+1 & \end{array}$$

$$(*) = \int (x+1)\,dx + \int \frac{x+1}{x^2(x^2+1)}\,dx =$$

La hacemos a parte

$$\frac{x+1}{\cancel{x^2(x^2+1)}} = \frac{A}{x} + \frac{B}{x^2} + \frac{Mx+N}{x^2+1} = \frac{Ax(x^2+1)+B(x^2+1)+Mx^3+Nx^2}{\cancel{x^2(x^2+1)}}$$

$$\Longrightarrow x+1 = Ax(x^2+1)+B(x^2+1)+Mx^3+Nx^2$$

Si $x=0 \longrightarrow 1 = B$

$$\left.\begin{array}{l} \text{Si } x=1 \longrightarrow 2 = 2A+2+M+N \\ \text{Si } x=-1 \longrightarrow 0 = -2A+2-M+N \\ \text{Si } x=2 \longrightarrow 3 = 10A+5+8M+4N \end{array}\right\} \left\{\begin{array}{l} 2A+M+N = 0 \\ -2A-M+N = -2 \\ 10A+8M+4N = -2 \end{array}\right\}$$

$$\left.\begin{array}{c} \text{Ecuación 1} \\ + \\ \text{Ecuación 2} \end{array}\right\} \Rightarrow 2N = -2 \Rightarrow N = -1$$

$$\left.\begin{array}{l} \text{Ecuación 1} \\ \text{Ecuación 3} \end{array}\right\} \begin{array}{l} 2A+M = 1 \\ 10A+8M = 2 \end{array} \left\{\begin{array}{r} -10A-5M = -5 \\ 10A+8M = 2 \\ \hline 3M = -3 \Rightarrow M = -1 \end{array}\right.$$

$$A = 1$$

$$\int \frac{x+1}{x^2(x^2+1)}\,dx = \int \frac{1}{x}\,dx + \int \frac{1}{x^2}\,dx + \int \frac{-x-1}{x^2+1}\,dx =$$

$$= \ln|x| - \frac{1}{x} - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx =$$

$$= \ln|x| - \frac{1}{x} - \frac{1}{2}\ln|x^2+1| - \operatorname{arctg}(x)$$

 $= \frac{x^2}{2} + x + \ln|x| - \frac{1}{x} - \frac{1}{2}\ln|x^2+1| - \operatorname{arctg}(x) + C$

245) $\int x \cdot 2^{x^2-3}\,dx = \frac{1}{2}\int 2x \cdot 2^{x^2-3}\,dx = \frac{1}{2}\cdot\frac{2^{x^2-3}}{\ln 2} = \frac{2^{x^2-4}}{\ln 2} + C$

246) $\int \frac{\ln^3 x}{2x}\,dx = \frac{1}{2}\int \frac{1}{x}\cdot(\ln x)^3\,dx = \frac{1}{2}\cdot\frac{(\ln x)^4}{4} + C$

247) $\int \underbrace{x \cdot \underbrace{\ln(1+x^2)}_{u}\,dx}_{dv} = \frac{x^2}{2}\ln(1+x^2) - \int \frac{x^3}{1+x^2}\,dx = ⊛$

$u = \ln(1+x^2) \quad du = \frac{2x}{1+x^2}\,dx$

$dv = x \cdot dx \quad v = \frac{x^2}{2}$

$$\begin{array}{r|l} x^3 & x^2+1 \\ \cline{2-2} -x^3 - x & x \\ \hline -x & \end{array}$$

$⊛ = \frac{x^2}{2}\ln(1+x^2) - \left[\int x\,dx - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx\right] =$

$= \frac{x^2}{2}\ln(1+x^2) - \frac{x^2}{2} + \frac{1}{2}\ln(1+x^2) + C$

248) $\int \frac{1}{e^x+e^{-x}}\,dx = \int \frac{1}{e^x+\frac{1}{e^x}}\,dx = \int \frac{1}{\frac{(e^x)^2+1}{e^x}}\,dx$

$= \int \frac{e^x}{1+(e^x)^2}\,dx = \operatorname{arctg}(e^x) + C'$

249) $\int \frac{x^2+2}{\sqrt{x^3+6x}}\,dx = \frac{1}{3}\int 3(x^2+2)\cdot(x^3+6x)^{-1/3}\,dx = \frac{1}{\cancel{3}}\cdot\frac{(x^3+6x)^{2/3}}{2/\cancel{3}} + C' =$

$= \frac{1}{2}\sqrt[3]{(x^3+6x)^2} + C'$

250) $\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\int \frac{2x}{1+(x^2)^2}\,dx = \frac{1}{2}\operatorname{arctg}(x^2) + C'$

251) $\int \frac{\operatorname{arctg}(x)}{1+x^2}\,dx = \int \frac{1}{1+x^2}\cdot \operatorname{arctg}(x)\,dx = \frac{(\operatorname{arctg} x)^2}{2} + C'$

252) $\int \frac{dx}{(\operatorname{arcsen} x)^5\cdot\sqrt{1+x^2}} = \int \frac{1}{\sqrt{1+x^2}}\cdot(\operatorname{arcsen} x)^{-5}\,dx = \frac{(\operatorname{arcsen} x)^{-4}}{-4} + C'$

253) $\int \operatorname{sen}^5 x \cdot \cos^2 x \, dx = \int \operatorname{sen} x \cdot \operatorname{sen}^2 x \cdot \operatorname{sen}^2 x \cdot \cos^2 x \, dx =$

$$= \int \operatorname{sen} x \cdot (1-\cos^2 x) \cdot (1-\cos^2 x) \cdot \cos^2 x \, dx =$$

$$= \int \operatorname{sen} x \cdot (1 - 2\cos^2 x + \cos^4 x) \cdot \cos^2 x \, dx =$$

$$= -\int -\operatorname{sen} x \cdot \cos^2 x \, dx + 2\int -\operatorname{sen} x \cdot \cos^4 x \, dx + (-1)\int -\operatorname{sen} x \cdot \cos^6 x \, dx =$$

$$= -\frac{\cos^3 x}{3} + 2\frac{\cos^5 x}{5} - \frac{\cos^7 x}{7} + C$$

254) $\int \sqrt{4-x^2}\, dx = \int \sqrt{4-(2\operatorname{sen} t)^2} \cdot 2\cos t \, dt =$

$x = 2\operatorname{sen} t$

$dx = 2\cos t \, dt$

$$= \int \sqrt{4(1-\operatorname{sen}^2 t)} \cdot 2\cos t \, dt =$$

$$= \int 2\cancel{\sqrt{\cos^2 t}} \cdot 2 \cdot \cos t \, dt = \int 4\cos^2 t \, dt = \int (2 + 2\cos(2t))\, dt =$$

$\cos^2 t + \operatorname{sen}^2 t = 1$

$\cos^2 t - \operatorname{sen}^2 t = \cos(2t)$

$2\cos^2 t = 1 + \cos(2t)$

$$= 2t + \operatorname{sen}(2t) = 2t + 2\operatorname{sen} t \cos t = 2\arcsin\left(\frac{x}{2}\right) + x \cdot \sqrt{1-\frac{x^2}{4}} =$$

$\operatorname{sen}(2t) = 2\operatorname{sen} t \cos t$

$$= 2\arcsin\left(\frac{x}{2}\right) + \frac{x}{2} \cdot \sqrt{4-x^2} + C$$

255) $\int \operatorname{sen}^2 x \, dx = \int \frac{1-\cos(2x)}{2} dx = \int \frac{1}{2} dx - \frac{1}{2}\cdot\frac{1}{2}\int 2\cos(2x) dx$

$$\cos^2 x + \operatorname{sen}^2 x = 1$$
$$\cos^2 x - \operatorname{sen}^2 x = \cos(2x)$$

Restando: $2\operatorname{sen}^2 x = 1 - \cos(2x)$

$\Downarrow$

$$\operatorname{sen}^2 x = \frac{1-\cos(2x)}{2}$$

$$= \frac{x}{2} - \frac{1}{4}\operatorname{sen}(2x) + C$$

256) $\int \frac{\sqrt{2-x^2}}{4} dx = \frac{1}{4}\int \sqrt{2-x^2}\, dx = \frac{1}{4}\int \sqrt{2-2\operatorname{sen}^2 t}\cdot\sqrt{2}\cdot\cos t\, dt$

$$\left.\begin{aligned} x &= \sqrt{2}\operatorname{sen} t \\ dx &= \sqrt{2}\cdot\cos t\, dt \end{aligned}\right\}$$

$$= \frac{1}{4}\int \sqrt{2}\cdot\sqrt{1-\operatorname{sen}^2 t}\cdot\sqrt{2}\cdot\cos t\, dt = \frac{1}{4}\int 2\cos^2 t\, dt = \frac{1}{2}\int \cos^2 t\, dt =$$

$$\cos^2 t + \operatorname{sen}^2 t = 1$$
$$\cos^2 t - \operatorname{sen}^2 t = \cos(2t)$$
$$\cos^2 t = \frac{1+\cos(2t)}{2}$$

$$= \frac{1}{2}\int \frac{1+\cos(2t)}{2} dt =$$

$$= \frac{1}{4}\left[t + \frac{1}{2}\operatorname{sen}(2t)\right] = \frac{1}{4}\left[t + \cancel{\frac{1}{2}}\,\cancel{2}\operatorname{sen} t\cos t\right] =$$

$$= \frac{1}{4}\left[\operatorname{arcsen}\left(\frac{x}{\sqrt{2}}\right) + \frac{x}{\sqrt{2}}\cdot\sqrt{1-\frac{x^2}{2}}\right] + C$$

257) $\int (2x-3)\,dx = x^2+3x+C$

258) $\int (3x^2+4x-2)\,dx = x^3+2x^2-2x+C$

259) $\int \left(\frac{3}{4}x^3-3x^2+6x-1\right)dx = \frac{3x^4}{16}-x^3+3x^2-x+C$

260) $\int \frac{7}{\operatorname{sen}^2(3x)}\,dx = 7\cdot\frac{-1}{3}\int -3\operatorname{cosec}^2(3x)\,dx = \frac{-7}{3}\operatorname{cotg}(3x)+C$

Ojo! $\rightarrow \frac{1}{\operatorname{sen}^2 x} = \operatorname{cosec}(x)$; $f(x)=\operatorname{cotg}(x) \rightarrow f'(x) = -\operatorname{cosec}^2(x)$

261) $\int 3\sec^2\left(\frac{1}{5}x\right)dx = 3\cdot 5\int \frac{1}{5}\sec^2\left(\frac{1}{5}x\right)dx = 15\operatorname{tg}\left(\frac{1}{5}x\right)+C$

Ojo! $\rightarrow f(x)=\operatorname{tg}(x) \rightarrow f'(x) = \frac{1}{\cos^2 x} = \sec^2 x$

262) $\int \frac{2x+\sqrt{x}}{x^2}\,dx = \int \frac{2\cancel{x}}{x^{\cancel{2}}}\,dx + \int \frac{\sqrt{x}}{x^2}\,dx = 2\ln|x| + \int x^{-3/2}\,dx$

$= 2\ln|x| + \frac{x^{-1/2}}{-1/2} = 2\ln|x| - \frac{2}{\sqrt{x}} + C$

263) $\int \frac{1-\operatorname{sen}x}{2x+2\cos x}\,dx = \frac{1}{2}\int \frac{2(1-\operatorname{sen}x)}{2x+2\cos x}\,dx = \frac{1}{2}\ln|2x+2\cos x| + C$

264) $\displaystyle\int \ln\left(\frac{x+1}{x-1}\right)^{x} dx = \int x \cdot \underbrace{\ln\left(\frac{x+1}{x-1}\right)}_{u} dx = \circledast$

dv

$u = \ln\left(\frac{x+1}{x-1}\right) \qquad du = \dfrac{1}{\frac{x+1}{\cancel{x-1}}} \cdot \dfrac{1(x-1)-(x+1)\cdot 1}{(x-1)^{\cancel{2}}} = \dfrac{-2}{x^2-1}\,dx$

$dv = x\,dx \qquad v = \dfrac{x^2}{2}$

$\circledast = \dfrac{x^2}{2}\ln\left(\dfrac{x+1}{x-1}\right) + \displaystyle\int \frac{x^2}{x^2-1}\,dx = \frac{x^2}{2}\ln\left(\frac{x+1}{x-1}\right) + \int 1\,dx + \int \frac{1}{x^2-1}\,dx =$

$$\begin{array}{r|l} x^2 & x^2-1 \\ \cline{2-2} -x^2+1 & 1 \\ \hline 1 & \end{array}$$

$\dfrac{1}{\cancel{x^2-1}} = \dfrac{A}{x+1} + \dfrac{B}{x-1} = \dfrac{A(x-1)+B(x+1)}{\cancel{(x+1)(x-1)}}$

$\Rightarrow 1 = A(x-1)+B(x+1)$

Si $x = 1 \Rightarrow 1 = 2B \Rightarrow B = 1/2$

Si $x = -1 \Rightarrow 1 = -2A \Rightarrow A = -1/2$

$\circledast = \dfrac{x^2}{2}\ln\left(\dfrac{x+1}{x-1}\right) + x + \displaystyle\int \frac{-1/2}{x+1}\,dx + \int \frac{1/2}{x-1}\,dx =$

$= \dfrac{x^2}{2}\ln\left(\dfrac{x+1}{x-1}\right) + x - \dfrac{1}{2}\ln|x+1| + \dfrac{1}{2}\ln|x-1| + C$

265) $\int x\cdot(\ln x)^2 dx = \frac{x^2}{2}(\ln x)^2 - \int x \ln x\, dx = (*)$

$u=(\ln x)^2 \quad du = \frac{2\ln x}{x}dx$ || $u=\ln x \quad du=\frac{1}{x}dx$

$dv = x\,dx \quad v=\frac{x^2}{2}$ || $dv = x\,dx \quad v=\frac{x^2}{2}$

$(*) = \frac{x^2}{2}(\ln x)^2 - \left[\frac{x^2}{2}\ln x - \int \frac{x}{2}dx\right] =$

$= \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C$

266) $\int \underbrace{x}_{dv}\cdot \underbrace{\operatorname{arctg}(x+1)}_{u}\,\underbrace{dx}_{dv} = \frac{x^2}{2}\operatorname{arctg}(x+1) - \int \frac{x^2}{2x^2+4x+4}dx = (*)$

$u = \operatorname{arctg}(x+1) \quad du = \frac{1}{1+(x+1)^2}dx = \frac{1\,dx}{x^2+2x+2}$

$dv = x\,dx \quad v=\frac{x^2}{2}$

$$\begin{array}{r|l} x^2+0x+0 & 2x^2+4x+4 \\ \cline{2-2} -x^2-2x-2 & 1/2 \\ \hline -2x-2 & \end{array}$$

|| $(*) = \frac{x^2}{2}\operatorname{arctg}(x+1) - \int \frac{1}{2}dx + \int \frac{2x+2}{2x^2+4x+4}$

$= \frac{x^2}{2}\operatorname{arctg}(x+1) - \frac{x}{2} + \frac{1}{2}\int \frac{2(2x+2)}{2x^2+4x+4}dx =$

$= \frac{x^2}{2}\operatorname{arctg}(x+1) - \frac{x}{2} + \frac{1}{2}\ln|2x^2+4x+4| + C$

267) $\int \frac{x \cdot \arcsin(x)}{\sqrt{1-x^2}}dx = -\sqrt{1-x^2} \cdot \arcsin(x) + \int 1dx = (*)$

$u = \arcsin(x) \qquad du = \frac{1}{\sqrt{1-x^2}}dx$

$dv = \frac{x}{\sqrt{1-x^2}}dx \qquad v = -\sqrt{1-x^2}$

$(*) = -\sqrt{1-x^2} \cdot \arcsin(x) + x + C$

268) $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = \int \sqrt{t^2} \cdot 4tdt = \int 4t^2 dt = \frac{4t^3}{3} + k =$

$1+\sqrt{x} = t^2$

$\frac{1}{2\sqrt{x}}dx = 2tdt$

$\frac{dx}{\sqrt{x}} = 4tdt$

$= \frac{4}{3}\sqrt{(1+\sqrt{x})^3} + C$

269) $\int \frac{x^4}{\sqrt{1-x^{10}}}dx = \frac{1}{5}\int \frac{5x^4 dx}{\sqrt{1-(x^5)^2}} = \frac{1}{5}\arcsin(x^5) + C$

270) $\int e^x \cdot \sqrt{(e^x+1)^3}dx = \int e^x \cdot (e^x+1)^{3/2}dx =$

$= \frac{(e^x+1)^{5/2}}{5/2} + C = \frac{2}{5}\sqrt{(e^x+1)^5} + C$

271) $\int \cos^3 x \ \text{sen}^5 x\, dx = \int \cos x \cdot \cos^2 x \cdot \text{sen}^5 x\, dx =$

$= \int \cos x \left(1 - \text{sen}^2 x\right) \cdot \text{sen}^5 dx = \int \cos x \left(\text{sen}^5 x - \text{sen}^7 x\right) dx =$

$= \int \cos x \cdot \text{sen}^5 x\, dx - \int \cos x \cdot \text{sen}^7 x\, dx = \frac{\text{sen}^6 x}{6} - \frac{\text{sen}^8 x}{8} + C$

272) $\int \text{sen}^3 x \cdot \cos^{15} x\, dx = \int \text{sen} x \cdot \text{sen}^2 x \cdot \cos^{15} x\, dx =$

$= \int \text{sen} x \cdot \left(1 - \cos^2 x\right) \cdot \cos^{15} x\, dx = \int \text{sen} x \left(\cos^{15} x - \cos^{17} x\right) dx =$

$= -\int -\text{sen} x \cos^{15} x\, dx + \int -\text{sen} x \cos^{17} x\, dx =$

$= -\frac{\cos^{16} x}{16} + \frac{\cos^{18} x}{18} + C$

273) $\int \frac{\text{sen}^3 x}{\cos^2 x} dx = \int \frac{\text{sen} x \cdot \text{sen}^2 x}{\cos^2 x} dx = \int \frac{\text{sen} x \cdot \left(1 - \cos^2 x\right)}{\cos^2 x} dx =$

$= \int \frac{\text{sen} x - \text{sen} x \cdot \cos^2 x}{\cos^2 x} dx = \int \frac{\text{sen} x}{\cos^2 x} dx - \int \frac{\text{sen} x \cdot \cancel{\cos^2 x}}{\cancel{\cos^2 x}} dx =$

$= -\int -\text{sen} x \cdot (\cos x)^{-2} dx - \int \text{sen} x\, dx = -\frac{(\cos x)^{-1}}{-1} + \cos x =$

$= \frac{1}{\cos x} + \cos x + C$

274) $\int \frac{1}{\cos^3 x \operatorname{sen} x} dx = \int \frac{\operatorname{sen}^2 x + \cos^2 x}{\cos^3 x \operatorname{sen} x} dx =$

$= \int \frac{\operatorname{sen}^{\cancel{2}} x}{\cos^3 x \cancel{\operatorname{sen} x}} dx + \int \frac{\cancel{\cos^2 x} dx}{\cos^{\cancel{3}} x \operatorname{sen} x} = \int \frac{\operatorname{sen} x}{\cos^3 x} dx + \int \frac{1 \, dx}{\cos x \cdot \operatorname{sen} x} =$

$= -\int -\operatorname{sen} x \cdot (\cos x)^{-3} dx + \int \frac{1}{\frac{\cos x \cdot \operatorname{sen} x \cdot \cos x}{\cos x}} dx =$

$= -\frac{(\cos x)^{-2}}{-2} + \int \frac{1/\cos^2 x}{\operatorname{tg} x} dx = \frac{1}{2\cos^2 x} + \ln|\operatorname{tg}(x)| + C$

275) $\int \operatorname{tg}^3 x \cdot \sec^3 x \, dx = \int \frac{\operatorname{sen}^3(x)}{\cos^3(x)} \cdot \frac{1}{\cos^3(x)} dx = \int \frac{\operatorname{sen}^3(x) dx}{\cos^6(x)} =$

$= \int \operatorname{sen}(x) \cdot \operatorname{sen}^2(x) \cdot \cos^{-6}(x) dx = \int \operatorname{sen}(x) \cdot (1 - \cos^2(x)) \cdot \cos^{-6}(x) dx =$

$= -\int -\operatorname{sen}(x) \cdot \cos^{-6}(x) dx + \int -\operatorname{sen}(x) \cdot \cos^{-4}(x) dx =$

$= -\frac{\cos^{-5}(x)}{-5} + \frac{\cos^{-3}(x)}{-3} + C = \frac{1}{5\cos^5(x)} - \frac{1}{3\cos^3(x)} + C$

276) $\displaystyle \underbrace{\int \frac{\cos x}{\operatorname{sen} x+\cos x}\,dx}_{=I} = \int \frac{\cos x + \operatorname{sen} x - \operatorname{sen} x}{\operatorname{sen} x + \cos x}\,dx =$

$$= \int \frac{\cos x+\operatorname{sen} x}{\operatorname{sen} x+\cos x}\,dx + \int \frac{-\operatorname{sen} x+\cos x-\cos x}{\operatorname{sen} x+\cos x}\,dx =$$

$$= \int 1\,dx + \int \frac{\cos x-\operatorname{sen} x}{\operatorname{sen} x+\cos x}\,dx - \int \frac{\cos x}{\operatorname{sen} x+\cos x}\,dx =$$

$$= x + \ln|\operatorname{sen} x+\cos x| - \underbrace{\int \frac{\cos x}{\operatorname{sen} x+\cos x}\,dx}_{=I}$$

$$\Rightarrow 2I = x + \ln|\operatorname{sen} x+\cos x|$$

$$\Rightarrow \int \frac{\cos x}{\operatorname{sen} x+\cos x}\,dx = \frac{1}{2}\left[x+\ln|\operatorname{sen} x+\cos x|\right] + C$$

277) $\displaystyle \int \frac{\cos x}{2\operatorname{sen} x\cos^2 x+\operatorname{sen}^3 x}\,dx = \int \frac{dt}{2t(1-t^2)+t^3} = (*)$

$\operatorname{sen} x = t \rightarrow \cos^2 x = 1-\operatorname{sen}^2 x = 1-t^2$

$\cos x\,dx = dt$

$$(*) = \int \frac{dt}{2t-2t^3+t^3} = \int \frac{dt}{2t-t^3} = (*)$$

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$$2t-t^3=0 \Rightarrow t(2-t^2)=0 \begin{cases} t=0 \\ 2-t^2=0 \begin{cases} t=\sqrt{2} \\ t=-\sqrt{2} \end{cases} \end{cases}$$

$$\frac{1}{\cancel{2t-t^3}} = \frac{A}{t} + \frac{B}{\sqrt{2}-t} + \frac{C}{\sqrt{2}+t} = \frac{A(2-t^2)+Bt(\sqrt{2}+t)+Ct(\sqrt{2}-t)}{\cancel{t(\sqrt{2}-t)(\sqrt{2}+t)}}$$

$$\Rightarrow 1 = A(2-t^2)+Bt(\sqrt{2}+t)+Ct(\sqrt{2}-t)$$

Si $t=0 \Rightarrow 1=2A \Rightarrow A=1/2$

Si $t=\sqrt{2} \Rightarrow 1=4B \Rightarrow B=1/4$

Si $t=-\sqrt{2} \Rightarrow 1=-4C \Rightarrow C=-1/4$

$$(*) = \int \frac{1/2}{t}\,dt + -\int \frac{1/4}{\sqrt{2}-t}\,dt + \int \frac{-1/4}{\sqrt{2}+t}\,dt =$$

$$= \frac{1}{2}\ln|t| - \frac{1}{4}\ln|\sqrt{2}-t| - \frac{1}{4}\ln|\sqrt{2}+t| + K =$$

$$= \frac{1}{2}\ln|\operatorname{sen} x| - \frac{1}{4}\ln|\sqrt{2}-\operatorname{sen} x| - \frac{1}{4}\ln|\sqrt{2}+\operatorname{sen} x| + C_1 =$$

$$= \frac{1}{2}\ln|\operatorname{sen} x| - \frac{1}{4}\left[\ln|\sqrt{2}-\operatorname{sen} x| + \ln|\sqrt{2}+\operatorname{sen} x|\right] + C_1 =$$

$$= \frac{1}{2}\ln|\operatorname{sen} x| - \frac{1}{4}\ln\left[(\sqrt{2}-\operatorname{sen} x)(\sqrt{2}+\operatorname{sen} x)\right] + C_1 =$$

$$= \frac{1}{2}\ln|\operatorname{sen} x| - \frac{1}{4}\ln(2-\operatorname{sen}^2 x) = \frac{1}{2}\ln|\operatorname{sen} x| - \frac{1}{4}\ln|1+\cos^2 x| + C$$

278) $$\int \frac{1+\sqrt{x+1}}{1+\sqrt[3]{x+1}}\,dx = \int \frac{1+\sqrt{t^6}}{1+\sqrt[3]{t^6}}\cdot 6t^5dt =$$

$$\left.\begin{array}{l} x+1 = t^6 \\ dx = 6t^5dt \end{array}\right\| = \int \frac{1+t^3}{1+t^2}\cdot 6t^5dt =$$

$$= \int \frac{6t^8+6t^5}{1+t^2}dt = \circledast$$

$$\begin{array}{l|l}
6t^8+0t^7+0t^6+6t^5+0t^4+0t^3+0t^2+0t+0 & t^2+1 \\ \hline
-6t^8 \qquad -6t^6 & 6t^6-6t^4+6t^3+6t^2-6t \\
\quad -6t^6+6t^5+0t^4 & -6 \\
\quad +6t^6 \qquad +6t^4 & \\
\qquad 6t^5+6t^4+0t^3 & \\
\qquad -6t^5 \qquad -6t^3 & \\
\qquad\quad 6t^4-6t^3+0t^2 & \\
\qquad\quad -6t^4 \qquad -6t^2 & \\
\qquad\qquad -6t^3-6t^2+0t & \\
\qquad\qquad +6t^3 \qquad +6t & \\
\qquad\qquad\quad -6t^2+6t+0 & \\
\qquad\qquad\quad +6t^2 \qquad +6 & \\
\qquad\qquad\qquad 6t+6 &
\end{array}$$

$$\circledast = \int (6t^6-6t^4+6t^3+6t^2-6t-6)\,dt + \int \frac{6t+6}{1+t^2}dt =$$

$$= \frac{6t^7}{7} - \frac{6t^5}{5} + \frac{6t^4}{4} + \frac{6t^3}{3} - \frac{6t^2}{2} - 6t + \frac{6}{2}\int \frac{2t}{1+t^2}dx + 6\int\frac{1}{1+t^2}dt$$

$$= \frac{6}{7}t^7 - \frac{6}{5}t^5 + \frac{3}{2}t^4 + 2t^3 - 3t^2 - 6t + 3\ln|1+t^2| + 6\,\mathrm{arctg}(t) + K =$$

$$= \frac{6}{7}\sqrt[6]{(x+1)^7} - \frac{6}{5}\sqrt[6]{(x+1)^5} + \frac{3}{2}\sqrt[6]{(x+1)^4} + 2\sqrt[6]{(x+1)^3} - 3\sqrt[6]{(x+1)^2} -$$

$$- 6\sqrt[6]{(x+1)} + 3\ln\left|1+\sqrt[6]{(x+1)^2}\right| + 6\,\mathrm{arctg}\left(\sqrt[6]{x+1}\right) + C$$

279) $$\int \frac{1+x+\sqrt{x+1}}{(x+1)\cdot\sqrt[3]{x+1}}\,dx = \int \frac{t^6+\sqrt{t^6}}{t^6\cdot\sqrt[3]{t^6}}\,6t^5 dt =$$

$x+1 = t^6$
$dx = 6t^5\cdot dt$

$$= \int \frac{t^6+t^3}{t^{\cancel{8}\,3}}\cdot 6\cancel{t^5}dt =$$

$$= \int \frac{\cancel{t^3}(t^3+1)}{\cancel{t^3}}\cdot 6\cdot dt = \int (6t^3+6)dt = \frac{6t^4}{4} + 6t + K =$$

$$= \frac{3}{2}t^4 + 6t + K = \frac{3}{2}\sqrt[6]{(x+1)^4} + 6\sqrt[6]{x+1} + C$$

280) $$\int \frac{1}{(1-e^x)^2}dx = \int \frac{1}{(1-t)^2}\cdot\frac{dt}{t} = \int \frac{dt}{t(1-t)^2} = (*)$$

$e^x = t$
$e^x dx = dt \longrightarrow dx = \frac{dt}{e^x} = \frac{dt}{t}$

$$\frac{1}{t(\cancel{1-t})^2} = \frac{A}{t} + \frac{B}{(1-t)} + \frac{C}{(1-t)^2} = \frac{A(1-t)^2 + Bt(1-t) + Ct}{\cancel{t(1-t)^2}}$$

$$\Rightarrow 1 = A(1-t)^2 + Bt(1-t) + Ct$$

Si $t=0 \Rightarrow 1 = A$

Si $t=1 \Rightarrow 1 = C$

Si $t=2 \Rightarrow 1 = 1 - 2B + 2 \Rightarrow B = 1$

$$(*) = \int \frac{1}{t}\,dt + \int \frac{1}{1-t}\,dt + \int \frac{1}{(1-t)^2}\,dt =$$

$$= \ln|t| - \ln|1-t| + (1-t)^{-1} + k =$$

$$= \ln|t| - \ln|1-t| + \frac{1}{1-t} + k =$$

$$= \ln e^x - \ln|1-e^x| + \frac{1}{1-e^x} + C' =$$

$$= x - \ln|1-e^x| + \frac{1}{1-e^x} + C'$$

281) $$\int \frac{1+\sqrt{e^x}}{(1-\sqrt[4]{e^x})^2}\cdot dx = \int \frac{1+\sqrt{t^4}}{(1-\sqrt[4]{t^4})^2}\cdot \frac{4dt}{t} = (*)$$

$$e^x = t^4$$

$$e^x dx = 4t^3 \cdot dt \rightarrow dx = \frac{4t^3\cdot dt}{e^x} = \frac{4\cancel{t^3}\cdot dt}{t^{\cancel{4}}} = \frac{4dt}{t}$$

$$(*) = \int \frac{(1+t^2)\cdot 4}{(1-t)^2\cdot t}\,dt = \int \frac{4t^2+4}{t(1-t)^2}\,dt = (*)$$

$$\frac{4t^2+4}{t(1-t)^2} = \frac{A}{t} + \frac{B}{(1-t)} + \frac{C}{(1-t)^2} = \frac{A(1-t)^2 + Bt(1-t) + Ct}{t(1-t)^2}$$

$$\Rightarrow 4t^2+4 = A(1-t)^2 + B\cdot t(1-t) + Ct$$

si $t=0 \rightarrow 4 = A$

si $t=1 \rightarrow 8 = C$

si $t=2 \rightarrow 20 = 4 - 2B + 16 \Rightarrow B = 0$

$$(*) = \int \frac{4}{t}\,dt + \int \frac{8}{(1-t)^2}\,dt = 4\ln|t| - 8\cdot\frac{(1-t)^{-1}}{-1} + K =$$

$$= 4\ln|t| + \frac{8}{1-t} + K = 4\ln\sqrt[4]{e^x} + \frac{8}{1-\sqrt[4]{e^x}} + C' =$$

$$= 4\ln(e^x)^{1/4} + \frac{8}{1-\sqrt[4]{e^x}} + C' = 4\ln e^{\frac{1}{4}x} + \frac{8}{1-\sqrt[4]{e^x}} + C' =$$

$$= 4\cdot\frac{1}{4}x + \frac{8}{1-\sqrt[4]{e^x}} + C' = x + \frac{8}{1-\sqrt[4]{e^x}} + C'$$

282) $\displaystyle\int \frac{1}{e^x+e^{-x}+1}\,dx = \int \frac{1}{e^x+\frac{1}{e^x}+1}\,dx = \int \frac{1}{\frac{(e^x)^2+1+e^x}{e^x}}\,dx =$

$$= \int \frac{e^x}{(e^x)^2+e^x+1}\,dx = (*)$$

$$e^x = t \qquad e^x\,dx = dt \qquad \circledast = \int \frac{dt}{t^2+t+1} = \circledast$$

$$t^2+t+1=0$$

$$t = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$$t^2+t+1 = \left(t+\frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\circledast = \int \frac{dt}{\frac{3}{4}\cdot\frac{4}{3}\left[\frac{3}{4}+\left(t+1/2\right)^2\right]} = \frac{4}{3}\frac{\sqrt{3}}{2}\int \frac{dt\cdot 2/\sqrt{3}}{1+\left(\frac{2\left(t+1/2\right)}{\sqrt{3}}\right)^2} =$$

$$= \frac{2\sqrt{3}}{3}\operatorname{arctg}\left(\frac{2}{\sqrt{3}}t+\frac{1}{\sqrt{3}}\right) = \frac{2\sqrt{3}}{3}\operatorname{arctg}\left(\frac{2e^x+1}{\sqrt{3}}\right) + C$$

283) $$\int \frac{e^{3x}}{e^{2x}-3e^x+2}\,dx = \int \frac{e^{2x}\cdot e^x\,dx}{e^{2x}-3e^x+2} = \int \frac{t^2\,dt}{t^2-3t+2} = \circledast$$

$$e^x = t \longrightarrow e^{2x} = (e^x)^2 = t^2$$

$$e^x\,dx = dt$$

$$\begin{array}{r|l} t^2+0t+0 & t^2-3t+2 \\ -t^2+3t-2 & 1 \\ \hline 3t-2 & \end{array}$$

$$= \int 1\,dt + \int \frac{3t-2}{t^2-3t+2}\,dt = ☺$$

la hacemos a parte

$$t^2-3t+2=0 \quad \begin{cases} \to t=2 \\ \to t=1 \end{cases}$$

$$\frac{3t-2}{\cancel{t^2-3t+2}} = \frac{A}{t-2} + \frac{B}{t-1} = \frac{A(t-1)+B(t-2)}{\cancel{(t-2)(t-1)}}$$

$$\Rightarrow \quad 3t-2 = A(t-1)+B(t-2)$$

$$\text{Si } t=1 \longrightarrow 1=-B \Rightarrow B=-1$$

$$\text{Si } t=2 \longrightarrow 4=A$$

$$\int \frac{3t-2}{t^2-3t+2}\,dt = \int \frac{4}{t-2}\,dt + \int \frac{-1}{t-1}\,dt = 4\ln|t-2| - \ln|t-1| + K$$

$\Rightarrow$  $= t + 4\ln|t-2| - \ln|t-1| + K =$

$$= e^x + 4\ln|e^x-2| - \ln|e^x-1| + C'$$

284) $\displaystyle\int (1-x^2)^{-3/2}\,dx = \int (1-\operatorname{sen}^2 t)^{-3/2} \cdot \cos t\,dt =$

$$\begin{array}{l} x = \operatorname{sen} t \\ dx = \cos t\,dt \end{array} \quad \Bigg\| \quad = \int (\cos^2 t)^{-3/2} \cdot \cos t\,dt =$$

$$= \int \cos^{-3} t \cdot \cos t\,dt = \int \cos^{-2} t\,dt = \int \frac{1}{\cos^2 t}\,dt =$$

$$= tg(t) + K = \frac{x}{\sqrt{1-x^2}} + C'$$

$$tg(t) = \frac{sen(t)}{cos(t)} = \frac{x}{\sqrt{1-x^2}}$$

285) $\int (1-(2x+1)^2)^{-1/2}\, dx = \int \frac{1}{\sqrt{1-(2x+1)^2}}\, dx = (*)$

$2x+1 = sen\, t$

$2dx = cos\, t\, dt \longrightarrow dx = \frac{1}{2} cos\, t\, dt$

$$(*) = \int \frac{1}{\sqrt{1-sen^2 t}} \cdot \frac{1}{2} cos\, t\, dt = \int \frac{1}{\cancel{cos\, t}} \cdot \frac{1}{2} \cdot \cancel{cos\, t}\, dt = \frac{1}{2} t + K =$$

$$= \frac{1}{2} arcsen(2x+1) + C$$

286) $\int \frac{2^{3x}}{2^x - 4}\, dx = \int \frac{2^{2x} \cdot 2^x \cdot dx}{2^x - 4} = \int \frac{t^2\, dt}{ln 2\,(t-4)} = (*)$

$2^x = t \longrightarrow 2^{2x} = t^2$

$2^x \cdot ln 2\, dx = dt \longrightarrow 2^x dx = \frac{dt}{ln 2}$

$$(*) = \frac{1}{ln 2} \int \frac{t^2}{t-4}\, dt = \left\{ \begin{array}{ll} t^2 + 0t + 0 & \underline{|t-4} \\ -t^2 + 4t & t+4 \\ \hline \quad -4t + 16 & \\ \hline \quad\quad 16 & \end{array} \right\} = (*)$$

$$(*) = \frac{1}{\ln 2}\cdot\left[\int (t+4)\,dt + \int \frac{16}{t-4}\,dt\right] =$$

$$= \frac{1}{\ln 2}\cdot\left[\frac{t^2}{2} + 4t + 16\ln|t-4|\right] + K =$$

$$= \frac{1}{\ln 2}\cdot\left[\frac{2^{2x}}{2} + 4\cdot 2^x + 16\ln|2^x-4|\right] + C' =$$

$$= \frac{1}{\ln 2}\cdot\left(2^{2x-1} + 2^{x+2} + 16\ln|2^x-4|\right) + C'$$

287) $\displaystyle\int \frac{\operatorname{sen}(2x)+\cos x}{\cos x}\,dx = \int \frac{2\operatorname{sen}x\cdot\cos x + \cos x}{\cos x}\,dx =$

$$= \int \frac{2\operatorname{sen}x\,\cancel{\cos x}}{\cancel{\cos x}}\,dx + \int \frac{\cancel{\cos x}^{\,1}}{\cancel{\cos x}}\,dx = -2\cos x + x + C'$$

288) $\displaystyle\int \frac{7}{x\sqrt[3]{\ln x}}\,dx = 7\int \frac{1}{x}\cdot(\ln x)^{-1/3}\,dx = 7\cdot\frac{(\ln x)^{2/3}}{2/3} + C' =$

$$= \frac{21}{2}\cdot\sqrt[3]{(\ln x)^2} + C'$$

289) $\displaystyle\int \frac{(\ln x)^2 + x}{x}\,dx = \int \frac{1}{x}(\ln x)^2\,dx + \int 1\,dx =$

$$= \frac{(\ln x)^3}{3} + x + C'$$

290) $\displaystyle\int \frac{1}{\sqrt{x+2}+\sqrt{x-2}}\,dx = \int \frac{1}{4}\left(\sqrt{x+2}-\sqrt{x-2}\right)dx = (*)$

$$\frac{1}{\sqrt{x+2}+\sqrt{x-2}}\cdot\frac{(\sqrt{x+2}-\sqrt{x-2})}{(\sqrt{x+2}-\sqrt{x-2})} = \frac{(\sqrt{x+2}-\sqrt{x-2})}{(\cancel{\sqrt{x+2}})^{\cancel{2}}-(\cancel{\sqrt{x-2}})^{\cancel{2}}} = \frac{1}{4}\left(\sqrt{x+2}-\sqrt{x-2}\right)$$

$$(*) = \frac{1}{4}\int (x+2)^{1/2}dx - \frac{1}{4}\int (x-2)^{1/2}dx =$$

$$= \frac{1}{4}\,\frac{(x+2)^{3/2}}{3/2} - \frac{1}{4}\,\frac{(x-2)^{3/2}}{3/2} + C_1 = \frac{1}{6}\sqrt{(x+2)^3} - \frac{1}{6}\sqrt{(x-2)^3} + C_1$$

291) $\displaystyle\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx + \int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx =$

$$= \int \sqrt{\frac{1+x^2}{1-x^4}}\,dx + \int \sqrt{\frac{1-x^2}{1-x^4}}\,dx = \int \sqrt{\frac{\cancel{1+x^2}}{(\cancel{1+x^2})(1-x^2)}}\,dx + \int \sqrt{\frac{\cancel{1-x^2}}{(1+x^2)(\cancel{1-x^2})}}\,dx$$

$$= \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arcsen}(x) + \int \frac{1}{\sqrt{1+x^2}}\,dx =$$ 

la hacemos a parte

$$\int \frac{1}{\sqrt{1+x^2}}\,dx \longrightarrow \sqrt{1+x^2} = t - x$$

$$(\cancel{\sqrt{1+x^2}})^{\cancel{2}} = (t-x)^2 \Longrightarrow 1+\cancel{x^2} = t^2 - 2tx + \cancel{x^2}$$

$$\Longrightarrow 2tx = t^2 - 1 \Longrightarrow x = \frac{t^2-1}{2t}$$

$$x = \frac{t^2-1}{2t} \Rightarrow dx = \frac{4t^2-2t^2+2}{(2t)^2}dt = \frac{2t^2+2}{(2t)^2}dt$$

$$\int \frac{1}{\sqrt{1+x^2}}dx = \int \frac{1}{t-\left(\frac{t^2-1}{2t}\right)} \cdot \frac{2(t^2+1)}{(2t)^2} \cdot dt =$$

$$= \int \frac{1}{\frac{2t^2-t^2+1}{\cancel{2t}}} \cdot \frac{2(t^2+1)}{(2t)^{\cancel{2}}}dt = \int \frac{1}{\cancel{t^2+1}} \cdot \frac{\cancel{2}\,\cancel{(t^2+1)}}{\cancel{2}t}dt =$$

$$= \int \frac{1}{t}dt = \ln|t| + k = \ln\left|x + \sqrt{1+x^2}\right| + C'$$

$\Rightarrow$  $= \arcsen(x) + \ln\left|x+\sqrt{1+x^2}\right| + C'$

Nota:

En realidad la integral $\int \frac{1}{\sqrt{1+x^2}}dx$ es inmediata, siendo $\int \frac{1}{\sqrt{1+x^2}}dx = \operatorname{argsenh}(x)$. Pero al no darse las funciones hiperbólicas en bachillerato, se ha resuelto por cambio de variable para obtener su forma logarítmica.

Obviamente, $\operatorname{argsenh}(x) = \ln\left(x+\sqrt{x^2+1}\right)$

292) $\int \frac{x-1}{\sqrt{2x}-\sqrt{x+1}}\,dx = \int \left(\sqrt{2x}+\sqrt{x+1}\right)dx = \circledast$

$$\frac{(x-1)}{\sqrt{2x}-\sqrt{x+1}} \cdot \frac{\sqrt{2x}+\sqrt{x+1}}{\sqrt{2x}+\sqrt{x+1}} = \frac{(x-1)\left(\sqrt{2x}+\sqrt{x+1}\right)}{(\sqrt{2x})^2-(\sqrt{x+1})^2} = \frac{\cancel{(x-1)}\left(\sqrt{2x}+\sqrt{x+1}\right)}{\cancel{(x-1)}}$$

$$\circledast \int \sqrt{2x}\,dx + \int \sqrt{x+1}\,dx = \frac{1}{2}\int 2\,(2x)^{1/2}dx + \int (x+1)^{1/2}dx =$$

$$= \frac{1}{2}\cdot\frac{(2x)^{3/2}}{3/2} + \frac{(x+1)^{3/2}}{3/2} = \frac{1}{3}\sqrt{(2x)^3} + \frac{2}{3}\sqrt{(x+1)^3} + C$$

293) $\int e^{2x}\cdot \operatorname{sen}(e^x)\,dx = \int \underbrace{e^x}_{u}\cdot \underbrace{e^x\cdot \operatorname{sen}(e^x)\,dx}_{dv} = \circledast$

$u = e^x \qquad du = e^x dx$

$dv = e^x \operatorname{sen}(e^x)dx \qquad v = -\cos(e^x)$

$$\circledast = -e^x\cos(e^x) + \int e^x\cos(e^x)\,dx = -e^x\cos(e^x) + \operatorname{sen}(e^x) + C$$