ECUACIONES TRIGONOMÉTRICAS

Resolver las siguientes ecuaciones:

1) $\text{sen } 2x = \text{sen } x$

2) $\cos 2x - \cos x = \text{sen}\left(\frac{x}{2}\right)$

3) $\text{sen } x + \text{sen } 2x + \text{sen } 3x = 0$

4) $\text{tag}^2\, 2x + \text{sen}^2\, 2x = 3$

5) $\cos x + \sqrt{3}\ \text{sen} x = 1$

6) $\text{sen} x \cdot \cos x = \frac{1}{2}$

7) $\cos 2\pi + 8 \cos x + 3 = 0$

8) $\cos 2x - \cos 6x = \text{sen } 5x + \text{sen } 3x$

9) $\text{sen}^4 x - 2\cos^4 x + 1 = 0$

10) $4 \cdot \text{sen}\left(x - \frac{\pi}{6}\right) \cdot \cos\left(x - \frac{\pi}{6}\right) = \sqrt{3}$

11) $4 \cdot \text{sen}\left(\frac{x}{2}\right) + 2\cos x = 3$

12) $8 \cdot \text{tag}^2\left(\frac{x}{2}\right) = 1 + \sec x$

13) $\text{sen } x + \cos x = \cos x\, (\text{sen } x + \cos x)$

14) $\text{sen} 2x = \cos \frac{\pi}{3}$

15) $\text{tag } 2x = -\text{tag } x$

16) $(\cos^2 x - \text{sen}^2 x)^2 = \text{sen } 2x$

1) $\operatorname{sen} 2x = \operatorname{sen} x \rightarrow 2 \cdot \operatorname{sen} x \cdot \cos x - \operatorname{sen} x = 0 \rightarrow$

$\operatorname{sen} x\,(2\cos x - 1) = 0 \rightarrow \operatorname{sen} x = 0 \rightarrow x_1 = 0^\circ + k360^\circ$

$\rightarrow x_2 = 180^\circ + k360^\circ$

$\rightarrow 2\cos x - 1 = 0$

$\downarrow$

$\cos x = \frac{1}{2} \begin{cases} x_3 = 60^\circ \\ x_4 = 300^\circ \end{cases}$

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2) $\cos 2x - \cos x = \operatorname{sen}\left(\frac{x}{2}\right)$

aplicando $\cos A - \cos B = -2\operatorname{sen}\left(\frac{A+B}{2}\right)\operatorname{sen}\left(\frac{A-B}{2}\right)$

$-2\operatorname{sen}\left(\frac{3x}{2}\right)\operatorname{sen}\left(\frac{x}{2}\right) = \operatorname{sen}\left(\frac{x}{2}\right)$

$0 = \operatorname{sen}\left(\frac{x}{2}\right) + 2\operatorname{sen}\left(\frac{3x}{2}\right)\operatorname{sen}\left(\frac{x}{2}\right)$

$0 = \underbrace{\operatorname{sen}\left(\frac{x}{2}\right)}_{0}\underbrace{\left[1 + 2\operatorname{sen}\left(\frac{3x}{2}\right)\right]}_{0}$

$\operatorname{sen}\frac{x}{2} = 0 \rightarrow \frac{x}{2} = 0 \rightarrow x_1 = 0^\circ$

$\rightarrow \frac{x}{2} = 180 \rightarrow x_2 = 360^\circ$

$1 + 2\operatorname{sen}\left(\frac{3x}{2}\right) = 0 \rightarrow \operatorname{sen}\left(\frac{3x}{2}\right) = -\frac{1}{2} \begin{cases} \frac{3x}{2} = 210 \rightarrow x_3 = 140^\circ \\ \frac{3x}{2} = 330 \rightarrow x_4 = 220^\circ \end{cases}$

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3) $\operatorname{sen}x + \operatorname{sen}2x + \operatorname{sen}3x = 0$

$\downarrow$

$\underbrace{\operatorname{sen}3x + \operatorname{sen}x} + \operatorname{sen}2x = 0$

Aplicando $\operatorname{sen}A + \operatorname{sen}B = 2\operatorname{sen}\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$2\operatorname{sen}2x \cdot \cos x + \operatorname{sen}2x = 0$

$\operatorname{sen}2x\,(2\cdot\cos x + 1) = 0$

$\overset{\shortparallel}{0} \qquad \overset{\shortparallel}{0}$

$\operatorname{sen}2x = 0 \to 2x = 0 \to x_1 = 0°$

$\to 2x = 180 \to x_2 = 90°$

$2\cos x + 1 = 0 \to \cos x = \frac{-1}{2}$ $\begin{cases} x_3 = 120° \\ x_4 = 240° \end{cases}$

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6) $\operatorname{sen}x \cdot \cos x = \frac{1}{2} \to 2\cdot\operatorname{sen}x\cdot\cos x = 1 \to$

$\operatorname{sen}(2x) = 1 \to 2x = 90° \to \underline{\underline{x = 45°}}$

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4) $\tan^2 2x + \operatorname{sen}^2 2x = 3$

$2x = A \rightarrow tg^2 A + \operatorname{sen}^2 A = 3 \rightarrow \frac{\operatorname{sen}^2 A}{\cos^2 A} + \operatorname{sen}^2 A = 3$

$\rightarrow \operatorname{sen}^2 A + \operatorname{sen}^2 A \cos^2 A = 3\cos^2 A$

$\operatorname{sen}^2 A + \operatorname{sen}^2 A (1 - \operatorname{sen}^2 A) = 3(1 - \operatorname{sen}^2 A)$

$\operatorname{sen}^2 A + \operatorname{sen}^2 A - \operatorname{sen}^4 A = 3 - 3\operatorname{sen}^2 A$

$\operatorname{sen}^4 A - 5\operatorname{sen}^2 A + 3 = 0$

$\operatorname{sen}^2 A = \frac{5 \pm \sqrt{25-12}}{2} = \frac{5 \pm \sqrt{13}}{2} \left\langle \begin{array}{l} 4'3 \text{ No vale} \\ 0'697 \end{array} \right.$

$\operatorname{sen} A = \pm\sqrt{0'697} = \pm 0'835 \rightarrow A = 56'6° = 2x$

$x = 28'3°$

$\Rightarrow \begin{cases} x_1 = 28'3° \\ x_2 = 151'7° \\ x_3 = 208'3° \\ x_4 = 331'7° \end{cases}$

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5) $\cos x + \sqrt{3}\,\text{sen}\,x = 1 \rightarrow \sqrt{3}\,\text{sen}\,x = 1 - \cos x$

$(\sqrt{3}\,\text{sen}\,x)^2 = (1-\cos x)^2$

$3 \cdot \text{sen}^2 x = 1 - 2\cos x + \cos^2 x$

$3(1-\cos^2 x) = 1 - 2\cos x + \cos^2 x$

$4\cos^2 x - 2\cos x - 2 = 0 \rightarrow 2\cos^2 x - \cos x - 1 = 0$

$\cos x = \dfrac{1 \pm \sqrt{1+8}}{4} = \dfrac{1 \pm 3}{4} < \begin{matrix} 1 \\ -\frac{1}{2} \end{matrix}$

$\cos x = 1 \rightarrow x_1 = 0°$

$\cos x = -\frac{1}{2} \rightarrow x_2 = 120°\ ,\ x_3 = 240°$

7) $\cos(2\pi) + 8\cos x + 3 = 0 \rightarrow 1 + 8\cos x + 3 = 0$

$8\cos x = -4 \rightarrow \cos x = -\frac{1}{2} \rightarrow x_1 = 120°$

$\searrow x_2 = 240°$

8) $\cos 2x - \cos 6x = \text{sen}\, 5x + \text{sen}\, 3x$

$-(\cos 6x - \cos 2x) = \text{sen}\, 5x + \text{sen}\, 3x$

aplicando $\cos A - \cos B = -2\,\text{sen}\left(\frac{A+B}{2}\right)\text{sen}\left(\frac{A-B}{2}\right)$

y $\text{sen}\, A + \text{sen}\, B = 2\,\text{sen}\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$-\left(-2\,\text{sen}(4x)\cdot \text{sen}\, 2x\right) = 2\,\text{sen}(4x)\cos x$

$\cancel{2}\cdot \text{sen}(4x)\,\text{sen}\, 2x = \cancel{2}\,\text{sen}(4x)\cos x$

$\text{sen}(4x)\,\text{sen}\, 2x - \text{sen}(4x)\cos x = 0$

$\text{sen}(4x)\cdot 2\cdot \text{sen}\, x\cdot \cos x - \text{sen}(4x)\cos x = 0$

$\underbrace{\text{sen}(4x)}_{=0}\cdot \underbrace{\cos x}_{=0}\cdot \underbrace{[2\,\text{sen}\, x - 1]}_{=0} = 0$

$\text{sen}\, 4x = 0 \rightarrow \begin{cases} 4x = 0 \rightarrow x_1 = 0^\circ \\ 4x = 180 \rightarrow x_2 = 45^\circ \end{cases}$

$\cos x = 0 \rightarrow \begin{cases} x_3 = 90^\circ \\ x_4 = 270^\circ \end{cases}$

$2\,\text{sen}\, x - 1 = 0 \rightarrow \text{sen}\, x = \frac{1}{2} \rightarrow \begin{cases} x_5 = 30^\circ \\ x_6 = 150^\circ \end{cases}$

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9) $\operatorname{sen}^4 x - 2\cos^4 x + 1 = 0$

$$\underbrace{\operatorname{sen}^4 x - \cos^4 x} - \underbrace{\cos^4 x + 1} = 0$$

$$\underbrace{(\operatorname{sen}^2 x + \cos^2 x)}_{1}(\operatorname{sen}^2 x - \cos^2 x) + \operatorname{sen}^4 x = 0$$

$$\operatorname{sen}^2 x - \cos^2 x + \operatorname{sen}^4 x = 0$$

$$\operatorname{sen}^2 x - (1 - \operatorname{sen}^2 x) + \operatorname{sen}^4 x = 0$$

$$\operatorname{sen}^4 x + 2\operatorname{sen}^2 x - 1 = 0$$

$$\operatorname{sen}^2 x = \frac{2 \pm \sqrt{44}}{2} = 1 \rightarrow \operatorname{sen} x = \pm\sqrt{1} = \pm 1$$

$$\rightarrow x_1 = 90 \ , \ x_2 = 270^\circ$$

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10) $4 \cdot \operatorname{sen}\left(x - \frac{\pi}{6}\right) \cdot \cos\left(x - \frac{\pi}{6}\right) = \sqrt{3}$

$$2 \cdot \underbrace{2 \cdot \operatorname{sen}\left(x - \frac{\pi}{2}\right) \cdot \cos\left(x - \frac{\pi}{6}\right)}_{\text{ángulo doble}} = \sqrt{3}$$

$$2 \cdot \operatorname{sen} 2\left[x - \frac{\pi}{2}\right] = \sqrt{3} \rightarrow \operatorname{sen}\left[2\left(x - \frac{\pi}{2}\right)\right] = \frac{\sqrt{3}}{2} \rightarrow$$

$$\rightarrow \begin{cases} 2\left[x - \frac{\pi}{2}\right] = 60^\circ = \frac{\pi}{3} \rightarrow x_1 = \frac{2\pi}{3} \\ 2\left(x - \frac{\pi}{2}\right) = 120^\circ = \frac{2\pi}{3} \rightarrow x_2 = \frac{5\pi}{6} \end{cases}$$

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11) $4 \cdot \text{sen}\left(\frac{x}{2}\right) + 2\cos x = 3 \rightarrow \text{sen}\left(\frac{x}{2}\right) = \frac{3}{4} - \frac{1}{2}\cos x$

$$\pm\sqrt{\frac{1-\cos x}{2}} = \frac{3}{4} - \frac{1}{2}\cos x$$

$$\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2 = \left(\frac{3}{4} - \frac{1}{2}\cos x\right)^2$$

$$\frac{1-\cos x}{2} = \frac{9}{16} + \frac{1}{4}\cos^2 x - \frac{3}{4}\cos x$$

$$\vdots$$

$$4\cos^2 x - 4\cos x + 1 = 0$$

$$\cos x = \frac{4 \pm \sqrt{16-16}}{8} = \frac{1}{2} \quad \rightarrow x_1 = 60^\circ$$

$$\rightarrow x_2 = 300^\circ$$

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13) $\text{sen}\,x + \cos x = \cos x(\text{sen}\,x + \cos x) \rightarrow$

$$\frac{\text{sen}\,x + \cos x}{(\text{sen}\,x + \cos x)} = \cos x \rightarrow \quad \cos x = 1 \quad x = 0^\circ$$

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14) $\text{sen}\,2x = \frac{1}{2} \rightarrow 2x = 30^\circ \rightarrow x_1 = 15^\circ$

$$\rightarrow 2x = 150^\circ \rightarrow x_2 = 75^\circ$$

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12) $8\cdot tg^2\left(\frac{x}{2}\right)=1+\sec x$

$$8\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)^2=1+\frac{1}{\cos x}$$

$$8\left(\frac{1-\cos x}{1+\cos x}\right)=\frac{1+\cos x}{\cos x}$$

$$8\cos x(1-\cos x)=(1+\cos x)^2$$

$$8\cos x-8\cos^2 x=1+\cos^2 x+2\cos x$$

$$9\cos^2 x-6\cos x+1=0$$

$$\cos x=\frac{6\pm\sqrt{36-36}}{18}=\frac{1}{3}\rightarrow x=\text{arcocos}\,\frac{1}{3}$$

$$\rightarrow\begin{cases}x_1=70'5^\circ\\ x_2=289'5^\circ\end{cases}$$

15) $tg2x=-tgx\rightarrow\dfrac{2tgx}{1-tg^2x}=-tgx\rightarrow$

$2tgx=-tgx+tg^3x\rightarrow tg^3x-3tgx=0\rightarrow$

$\underset{0}{\underset{\shortparallel}{tgx}}\,(\underset{0}{\underset{\shortparallel}{tg^2x-3}})=0\rightarrow tgx=0\rightarrow\begin{cases}x_1=0\\ x_2=180^\circ\end{cases}$

$\longrightarrow tgx=\pm\sqrt{3}\rightarrow\begin{cases}x_3=60^\circ\\ x_4=120^\circ\\ x_5=240^\circ\\ x_6=300^\circ\end{cases}$

16) $\underbrace{(\cos^2 x - \operatorname{sen}^2 x)}_{\text{ángulo doble}}{}^2 = \operatorname{sen} 2x$

$(\cos 2x)^2 = \operatorname{sen} 2x \rightarrow \cos^2(2x) = \operatorname{sen}(2x)$

$1 - \operatorname{sen}^2(2x) = \operatorname{sen} 2x \rightarrow \operatorname{sen}^2(2x) + \operatorname{sen}(2x) - 1 = 0$

$\operatorname{sen} 2x = \dfrac{-1 \pm \sqrt{5}}{2} \begin{cases} \dfrac{-1+\sqrt{5}}{2} = 0'618 \\ \dfrac{-1-\sqrt{5}}{2} = -1'618 \leftarrow \text{No vale} \end{cases}$

$\rightarrow 2x = \arcsin 0'618 \rightarrow 2x = 38'17° \rightarrow x_1 = 19'08°$

$\hookrightarrow 2x = 141'83 \rightarrow x_2 = 70'915°$