

1. Dada la función:

$$y = F(x) = \begin{cases} x^2 - 5 & \text{si } x < 1 \\ 3x + 2 & \text{si } 1 \leq x < 4 \\ 4x - 2 & \text{si } x \geq 4 \end{cases}$$

¿Existe el límite de $F(x)$ para $x \rightarrow 1$? ¿Y para $x \rightarrow 4$? Razona tu respuesta.
Dibuja la gráfica de $F(x)$.

2. Calcular b para que

$$\lim_{x \rightarrow +\infty} \frac{-x^2 + 2x}{bx(1+x)} = \frac{1}{5}$$

3. Calcular los siguientes límites:

$$\text{a) } \lim_{x \rightarrow +\infty} \frac{2x^2 - 5x + 6}{x^3 - x^2 + 8}$$

$$\text{b) } \lim_{x \rightarrow +\infty} \frac{4x^4 - 2x}{x(x^2 - 1)}$$

$$\text{c) } \lim_{x \rightarrow +\infty} \left(\frac{2x^2 + 1}{x^2 + 5} \right)^{\frac{3x+1}{x+2}}$$

$$\text{d) } \lim_{x \rightarrow +\infty} (\sqrt{x^3 + 2} - 5x)$$

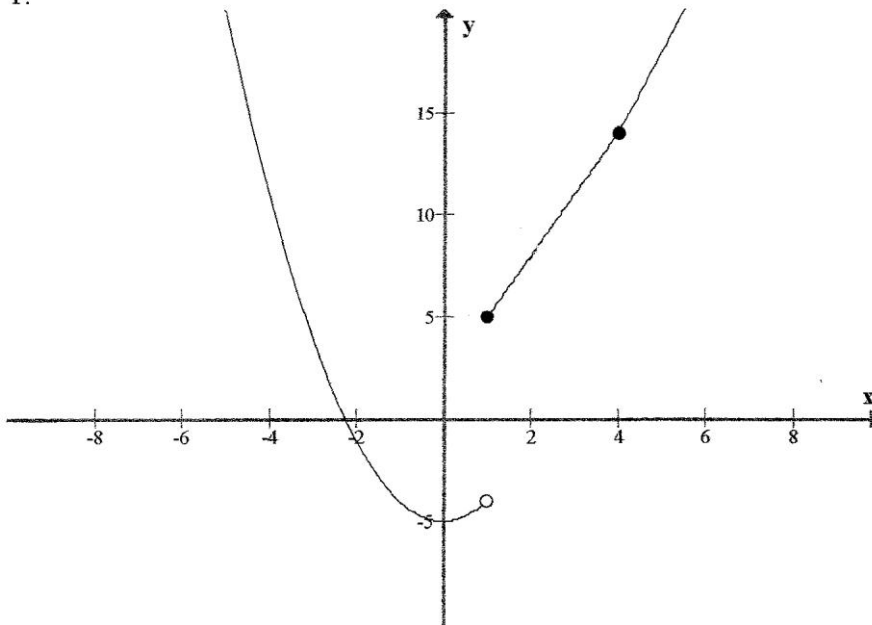
4. Calcular los siguientes límites del número e :

$$\text{a) } \lim_{x \rightarrow 2} \left(\frac{x+3}{2x+1} \right)^{2/(x-2)}$$

$$\text{b) } \lim_{x \rightarrow +\infty} \left(\frac{3x+1}{3x+4} \right)^{2x}$$

SOLUCIONES

1.



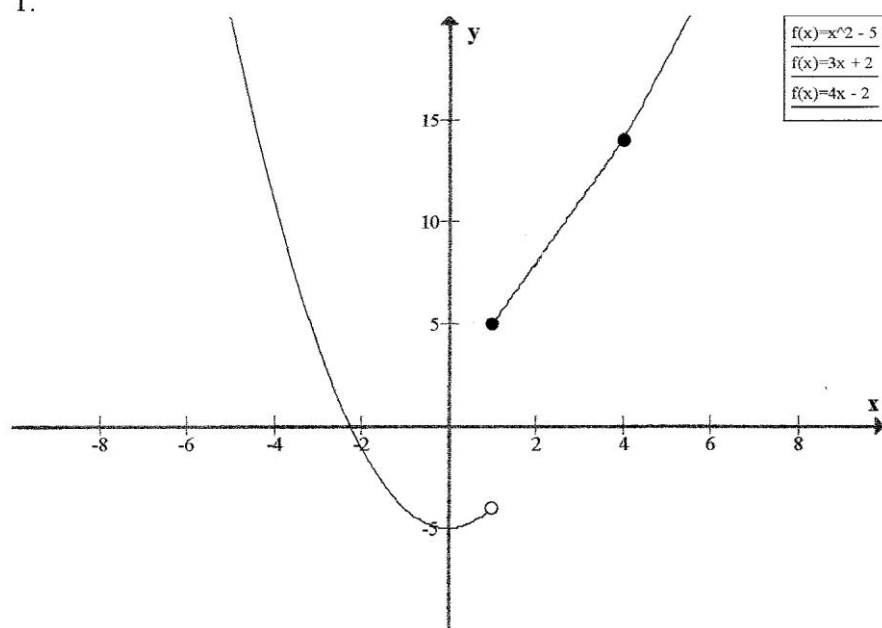
No existe límite para $x = 1$, discontinuidad de salto finito. Sí existe límite para $x = 4$, vale 14, es continua.

2. $b = -5$

3. a) 0 b) $\pm\infty$ c) 8 d) $\pm\infty$

4. a) $e^{-2/5}$ b) e^{-2}

1. 1.



$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 - 5 = -4 \quad \neq \quad \nexists \text{ limite en } x=1$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 3x + 2 = 5 \quad \text{discontinua de salto finito}$$

$$\lim_{x \rightarrow 4^-} 3x + 2 = 14 = \lim_{x \rightarrow 4^+} 4x - 2 = f(4)$$

\exists limite y es continua en $x=4$

— o —

$$2. \lim_{x \rightarrow \infty} \frac{-x^2 + 2x}{bx(1+x)} = \frac{1}{5}$$

$$\lim_{x \rightarrow \infty} \frac{-x^2 + 2x}{bx(1+x)} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{-x^2 + 2x}{bx^2 + bx} =$$

$$\lim_{x \rightarrow \infty} \frac{-\frac{x^2}{x^2} + \frac{2x}{x^2}}{\frac{bx^2}{x^2} + \frac{bx}{x^2}} = \lim_{x \rightarrow \infty} \frac{-1 + \frac{2}{x} \rightarrow 0}{b + \frac{b}{x} \rightarrow 0} = \frac{-1}{b} = \frac{1}{5}$$

\Downarrow
 $b = -5$

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$$3. a) \lim_{x \rightarrow \infty} \frac{2x^2 - 5x + 6}{x^3 - x^2 + 8} = \left[\frac{\infty}{\infty} \right] =$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{2x^2}{x^3} - \frac{5x}{x^3} + \frac{6}{x^3}}{\frac{x^3}{x^3} - \frac{x^2}{x^3} + \frac{8}{x^3}} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x} - \frac{5}{x^2} + \frac{6}{x^3}}{1 - \frac{1}{x} + \frac{8}{x^3}} =$$

$= \frac{0}{1} = 0$

$$b) \lim_{x \rightarrow \infty} \frac{4x^4 - 2x}{x(x^2 - 1)} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{4x^4 - 2x}{x^3 - x} =$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{4x^4}{x^4} - \frac{2x}{x^4}}{\frac{x^3}{x^4} - \frac{x}{x^4}} = \lim_{x \rightarrow \infty} \frac{4 - \frac{2}{x^3} \rightarrow 0}{\frac{1}{x} - \frac{1}{x^3} \rightarrow 0} = \frac{4}{0} = \infty$$

$$3.c) \lim_{x \rightarrow \infty} \left(\frac{2x^2+1}{x^2+5} \right)^{\frac{3x+1}{x+2}} = \left(\frac{\infty}{\infty} \right)^{\left(\frac{\infty}{\infty} \right)} =$$

$$= \lim_{x \rightarrow \infty} \left(\frac{2x^2+1}{x^2+5} \right)^{\frac{3x+1}{x+2}} = 2^3 = 8$$

$$d) \lim_{x \rightarrow \infty} (\sqrt{x^3+2x} - 5x) = [\infty - \infty] =$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^3+2x} - 5x)(\sqrt{x^3+2x} + 5x)}{(\sqrt{x^3+2x} + 5x)} =$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^3+2x})^2 - (5x)^2}{\sqrt{x^3+2x} + 5x} = \lim_{x \rightarrow \infty} \frac{x^3+2x-25x^2}{\sqrt{x^3+2x} + 5x} =$$

$$= \lim_{x \rightarrow \infty} \frac{\overset{\text{grad } 3}{x^3+2x-25x^2}}{\underset{\text{grad } \frac{3}{2}}{\sqrt{x^3+2x} + 5x}} = \left[\frac{\infty}{\infty} \right] = \infty$$

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$$4. a) \lim_{x \rightarrow 2} \left(\frac{x+3}{2x+1} \right)^{\frac{2}{x-2}} = [1^\infty] =$$

$$= e^{\lim_{x \rightarrow 2} \frac{2}{x-2} \cdot \left(\frac{x+3}{2x+1} - 1 \right)} =$$

$$= e^{\lim_{x \rightarrow 2} \frac{2}{x-2} \cdot \frac{x+3-2x-1}{2x+1}} = e^{\lim_{x \rightarrow 2} \frac{2}{x-2} \cdot \frac{-x+2}{2x+1}} =$$

$$= e^{\lim_{x \rightarrow 2} \frac{2}{x-2} \cdot \frac{-(x-2)}{2x+1}} = e^{\lim_{x \rightarrow 2} \frac{-2}{2x+1}} = e^{-2/5}$$

$$b) \lim_{x \rightarrow \infty} \left(\frac{3x+1}{3x+4} \right)^{2x} = [1^\infty] =$$

$$= e^{\lim_{x \rightarrow \infty} 2x \left(\frac{3x+1}{3x+4} - 1 \right)} =$$

$$= e^{\lim_{x \rightarrow \infty} 2x \left(\frac{3x+1-3x-4}{3x+4} \right)} = e^{\lim_{x \rightarrow \infty} 2x \left(\frac{-3}{3x+4} \right)} =$$

$$= e^{\lim_{x \rightarrow \infty} \frac{-6x}{3x+4}} = e^{-2}$$

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