1.- a) **(1 pto)** Demuestra la propiedad de los logaritmos que dice que:

$$log_a\left(\frac{M}{N}\right) = log_a M - log_a N$$

b) **(1 pto)** Calcula $\sqrt{2\cdot\sqrt{2\cdot\sqrt{log_2\left(\frac{0,16}{10^{-2}}\right)}}}$

2.- a) **(2 ptos)** Desarrollar por el Binomio de Newton: $\left(3x^4y^2 - \frac{1}{6}y^3x^5\right)^5 =$

b) **(1 pto)** Racionalizar: $\frac{4}{\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}}$

3.- a) **(1 pto)** Desarrolla el valor de: $2\cdot|3 - x| + 5x - 4 + 3\cdot|x + 1| =$

b) **(1 pto)** Resuelve la ecuación: $\frac{(x-1)^2 - (x-2)^2}{1-x^2} = \frac{x-1}{x+1} - \frac{x+1}{x-1}$

4.- a) **(2 ptos)** Resolver por el método de Gauss y clasifica: $\begin{cases} 2x - y - z = 0 \\ x - 2y + 7z = 3 \\ x - y + 2z = 1 \end{cases}$

b) **(1 pto)** Calcular: $\{\,|2x + 3| \leq 5\,\} \cap E\left(\frac{1}{2}, 3\right)$

① a) $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$ (Ver apuntes de teoría)

b) $\sqrt{2\sqrt{2\cdot\sqrt{\log_2\left(\frac{0{,}16}{10^{-2}}\right)}}} \overset{(*)}{=} \sqrt{2\cdot\sqrt{2\cdot\sqrt{4}}} = \sqrt{2\cdot\sqrt{2\cdot 2}} = \sqrt{2\cdot\sqrt{4}} = \sqrt{2\cdot 2} = \sqrt{4} = \boxed{2}$

(*) $\frac{0{,}16}{10^{-2}} = 0{,}16\cdot 10^2 = 16\cdot\underbrace{10^{-2}\cdot 10^2}_{10^0=1} = 16 = 2^4 \Rightarrow \log_2 2^4 = 4$

② a) $\left(3x^4y^2 - \frac{1}{6}y^3x^5\right)^5 = \binom{5}{0}(3x^4\cdot y^2)^5 - \binom{5}{1}(3x^4y^2)^4\left(\frac{1}{6}y^3x^5\right) +$

$+ \binom{5}{2}(3x^4y^2)^3\cdot\left(\frac{1}{6}y^3x^5\right)^2 - \binom{5}{3}(3x^4y^2)^2\left(\frac{1}{6}y^3x^5\right)^3 + \binom{5}{4}(3x^4y^2)\left(\frac{1}{6}y^3x^5\right)^4 - \binom{5}{5}\left(\frac{1}{6}y^3x^5\right)^5$

$= 1\cdot 243\cdot x^{20}y^{10} - 5\cdot 81\cdot x^{16}\cdot y^8\cdot\frac{1}{6}\cdot y^3\cdot x^5 + 10\cdot 27\cdot x^{12}\cdot y^6\cdot\frac{1}{36}y^6\cdot x^{10} - 10\cdot 9\cdot x^8 y^4\cdot\frac{1}{216}y^9\cdot x^{15} +$

$+ 5\cdot 3\cdot x^4\cdot y^2\cdot\frac{1}{1296}\cdot y^{12}\cdot x^{20} - 1\cdot\frac{1}{7.776}y^{15}\cdot x^{25} =$

$= \boxed{243x^{20}y^{10} - \frac{135}{2}x^{21}y^{11} + \frac{15}{2}x^{22}y^{12} - \frac{5}{12}\cdot x^{23}\cdot y^{13} + \frac{5}{432}x^{24}\cdot y^{14} - \frac{1}{7.776}x^{25}\cdot y^{15}}$

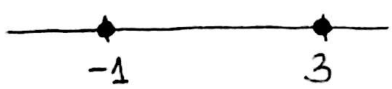
b) $\frac{4}{\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}} = \frac{4\cdot\left(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}}\right)}{\left(\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}\right)\left(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}}\right)} = \frac{4\cdot\left(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}}\right)}{\left(\sqrt{4+2\sqrt{3}}\right)^2 - \left(\sqrt{4-2\sqrt{3}}\right)^2}$

$= \frac{4\cdot\left(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}}\right)}{4+2\sqrt{3} - (4-2\sqrt{3})} = \frac{\cancel{4}\cdot\left(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}}\right)}{\cancel{4}\sqrt{3}} = \frac{\sqrt{3}\cdot\left(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}}\right)}{\sqrt{3}\cdot\sqrt{3}} =$

$= \boxed{\frac{\sqrt{12+6\sqrt{3}} + \sqrt{12-6\sqrt{3}}}{3}}$

③ a) $2\cdot|3-x| + 5x - 4 + 3\cdot|x+1| = *$

$|3-x| = \begin{cases} 3-x & \text{si } 3-x \geq 0 \Rightarrow 3 \geq x \Rightarrow x \leq 3 \\ x-3 & \text{si } 3-x < 0 \Rightarrow 3 < x \Rightarrow x > 3 \end{cases}$



$|x+1| = \begin{cases} x+1 & \text{si } x+1 \geq 0 \Rightarrow x \geq -1 \\ -x-1 & \text{si } x+1 < 0 \Rightarrow x < -1 \end{cases}$

$$=(*)=\begin{cases} 2\cdot(3-x)+5x-4+3(-x-1) & \text{si} \quad x<-1 \\ 2\cdot(3-x)+5x-4+3(x+1) & \text{si} \quad -1\le x\le 3 \\ 2\cdot(x-3)+5x-4+3(x+1) & \text{si} \quad x>3 \end{cases} \Rightarrow$$

$$=\boxed{\begin{cases} -1 & \text{si} \quad x<-1 \\ 6x+5 & \text{si} \quad -1\le x\le 3 \\ 10x-7 & \text{si} \quad x>3 \end{cases}}$$

b) $\dfrac{(x-1)^2-(x-2)^2}{(1-x^2)} = \dfrac{x-1}{x+1} - \dfrac{x+1}{x-1} \Rightarrow \dfrac{x^2-2x+1-x^2+4x-4}{(1-x^2)} = \dfrac{(x-1)^2-(x+1)^2}{x^2-1} \Rightarrow$

$\Rightarrow \dfrac{2x-3}{1-x^2} = \dfrac{x^2-2x+1-x^2-2x-1}{x^2-1} \Rightarrow \dfrac{2x-3}{1-x^2} = \dfrac{-4x}{x^2-1} \Rightarrow \dfrac{2x-3}{1-x^2} = \dfrac{4x}{1-x^2}$

$\Rightarrow (2x-3)(1-x^2) = 4x(1-x^2) \Rightarrow (2x-3)(1-x^2)-4x(1-x^2)=0$

$\Rightarrow (1-x^2)(2x-3-4x)=0 \Rightarrow (1-x^2)\cdot(-2x-3)=0 \Rightarrow$

$\Rightarrow \begin{cases} 1-x^2=0 \Rightarrow 1=x^2 \Rightarrow \boxed{x=\pm\sqrt{1}=\pm 1 \quad \text{NO VALE}} \\ -2x-3=0 \Rightarrow -3=2x \Rightarrow \boxed{x=-\frac{3}{2}} \end{cases}$ (harían 0 alguno de los 3 denominadores de la ec. de partida)

Comprobación:

Si $x=-\frac{3}{2} \Rightarrow \dfrac{\left(-\frac{5}{2}\right)^2-\left(-\frac{7}{2}\right)^2}{-\frac{5}{4}} = \dfrac{-\frac{5}{2}}{-\frac{1}{2}} - \dfrac{-\frac{1}{2}}{-\frac{5}{2}} \Rightarrow \dfrac{-\frac{24}{4}}{-\frac{5}{4}} = 5-\frac{1}{5} \Rightarrow$

$\frac{24}{5} = \frac{24}{5} \Rightarrow$ sí es solución

4) a) $\begin{cases} 2x-y-z=0 \\ x-2y+7z=3 \\ x-y+2z=1 \end{cases} \xrightarrow{E_1 \leftrightarrow E_3} \begin{cases} x-y+2z=1 \\ x-2y+7z=3 \\ 2x-y-z=0 \end{cases} \xrightarrow[E_3-2E_1]{E_2-E_1} \begin{cases} x-y+2z=1 \\ -y+5z=2 \\ y-5z=-2 \end{cases} \xrightarrow{E_3+E_2} \begin{cases} x-y+2z=1 \\ -y+5z=2 \\ 0=0 \end{cases}$

Sistema Compatible Indeterminado: $z=t$

$\begin{cases} x-y+2t=1 \\ -y+5t=2 \Rightarrow 5t-2=y \end{cases} \Rightarrow x-(5t-2)+2t=1 \Rightarrow x-3t+2=1 \Rightarrow x=3t-1$

$$\boxed{S: (3t-1,\ 5t-2,\ t) \quad \forall t\in\mathbb{R}}$$

4 b) $\{|2x+3|\le 5\} \cap E\left(\frac{1}{2},3\right) = \boxed{\left(-\frac{5}{2}, 1\right]}$

$|2x+3|\le 5 \Rightarrow \left|x+\frac{3}{2}\right| \le \frac{5}{2} \Rightarrow E\left[-\frac{3}{2},\frac{5}{2}\right] = \left[-\frac{3}{2}-\frac{5}{2}, -\frac{3}{2}+\frac{5}{2}\right] =$

$= [-4, 1]$

$E\left(\frac{1}{2},3\right) = \left(\frac{1}{2}-3, \frac{1}{2}+3\right) = \left(-\frac{5}{2}, \frac{7}{2}\right)$

Graficamente: