

Resuelve las siguientes inecuaciones racionales:

a) $\frac{5x-2}{2x+1} \geq 0$

➤ Ceros

$$5x - 2 = 0 \Rightarrow x = \underline{\frac{2}{5}}$$

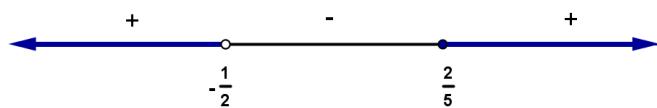
➤ Polos

$$2x + 1 = 0 \Rightarrow x = \underline{-\frac{1}{2}}$$

$$x = -1 \Rightarrow \frac{(-)}{(-)} = +$$

$$x = 0 \Rightarrow \frac{(-)}{(+)}) = -$$

$$x = 1 \Rightarrow \frac{(+)}{(+)} = +$$



Solución: $x \in \left(-\infty, -\frac{1}{2}\right) \cup \left[\frac{2}{5}, +\infty\right)$

b) $\frac{x^2-1}{x+2} \leq 0$

➤ Ceros: $x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow \begin{cases} x = -1 \\ x = 1 \end{cases}$

➤ Polos: $x + 2 = 0 \Rightarrow x = -2$

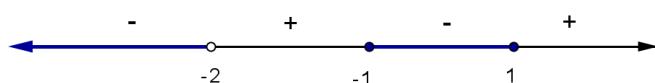
➤ Luego, factorizando, tenemos: $\frac{x^2-1}{x+2} < 0 \Leftrightarrow \frac{(x-1)(x+1)}{(x+2)} \leq 0$

$$x = -3 \Rightarrow \frac{(-)(-)}{(-)} = -$$

$$x = 0 \Rightarrow \frac{(-)(+)}{(+)}) = -$$

$$x = -1,5 \Rightarrow \frac{(-)(-)}{(+)}) = +$$

$$x = 2 \Rightarrow \frac{(+)(+)}{(+)}) = +$$



Solución: $x \in (-\infty, -2) \cup [-1, 1]$

c) $\frac{x^2 - 5x + 4}{x^2 - 5x + 6} \geq 0$

➤ Ceros: $x^2 - 5x + 4 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2} = \begin{cases} x = 4 \\ x = 1 \end{cases}$

➤ Polos: $x^2 - 5x + 6 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2} = \begin{cases} x = 3 \\ x = 2 \end{cases}$

➤ Luego, factorizando, tenemos: $\frac{x^2 - 5x + 4}{x^2 - 5x + 6} \geq 0 \Leftrightarrow \frac{(x-4)(x-1)}{(x-3)(x-2)} \geq 0$

$$x = 0 \Rightarrow \frac{(-)(-)}{(-)(-)} = +$$

$$x = 3,5 \Rightarrow \frac{(-)(+)}{(+)(+)} = -$$

$$x = 1,5 \Rightarrow \frac{(-)(+)}{(-)(-)} = -$$

$$x = 4 \Rightarrow \frac{(+)(+)}{(+)(+)} = +$$

$$x = 2,5 \Rightarrow \frac{(-)(+)}{(-)(+)} = +$$



Solución: $x \in (-\infty, 1] \cup (2, 3) \cup [4, +\infty)$

d) $\frac{x^3 + x^2 - 5x + 3}{x^3 + 5x^2 + 3x - 9} \leq 0$

➤ Ceros: $x^3 + x^2 - 5x + 3 = 0$

Posibles raíces enteras = {divisores de 3} = {±1, ±3}

	1	+1	-5	+3	
	1	+1	+2	-3	
	1	+2	-3	0	

$\Rightarrow x^3 + x^2 - 5x + 3 = (x-1)(x^2 + 2x - 3)$

Luego $x^3 + x^2 - 5x + 3 = 0 \Leftrightarrow (x-1)(x^2 + 2x - 3) = 0 \Leftrightarrow \begin{cases} x-1=0 \Rightarrow x=1 \\ x^2 + 2x - 3 = 0 \quad (*) \end{cases}$

(*) $x^2 + 2x - 3 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4+12}}{2} = \frac{-2 \pm 4}{2} = \begin{cases} x=1 \\ x=-3 \end{cases}$

Por tanto, Ceros $\rightarrow x = 1(\text{doble}) \quad x = -3$

➤ Polos: $x^3 + 5x^2 + 3x - 9 = 0$

Posibles raíces enteras = {divisores de -9 } = $\{\pm 1, \pm 3, \pm 9\}$

$$\begin{array}{c} \begin{array}{cccc} 1 & +5 & +3 & -9 \\ \hline 1 & +1 & +6 & +9 \\ \hline 1 & +6 & +9 & \boxed{0} \end{array} \\ \Rightarrow x^3 + 5x^2 + 3x - 9 = (x-1)(x^2 + 6x + 9) \end{array}$$

$$\text{Luego } x^3 + 5x^2 + 3x - 9 = 0 \Leftrightarrow (x-1)(x^2 + 6x + 9) = 0 \Leftrightarrow \begin{cases} x-1=0 \Rightarrow x=1 \\ x^2 + 6x + 9 = 0 \Rightarrow (x+3)^2 = 0 \Rightarrow x=-3 \text{ (doble)} \end{cases}$$

Por tanto, Polos $\rightarrow x = -3(\text{doble}) \quad x = 1$

$$\text{➤ Luego, factorizando, tenemos: } \frac{x^3 + x^2 - 5x + 3}{x^3 + 5x^2 + 3x - 9} \leq 0 \Leftrightarrow \frac{(x-1)^2(x+3)}{(x+3)^2(x-1)} \leq 0$$

$$x = -4 \Rightarrow \frac{(+)(-)}{(+)(-)} = +$$

$$x = 0 \Rightarrow \frac{(+)(+)}{(+)(-)} = -$$

$$x = 2 \Rightarrow \frac{(+)(+)}{(+)(+)} = +$$



Solución: $x \in (-3, -1)$

$$\text{e) } \frac{x^2 - 2x - 3}{x^2 + x - 2} < 0$$

$$\text{➤ Ceros: } x^2 - 2x - 3 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{cases} x = 3 \\ x = -1 \end{cases}$$

$$\text{➤ Polos: } x^2 + x - 2 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = \begin{cases} x = 1 \\ x = -2 \end{cases}$$

$$\text{➤ Luego, factorizando, tenemos: } \frac{x^2 - 2x - 3}{x^2 + x - 2} < 0 \Leftrightarrow \frac{(x-3)(x+1)}{(x-1)(x+2)} < 0$$

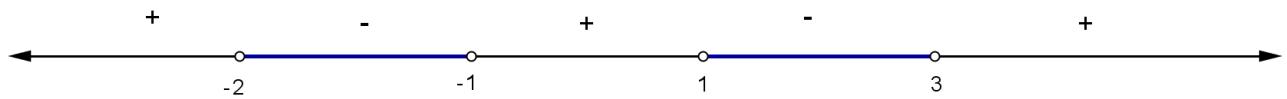
$$x = -3 \Rightarrow \frac{(-)(-)}{(-)(-)} = +$$

$$x = -1,5 \Rightarrow \frac{(-)(-)}{(-)(+)} = -$$

$$x = 0 \Rightarrow \frac{(-)(+)}{(-)(+)} = +$$

$$x = 2 \Rightarrow \frac{(-)(+)}{(+) (+)} = -$$

$$x = 4 \Rightarrow \frac{(+) (+)}{(+) (+)} = +$$



Solución: $x \in (-2, -1) \cup (1, 3)$

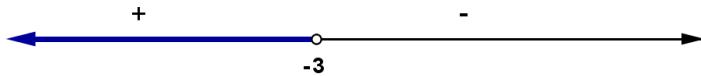
$$\mathbf{f)} \quad \frac{x-1}{x+3} - 1 > 0 \Rightarrow \frac{x-1-1 \cdot (x+3)}{x+3} > 0 \Rightarrow \frac{x-1-x-3}{x+3} > 0 \Rightarrow \frac{-4}{x+3} > 0$$

➤ Ceros: No tiene

➤ Polos: $x+3=0 \Rightarrow x=-3$

$$x = -4 \Rightarrow \frac{(-)}{(-)} = +$$

$$x = -2 \Rightarrow \frac{(-)}{(+) } = -$$



Solución: $x \in (-\infty, -3)$

$$\mathbf{g)} \quad \frac{x^2}{x^2-2} \leq 2 \Rightarrow \frac{x^2}{x^2-2} - 2 \leq 0 \Rightarrow \frac{x^2 - 2(x^2-2)}{x^2-2} \leq 0 \Rightarrow \frac{x^2 - 2x^2 + 4}{x^2-2} \leq 0 \Rightarrow \frac{-x^2 + 4}{x^2-2} \leq 0$$

➤ Ceros: $-x^2 + 4 = 0 \Rightarrow x^2 = 4 \Rightarrow \begin{cases} x = -2 \\ x = 2 \end{cases}$

➤ Polos: $x^2 - 2 = 0 \Rightarrow x^2 = 2 \Rightarrow \begin{cases} x = -\sqrt{2} \\ x = \sqrt{2} \end{cases}$

➤ Luego, factorizando, tenemos: $\frac{-x^2 + 4}{x^2 - 2} \leq 0 \Leftrightarrow \frac{-1(x-2)(x+2)}{(x-\sqrt{2})(x+\sqrt{2})} \leq 0 \xrightarrow{:(-1)} \frac{(x-2)(x+2)}{(x-\sqrt{2})(x+\sqrt{2})} \geq 0$

$$x = -3 \Rightarrow \frac{(-)(-)}{(-)(-)} = +$$

$$x = 1,5 \Rightarrow \frac{(-)(+)}{(+)(+)} = -$$

$$x = -1,5 \Rightarrow \frac{(-)(+)}{(-)(-)} = -$$

$$x = 3 \Rightarrow \frac{(+)(+)}{(+)(+)} = +$$

$$x = 0 \Rightarrow \frac{(-)(+)}{(-)(+)} = +$$



Solución: $x \in (-\infty, -2] \cup (-\sqrt{2}, \sqrt{2}) \cup [2, +\infty)$

h) $\frac{x^2 - 5}{x^2 + 3} < x \Rightarrow \frac{x^2 - 5}{x^2 + 3} - x < 0 \Rightarrow \frac{x^2 - 5 - x(x^2 + 3)}{x^2 + 3} < 0 \Rightarrow \frac{x^2 - 5 - x^3 - 3x}{x^2 + 3} < 0 \Rightarrow \frac{-x^3 + x^2 - 3x - 5}{x^2 + 3} < 0$

➤ Ceros: $-x^3 + x^2 - 3x - 5 = 0$

Posibles raíces enteras = {divisores de -5 } = $\{\pm 1, \pm 5\}$

$$\begin{array}{c|cccc} & -1 & +1 & -3 & -5 \\ \hline -1 & & +1 & -2 & +5 \\ & -1 & +2 & -5 & \boxed{0} \end{array} \Rightarrow -x^3 + x^2 - 3x - 5 = (x+1)(-x^2 + 2x - 5)$$

Luego $-x^3 + x^2 - 3x - 5 = 0 \Leftrightarrow (x+1)(-x^2 + 2x - 5) = 0 \Leftrightarrow \begin{cases} x+1=0 \Rightarrow x=-1 \\ -x^2 + 2x - 5 = 0 \end{cases} (*)$

$$(*) -x^2 + 2x - 5 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4 - 20}}{-2} \Rightarrow \text{No solución real}$$

Por tanto, Ceros $\rightarrow x = -1$

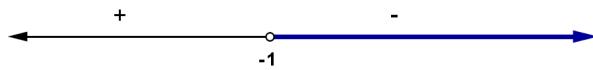
➤ Polos: $x^2 + 3 = 0 \Rightarrow x^2 = -3 \Rightarrow \text{No solución real}$

Por tanto, Polos \rightarrow No tiene

➤ Luego, factorizando, tenemos: $\frac{-x^3 + x^2 - 3x - 5}{x^2 + 3} < 0 \Leftrightarrow \frac{(x+1)(-x^2 + 2x - 5)}{x^2 + 3} < 0$

$$x = -2 \Rightarrow \frac{(-)(-)}{(+)} = +$$

$$x = 0 \Rightarrow \frac{(+)(-)}{(+)} = -$$



Solución: $x \in (-1, +\infty)$

i) $\frac{1}{x-1} < \frac{2}{x+4} \Rightarrow \frac{1}{x-1} - \frac{2}{x+4} < 0 \Rightarrow \frac{1(x+4) - 2(x-1)}{(x-1)(x+4)} < 0 \Rightarrow \frac{x+4 - 2x + 2}{(x-1)(x+4)} < 0 \Rightarrow \frac{6-x}{(x-1)(x+4)} < 0$

Por tanto, hay que resolver la inecuación: $\frac{6-x}{(x-1)(x+4)} < 0$

➤ Ceros: $6-x=0 \Rightarrow x=6$

➤ Polos: $(x-1)(x+4)=0 \Rightarrow \begin{cases} x=1 \\ x=-4 \end{cases}$

$$\text{➤ } \frac{6-x}{(x-1)(x+4)} < 0$$

$$x = -5 \Rightarrow \frac{(+)}{(-)(-)} = +$$

$$x = 2 \Rightarrow \frac{(+)}{(+)(+)} = +$$

$$x = 0 \Rightarrow \frac{(+)}{(-)(+)} = -$$

$$x = 7 \Rightarrow \frac{(-)}{(+)(+)} = -$$



Solución: $x \in (-4, 1) \cup (6, +\infty)$

$$\begin{aligned}
\mathbf{j)} \quad & \frac{3x-2}{x-1} - 1 \geq \frac{2x-1}{x+1} \Rightarrow \frac{3x-2}{x-1} - 1 - \frac{2x-1}{x+1} \geq 0 \Rightarrow \frac{(3x-2)(x+1) - (x-1)(x+1) - (2x-1)(x-1)}{(x-1)(x+1)} \geq 0 \Rightarrow \\
& \Rightarrow \frac{(3x^2 + 3x - 2x - 2) - (x^2 - 1) - (2x^2 - 2x - x + 1)}{(x-1)(x+1)} \geq 0 \Rightarrow \\
& \Rightarrow \frac{3x^2 + 3x - 2x - 2 - x^2 + 1 - 2x^2 + 2x + x - 1}{(x-1)(x+1)} \geq 0 \Rightarrow \frac{4x-2}{(x-1)(x+1)} \geq 0
\end{aligned}$$

Por tanto, hay que resolver la inecuación: $\frac{4x-2}{(x-1)(x+1)} \geq 0$

➤ Ceros: $4x-2=0 \Rightarrow x=\underline{\frac{1}{2}}$

➤ Polos: $(x-1)(x+1)=0 \Rightarrow \begin{cases} x=1 \\ x=-1 \end{cases}$

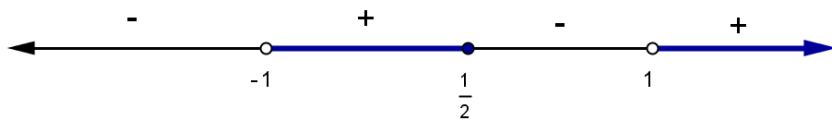
➤ $\frac{4x-2}{(x-1)(x+1)} \geq 0$

$$x=-2 \Rightarrow \frac{(-)}{(-)(-)} = -$$

$$x=0,75 \Rightarrow \frac{(+)}{(-)(+)} = -$$

$$x=0 \Rightarrow \frac{(-)}{(-)(+)} = +$$

$$x=2 \Rightarrow \frac{(+)}{(+)(+)} = +$$



Solución: $x \in \left(-1, \frac{1}{2}\right] \cup (1, +\infty)$

$$\mathbf{k)} \quad \frac{1}{x+3} - \frac{1}{x^2-9} \leq \frac{-1}{x-3} \Rightarrow \frac{1}{x+3} - \frac{1}{(x-3)(x+3)} + \frac{1}{x-3} \leq 0 \Rightarrow \frac{x-3-1+x+3}{(x-3)(x+3)} \leq 0 \Rightarrow \frac{2x-1}{(x-3)(x+3)} \leq 0$$

Por tanto, hay que resolver la inecuación: $\frac{2x-1}{(x-3)(x+3)} \leq 0$

➤ Ceros: $2x-1=0 \Rightarrow x=\underline{\frac{1}{2}}$

➤ Polos: $(x-3)(x+3)=0 \Rightarrow \begin{cases} x=3 \\ x=-3 \end{cases}$

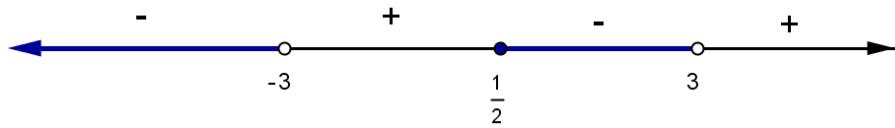
➤ $\frac{2x-1}{(x-3)(x+3)} \leq 0$

$$x = -4 \Rightarrow \frac{(-)}{(-)(-)} = -$$

$$x = 0 \Rightarrow \frac{(-)}{(-)(+)} = +$$

$$x = 1 \Rightarrow \frac{(+)}{(-)(+)} = -$$

$$x = 4 \Rightarrow \frac{(+)}{(+)(+)} = +$$



Solución: $x \in (-\infty, -3) \cup \left[\frac{1}{2}, 3 \right)$

I) $\frac{-x}{x-2} \leq \frac{x+1}{x+2} - \frac{6x+1}{x^2-4} \Rightarrow \frac{6x+1}{(x-2)(x+2)} - \frac{x+1}{x+2} - \frac{x}{x-2} \leq 0 \Rightarrow$

$$\Rightarrow \frac{6x+1-(x-2)(x+1)-x(x+2)}{(x-2)(x+2)} \leq 0 \Rightarrow \frac{6x+1-x^2+2x-x+2-x^2-2x}{(x-2)(x+2)} \leq 0 \Rightarrow \frac{-2x^2+5x+3}{(x-2)(x+2)} \leq 0$$

Por tanto, hay que resolver la inecuación: $\frac{-2x^2+5x+3}{(x-2)(x+2)} \leq 0$

➤ Ceros: $-2x^2 + 5x + 3 = 0$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4 \cdot (-2) \cdot 3}}{2 \cdot (-2)} = \frac{-5 \pm \sqrt{25 + 24}}{-4} = \frac{-5 \pm 7}{-4} = \begin{cases} x = -\frac{1}{2} \\ x = 3 \end{cases}$$

➤ Polos: $(x-2)(x+2) = 0 \Rightarrow \begin{cases} x = 2 \\ x = -2 \end{cases}$

➤ Luego, factorizando, tenemos: $\frac{-2x^2+5x+3}{(x-2)(x+2)} \leq 0 \Leftrightarrow \frac{-2\left(x+\frac{1}{2}\right)(x-3)}{(x-2)(x+2)} \leq 0$

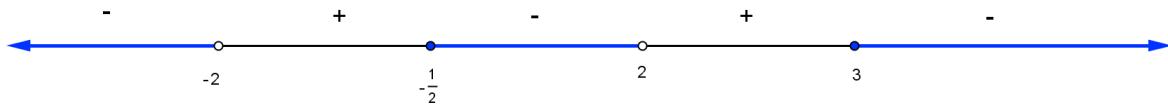
$$x = -3 \Rightarrow \frac{(-)(-)(-)}{(-)(-)} = -$$

$$x = -1 \Rightarrow \frac{(-)(-)(-)}{(-)(+)} = +$$

$$x = 0 \Rightarrow \frac{(-)(+)(-)}{(-)(+)} = -$$

$$x = 2,5 \Rightarrow \frac{(-)(+)(-)}{(+) (+)} = +$$

$$x = 4 \Rightarrow \frac{(-)(+)(+)}{(+) (+)} = -$$



Solución: $x \in (-\infty, -2) \cup \left[-\frac{1}{2}, 2\right) \cup [3, +\infty)$

$$\text{m)} \quad x^2 - \frac{64}{x^2} \geq -12 \Rightarrow x^2 - \frac{64}{x^2} + 12 \geq 0 \Rightarrow \frac{x^4 - 64 + 12x^2}{x^2} \geq 0 \Rightarrow \frac{x^4 + 12x^2 - 64}{x^2} \geq 0$$

Por tanto, hay que resolver la inecuación: $\frac{x^4 + 12x^2 - 64}{x^2} \geq 0$

➤ Ceros: $x^4 + 12x^2 - 64 = 0$

1) Hacemos el cambio de variable $x^2 = t$ y la ecuación se convierte en la ecuación de 2º grado:

$$t^2 + 12t - 64 = 0$$

2) Resolvemos la ecuación de segundo grado:

$$t^2 + 12t - 64 = 0 \Leftrightarrow t = \frac{-12 \pm \sqrt{144 + 256}}{2} = \frac{-12 \pm 20}{2} = \begin{cases} t = 4 \\ t = -16 \end{cases}$$

3) Deshacemos el cambio de variable

$$\bullet t = 4 \Rightarrow x^2 = 4 \Rightarrow x = \sqrt{4} \Rightarrow x = \pm 2$$

• $t = -16 \Rightarrow x^2 = -16 \Rightarrow x = \sqrt{-16} \Rightarrow$ no tiene solución real

➤ Polos: $x^2 = 0 \Rightarrow x = 0$

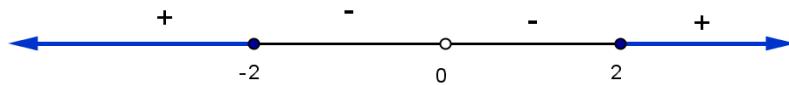
$$\bullet \frac{x^4 + 12x^2 - 64}{x^2} \geq 0$$

$$x = -3 \Rightarrow \frac{(+)}{(+)} = +$$

$$x = 1 \Rightarrow \frac{(-)}{(+)}) = -$$

$$x = -1 \Rightarrow \frac{(-)}{(+)} = -$$

$$x = 3 \Rightarrow \frac{(+)}{(+)} = +$$



Solución: $x \in (-\infty, -2] \cup [2, +\infty)$

$$\text{n)} \quad \frac{3x-3}{x-1} \leq \frac{7x+1}{x^2-1} - \frac{x^2+2}{x+1} \Rightarrow$$

$$\Rightarrow \frac{3x-3}{x-1} - \frac{7x+1}{(x-1)(x+1)} + \frac{x^2+2}{x+1} \leq 0 \Rightarrow \frac{(3x-3)(x+1) - (7x+1) + (x^2+2)(x-1)}{(x-1)(x+1)} \leq 0$$

$$\Rightarrow \frac{3x^2 + 3x - 3x - 3 - 7x - 1 + x^3 - x^2 + 2x - 2}{(x-1)(x+1)} \leq 0 \Rightarrow \frac{x^3 + 2x^2 - 5x - 6}{(x-1)(x+1)} \leq 0$$

Por tanto, hay que resolver la inecuación: $\frac{x^3 + 2x^2 - 5x - 6}{(x-1)(x+1)} \leq 0$

➤ Ceros: $x^3 + 2x^2 - 5x - 6 = 0$

$$\begin{array}{c|cccc} & 1 & +2 & -5 & -6 \\ \hline -1 & & -1 & -1 & +6 \\ \hline & 1 & +1 & -6 & \boxed{0} \end{array} \Rightarrow x^3 + 2x^2 - 5x - 6 = (x+1)(x^2 + x - 6)$$

$$x^3 + 2x^2 - 5x - 6 = 0 \Leftrightarrow (x+1)(x^2 + x - 6) = 0 \Leftrightarrow \begin{cases} x+1=0 \Rightarrow x=-1 \\ x^2+x-6=0 \quad (*) \end{cases}$$

$$(*) \quad x^2 + x - 6 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} = \begin{cases} x=2 \\ x=-3 \end{cases}$$

Por tanto, los ceros del polinomio son: $x = -1$, $x = 2$ y $x = -3$

$$➤ \text{Polos: } (x-1)(x+1) = 0 \Rightarrow \begin{cases} x=1 \\ x=-1 \end{cases}$$

$$➤ \text{Luego, factorizando, tenemos: } \frac{x^3 + 2x^2 - 5x - 6}{(x-1)(x+1)} \leq 0 \Leftrightarrow \frac{(x+1)(x-2)(x+3) \leq 0}{(x-1)(x+1)}$$

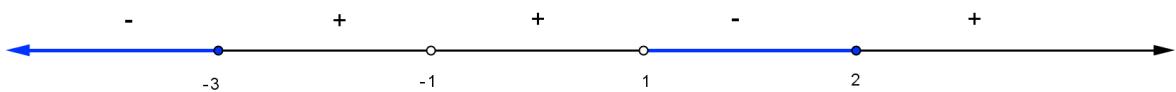
$$x = -4 \Rightarrow \frac{(-)(-)(-)}{(-)(-)} = -$$

$$x = 0 \Rightarrow \frac{(+)(-)(+)}{(-)(+)} = +$$

$$x = -2 \Rightarrow \frac{(-)(-)(+)}{(-)(-)} = +$$

$$x = 1,5 \Rightarrow \frac{(+)(-)(+)}{(+)(+)} = -$$

$$x = 3 \Rightarrow \frac{(+)(+)(+)}{(+)(+)} = +$$



Solución: $x \in (-\infty, -3] \cup (1, 2]$