Resuelve las siguientes integrales racionales:

1. $\int \frac{dx}{x^2-1}$

2. $\int \frac{x-1}{x^2+x-6}\,dx$

3. $\int \frac{x-5}{(x-1)^3}\,dx$

4. $\int \frac{x+2}{x^3-2x^2}\,dx$

5. $\int \frac{1}{(x-1)(x+2)^2}\,dx$

6. $\int \frac{x^3}{x-2}\,dx$

$$1.\quad \int \frac{1}{x^2-1}\,dx$$

$$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{A(x-1)+B(x+1)}{x^2-1}$$

$$1 = A(x-1) + B(x+1) \Rightarrow \begin{cases} A = -1/2 \\ B = 1/2 \end{cases}$$

$$\int \frac{1}{x^2-1}\,dx = \int \frac{-1/2}{x+1}\,dx + \int \frac{1/2}{x-1}\,dx = -\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C =$$

$$= \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C = \ln\sqrt{\left|\frac{x-1}{x+1}\right|} + C$$

$$2.\quad \int \frac{x-1}{x^2+x-6}\,dx = \int \frac{x-1}{(x-2)(x+3)}\,dx$$

$$\frac{x-1}{x^2+x-6} = \frac{x-1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{A(x+3)+B(x-2)}{(x-2)(x+3)}$$

$$x-1 = A(x+3) + B(x-2) \Rightarrow \begin{cases} A = 1/5 \\ B = 2/3 \end{cases}$$

$$\int \frac{x-1}{x^2+x-6}\,dx = \int \frac{1/5}{x-2}\,dx + \int \frac{2/3}{x+3}\,dx = \frac{1}{5}\ln|x-2| + \frac{2}{3}\ln|x+3| + C$$

$$3.\quad \int \frac{x-5}{(x-1)^3}\,dx$$

$$\frac{x-5}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} = \frac{A(x-1)^2 + B(x-1) + C}{(x-1)^3}$$

$$x-5 = A(x-1)^2 + B(x-1) + C \Rightarrow \begin{cases} A = -4 \\ B = 1 \\ C = -4 \end{cases}$$

$$\int \frac{x-5}{(x-1)^3}\,dx = \int \frac{-4}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx + \int \frac{-4}{(x-1)^3}\,dx =$$

$$= -4\ln|x-1| + \frac{(x-1)^{-1}}{-1} - \frac{4(x-1)^{-2}}{-2} + C = -4\ln|x-1| - \frac{1}{x-1} + \frac{2}{(x-1)^2} + C$$

$$4.\quad \int \frac{x+2}{x^3-2x^2}\,dx = \int \frac{x+2}{x^2(x-2)}\,dx$$

$$\frac{x+2}{x^3+2x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} = \frac{Ax(x-2) + B(x-2) + Cx^2}{x^2(x-2)}$$

$$x+2 = Ax(x-2) + B(x-2) + Cx^2 \Rightarrow \begin{cases} A = -1 \\ B = -1 \\ C = 1 \end{cases}$$

$$\int \frac{x+2}{x^3+2x^2}\,dx = \int \frac{-1}{x}\,dx + \int \frac{-1}{x^2}\,dx + \int \frac{1}{x-2}\,dx =$$

$$= -\ln|x| - \frac{x^{-1}}{-1} + \ln|x-2| = \ln\left|\frac{x-2}{x}\right| + \frac{1}{x} + C$$

5. $\displaystyle\int \frac{1}{(x-1)(x+2)^2}\,dx$

$$\frac{1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{A(x+2)^2 + B(x-1)(x+2) + C(x-1)}{(x-1)(x+2)^2}$$

$$1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1) \Rightarrow \begin{cases} A = 1/9 \\ B = -1/9 \\ C = -1/3 \end{cases}$$

$$\int \frac{1}{(x-1)(x+2)^2}\,dx = \int \frac{1/9}{x-1}\,dx - \int \frac{1/9}{x+2}\,dx - \int \frac{1/3}{(x+2)^2}\,dx =$$

$$= \frac{1}{9}\ln|x-1| - \frac{1}{9}\ln|x+2| - \frac{1}{3}\cdot\frac{(x+2)^{-1}}{-1} + C =$$

$$= \frac{1}{9}\ln\left|\frac{x-1}{x+2}\right| + \frac{1}{3(x+2)} + C$$

6. $\displaystyle\int \frac{x^3}{x-2}\,dx = \int \left(x^2+2x+4+\frac{8}{x-2}\right)dx = \frac{x^3}{3} + x^2 + 4x + 8\ln|x-2| + C$

$$\begin{array}{l|l} x^3 & x-2 \\ \hline -x^3+2x^2 & x^2+2x+4 \\ \hline \quad 2x^2 & \\ \quad -2x^2+4x & \\ \hline \qquad 4x & \\ \qquad -4x+8 & \\ \hline \qquad\quad 8 & \end{array}$$