06031

CALCULA LOS SIGUIENTES LÍMITES:

1) $$\lim_{x\to+\infty}\frac{2x^2-14x+12}{x^2-10x+4}=\frac{+\infty}{+\infty}(\mathrm{I})=\begin{cases}\lim\limits_{x\to+\infty}\dfrac{\frac{2x^2}{x^2}-\frac{14x}{x^2}+\frac{12}{x^2}}{\frac{x^2}{x^2}-\frac{10x}{x^2}+\frac{4}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{2-\frac{14}{x}+\frac{12}{x^2}}{1-\frac{10}{x}+\frac{4}{x^2}}=\dfrac{2-0+0}{1-0+0}=\dfrac{2}{1}=2\\ \lim\limits_{x\to+\infty}\dfrac{2x^2}{x^2}=\lim\limits_{x\to+\infty}2=2\end{cases}$$

2) $$\lim_{x\to+\infty}\frac{(2x^2-3)(5x-4)}{2x^3-2x^2+1}=\lim_{x\to+\infty}\frac{10x^3-8x^2-15x+12}{2x^3-2x^2+1}=\frac{+\infty}{+\infty}(\mathrm{I})=$$

$$\begin{cases}\lim\limits_{x\to+\infty}\dfrac{10x^3-8x^2-15x+12}{2x^3-2x^2+1}=\lim\limits_{x\to+\infty}\dfrac{\frac{10x^3}{x^3}-\frac{8x^2}{x^3}-\frac{15x}{x^3}+\frac{12}{x^3}}{\frac{2x^3}{x^3}-\frac{2x^2}{x^3}+\frac{1}{x^3}}=\lim\limits_{x\to+\infty}\dfrac{10-\frac{8}{x}-\frac{15}{x^2}+\frac{12}{x^3}}{2-\frac{2}{x}+\frac{1}{x^3}}=\dfrac{10-0-0+0}{2-0+0}=5\\ \lim\limits_{x\to+\infty}\dfrac{10x^3-8x^2-15x+12}{2x^3-2x^2+1}=\dfrac{+\infty}{+\infty}(\mathrm{I})=\lim\limits_{x\to+\infty}\dfrac{10x^3}{2x^3}=\lim\limits_{x\to+\infty}5=5\end{cases}$$

3) $$\lim_{x\to-1}\frac{x^3+1}{x^2+2x+1}=\frac{0}{0}(\mathrm{I})=\lim_{x\to-1}\frac{(x+1)(x^2-x+1)}{(x+1)^2}=\lim_{x\to-1}\frac{x^2-x+1}{x+1}=\frac{3}{0}=\begin{cases}\lim\limits_{x\to-1^-}\dfrac{x^2-x+1}{x+1}=\dfrac{3}{0^-}=-\infty\\ \lim\limits_{x\to-1^+}\dfrac{x^2-x+1}{x+1}=\dfrac{3}{0^+}=+\infty\end{cases}$$

$$\begin{array}{r|rrr|r} & 1 & 0 & 0 & +1\\ -1 & & -1 & +1 & -1\\ \hline & 1 & -1 & +1 & \boxed{0}\end{array}\Rightarrow x^3+1=(x-1)(x^2-x+1)$$

$$x^2+2x+1\underset{\substack{identidad\\notable}}{=}(x+1)^2$$

4) $$\lim_{x\to-1}\frac{x^2-1}{x^2+3x+2}=\frac{0}{0}(\mathrm{I})=\lim_{x\to-1}\frac{(x+1)(x-1)}{(x+1)(x+2)}=\lim_{x\to-1}\frac{x-1}{x+2}=\frac{-2}{1}=-2$$

$$\begin{array}{r|rr|r} & 1 & +3 & +2\\ -1 & & -1 & -2\\ \hline & 1 & +2 & \boxed{0}\end{array}\Rightarrow x^2+3x+2=(x+1)(x+2)$$

5) $$\lim_{x\to a}\frac{3x^2-2ax-a^2}{2x^2-3ax+a^2}=\frac{0}{0}(\mathrm{I})=\lim_{x\to a}\frac{(x-a)(3x+a)}{(x-a)(2x-a)}=\lim_{x\to a}\frac{3x+a}{2x-a}=\frac{3a+a}{2a-a}=\frac{4a}{a}=4 \qquad (a\neq 0)$$

$$\begin{array}{c|ccc} & 3 & -2a & -a^2 \\ a & & +3a & +a^2 \\ \hline & 3 & +a & \boxed{0} \end{array} \Rightarrow 3x^2-2ax-a^2=(x-a)(3x+a)$$

$$\begin{array}{c|ccc} & 2 & -3a & +a^2 \\ a & & +2a & -a^2 \\ \hline & 2 & -a & \boxed{0} \end{array} \Rightarrow 2x^2-3ax+a^2=(x-a)(2x-a)$$

6) $$\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^2}\right)=\infty-\infty(\mathrm{I})=\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{(1-x)(1+x)}\right)=\lim_{x\to 1}\frac{1+x-3}{(1-x)(1+x)}=\lim_{x\to 1}\frac{x-2}{(1-x)(1+x)}=\frac{-1}{0}=$$

$$\begin{cases}\displaystyle\lim_{x\to 1^-}\frac{x-2}{(1-x)(1+x)}=\frac{-1}{(0^+)(2)}=\frac{-1}{0^+}=-\infty \\ \displaystyle\lim_{x\to 1^+}\frac{x-2}{(1-x)(1+x)}=\frac{-1}{(0^-)(2)}=\frac{-1}{0^-}=+\infty\end{cases}$$

7) $$\lim_{x\to+\infty}\frac{\sqrt{x+4}}{x-2}=\frac{+\infty}{+\infty}(\mathrm{I})=\begin{cases}\displaystyle\lim_{x\to+\infty}\frac{\sqrt{\frac{x}{x^2}+\frac{4}{x^2}}}{\frac{x}{x^2}-\frac{2}{x^2}}=\lim_{x\to+\infty}\frac{\sqrt{\frac{1}{x}+\frac{4}{x^2}}}{1-\frac{2}{x}}=\frac{\sqrt{0+0}}{1-0}=\frac{0}{1}=0 \\ \displaystyle\lim_{x\to+\infty}\frac{\sqrt{x+4}}{x-2}=\lim_{x\to+\infty}\frac{\sqrt{x}}{x}=\lim_{x\to+\infty}\frac{x^{\frac{1}{2}}}{x}=\lim_{x\to+\infty}\frac{1}{x^{\frac{1}{2}}}=\lim_{x\to+\infty}\frac{1}{\sqrt{x}}=\frac{1}{+\infty}=0\end{cases}$$

8) $$\lim_{x\to 2}\frac{\sqrt{x+2}-2}{\sqrt{x+7}-3}=\frac{0}{0}(\mathrm{I})=\lim_{x\to 2}\frac{(\sqrt{x+2}-2)(\sqrt{x+2}+2)(\sqrt{x+7}+3)}{(\sqrt{x+7}-3)(\sqrt{x+7}+3)(\sqrt{x+2}+2)}=\lim_{x\to 2}\frac{[(\sqrt{x+2}-2)(\sqrt{x+2}+2)](\sqrt{x+7}+3)}{[(\sqrt{x+7}-3)(\sqrt{x+7}+3)](\sqrt{x+2}+2)}=$$

$$=\lim_{x\to 2}\frac{[(\sqrt{x+2})^2-(2)^2](\sqrt{x+7}+3)}{[(\sqrt{x+7})^2-(3)^2](\sqrt{x+2}+2)}=\lim_{x\to 2}\frac{(x+2-4)(\sqrt{x+7}+3)}{(x+7-9)(\sqrt{x+2}+2)}=\lim_{x\to 2}\frac{(x-2)(\sqrt{x+7}+3)}{(x-2)(\sqrt{x+2}+2)}=$$

$$=\lim_{x\to 2}\frac{(\sqrt{x+7}+3)}{(\sqrt{x+2}+2)}=\frac{\sqrt{2+7}+3}{\sqrt{2+2}+2}=\frac{3+3}{2+2}=\frac{6}{4}=\frac{3}{2}$$

9) $$\lim_{x\to 3}\frac{9-x^2}{\sqrt{3x}-3}=\frac{0}{0}(\mathrm{I})=\lim_{x\to 3}\frac{(9-x^2)(\sqrt{3x}+3)}{(\sqrt{3x}-3)(\sqrt{3x}+3)}=\lim_{x\to 3}\frac{(9-x^2)(\sqrt{3x}+3)}{(\sqrt{3x})^2-(3)^2}=\lim_{x\to 3}\frac{(9-x^2)(\sqrt{3x}+3)}{3x-9}\underset{\substack{\textit{factorizar}\\ \textit{polinomios}}}{=}$$

$$=\lim_{x\to 3}\frac{(3-x)(3+x)(\sqrt{3x}+3)}{3(x-3)}==\lim_{x\to 3}\frac{-(x-3)(3+x)(\sqrt{3x}+3)}{3(x-3)}==\lim_{x\to 3}\frac{-(3+x)(\sqrt{3x}+3)}{3}=\frac{-6\cdot 6}{3}=-12$$

10) $\lim\limits_{x\to+\infty}\left(\sqrt{x(x+1)}-x\right)=\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x}-x\right)=\lim\limits_{x\to+\infty}\dfrac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x}=\lim\limits_{x\to+\infty}\dfrac{(\sqrt{x^2+x})^2-(x)^2}{\sqrt{x^2+x}-x}=$

$$=\lim_{x\to+\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}=\lim_{x\to+\infty}\frac{x}{\sqrt{x^2+x}+x}=\frac{+\infty}{+\infty}(\mathrm{I})=\begin{cases} *\lim\limits_{x\to+\infty}\dfrac{x}{\sqrt{x^2+x}+x}=\lim\limits_{x\to+\infty}\dfrac{\frac{x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+\frac{x}{x}}= \\ =\lim\limits_{x\to+\infty}\dfrac{1}{\sqrt{1+\frac{1}{x}}+1}=\dfrac{1}{\sqrt{1+0}+1}=\dfrac{1}{2} \\ *\lim\limits_{x\to+\infty}\dfrac{x}{\sqrt{x^2}+x}=\lim\limits_{x\to+\infty}\dfrac{x}{2x}=\lim\limits_{x\to+\infty}\dfrac{1}{2}=\dfrac{1}{2}\end{cases}$$

11) $\lim\limits_{x\to4}\left(\dfrac{x+6}{x^2-16}-\dfrac{x+1}{x^2-4x}\right)=\infty-\infty(\mathrm{I})=\lim\limits_{x\to4}\left(\dfrac{x+6}{(x-4)(x+4)}-\dfrac{x+1}{x(x-4)}\right)=\lim\limits_{x\to4}\left(\dfrac{x(x+6)-(x+4)(x+1)}{x(x-4)(x+4)}\right)=$

$$=\lim_{x\to4}\left(\frac{x^2+6x-x^2-x-4x-4}{x(x-4)(x+4)}\right)=\lim_{x\to4}\frac{x-4}{x(x-4)(x+4)}=\lim_{x\to4}\frac{1}{x(x+4)}=\frac{1}{32}$$

12) $\lim\limits_{x\to0}\left(\dfrac{x^2+3}{x^3}-\dfrac{1}{x}\right)=\infty-\infty(\mathrm{I})=\lim\limits_{x\to0}\left(\dfrac{x^2+3-x^2}{x^3}\right)=\lim\limits_{x\to0}\left(\dfrac{3}{x^3}\right)=\dfrac{3}{0}=\begin{cases}\lim\limits_{x\to0^-}\dfrac{3}{x^3}=\dfrac{3}{0^-}=-\infty \\ \lim\limits_{x\to0^+}\dfrac{3}{x^3}=\dfrac{3}{0^+}=+\infty\end{cases}$

13) $\lim\limits_{x\to1}\left(\dfrac{2}{(x-1)^2}-\dfrac{1}{x(x-1)}\right)=\infty-\infty(\mathrm{I})=\lim\limits_{x\to1}\left(\dfrac{2x-(x-1)}{x(x-1)^2}\right)=\lim\limits_{x\to1}\dfrac{2x-x+1}{x(x-1)^2}=\lim\limits_{x\to1}\dfrac{x+1}{x(x-1)^2}=\dfrac{2}{0}=$

$$\begin{cases}\lim\limits_{x\to1^-}\dfrac{x+1}{x(x-1)^2}=\dfrac{2}{1\cdot(0^-)^2}=\dfrac{2}{0^+}=+\infty \\ \lim\limits_{x\to1^+}\dfrac{x+1}{x(x-1)^2}=\dfrac{2}{1\cdot(0^+)^2}=\dfrac{2}{0^+}=+\infty\end{cases}$$

14) $\lim\limits_{x\to+\infty}\left(\sqrt{\dfrac{x}{1+2x}}\right)^{\frac{x}{x+1}}=\lim\limits_{x\to+\infty}\left(\left(\dfrac{x}{1+2x}\right)^{\frac{1}{2}}\right)^{\frac{x}{x+1}}=\lim\limits_{x\to+\infty}\left(\dfrac{x}{1+2x}\right)^{\frac{x}{2x+2}}=\left[\lim\limits_{x\to+\infty}\left(\dfrac{x}{1+2x}\right)\right]^{\lim\limits_{x\to+\infty}\left(\frac{x}{2x+2}\right)}=$

$$=\left(\frac{1}{2}\right)^{\frac{1}{2}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

$$\circ\lim_{x\to+\infty}\left(\frac{x}{1+2x}\right)=\frac{+\infty}{+\infty}=\lim_{x\to+\infty}\frac{\frac{x}{x}}{\frac{1}{x}+\frac{2x}{x}}=\lim_{x\to+\infty}\frac{1}{\frac{1}{x}+2}=\frac{1}{0+2}=\frac{1}{2}$$

$$\circ\lim_{x\to+\infty}\left(\frac{x}{2x+2}\right)=\frac{1}{+\infty}=\lim_{x\to+\infty}\frac{\frac{x}{x}}{\frac{2x}{x}+\frac{2}{x}}=\lim_{x\to+\infty}\frac{1}{2+\frac{2}{x}}=\frac{1}{2+0}=\frac{1}{2}$$

15) $\lim\limits_{x\to2}\left(\dfrac{3}{x+1}\right)^{\frac{x^2+2x+5}{x}}=1^{\frac{13}{2}}=1$

16) $\lim\limits_{x\to+\infty}\left(\dfrac{x+1}{x-1}\right)^{x}=1^{\infty}=e^{\lim\limits_{x\to+\infty}\left(\frac{x+1}{x-1}-1\right)\cdot x}\underset{(*)}{=}e^{2}$

$$(*)\lim_{x\to+\infty}\left(\frac{x+1}{x-1}-1\right)\cdot x=\lim_{x\to+\infty}\left(\frac{x+1-x+1}{x-1}\right)\cdot x=\lim_{x\to+\infty}\left(\frac{2}{x-1}\right)\cdot x=\lim_{x\to+\infty}\frac{2x}{x-1}=\frac{+\infty}{+\infty}(\mathrm{I})=\begin{cases}\circ\lim\limits_{x\to+\infty}\dfrac{2x}{x-1}=\lim\limits_{x\to+\infty}\dfrac{\frac{2x}{x}}{\frac{x}{x}-\frac{1}{x}}=\\ \lim\limits_{x\to+\infty}\dfrac{2}{1-\frac{1}{x}}=\dfrac{2}{1-0}=2\\ \circ\lim\limits_{x\to+\infty}\dfrac{2x}{x}=\lim\limits_{x\to+\infty}2=2\end{cases}$$

17) $\lim\limits_{x\to1}\left(\dfrac{x^2+2x+1}{x+3}\right)^{\frac{x^3+2x^2+5}{x^2+4x-5}}=1^{\infty}(\mathrm{I})=e^{\lim\limits_{x\to1}\left(\frac{x^2+2x+1}{x+3}-1\right)\left(\frac{x^3+2x^2+5}{x^2+4x-5}\right)}\underset{(*)}{=}e^{1}=e$

$$(*)\lim_{x\to1}\left(\frac{x^2+2x+1}{x+3}-1\right)\cdot\left(\frac{x^3+2x^2+5}{x^2+4x-5}\right)=\lim_{x\to1}\left(\frac{x^2+2x+1-x-3}{x+3}\right)\cdot\left(\frac{x^3+2x^2+5}{x^2+4x-5}\right)=$$

$$=\lim_{x\to1}\left(\frac{x^2+x-2}{x+3}\right)\cdot\left(\frac{x^3+2x^2+5}{x^2+4x-5}\right)\underset{\substack{factorizar\\polinomios}}{=}=\lim_{x\to1}\frac{(x-1)(x+2)}{(x+3)}\cdot\frac{x^3+2x^2+5}{(x-1)(x+5)}=\lim_{x\to1}\frac{(x+2)(x^3+2x^2+5)}{(x+3)(x+5)}=\frac{3\cdot8}{4\cdot6}=1$$

18) $\lim\limits_{x\to+\infty}\left(1-\dfrac{1}{x}\right)^{x^2}=1^{\infty}(\mathrm{I})=e^{\lim\limits_{x\to+\infty}\left(1-\frac{1}{x}-1\right)\cdot x^2}\underset{(*)}{=}e^{-\infty}=0$

$$(*)\lim_{x\to+\infty}\left(1-\frac{1}{x}-1\right)\cdot x^2=\lim_{x\to+\infty}\left(-\frac{1}{x}\right)\cdot x^2=\lim_{x\to+\infty}\left(-\frac{x^2}{x}\right)=\lim_{x\to+\infty}(-x)=-\infty$$

19) $\lim\limits_{x\to+\infty}\left(\sqrt{x^3-x}-\sqrt{x^3-2x^2}\right)=\infty-\infty(\mathrm{I})=\lim\limits_{x\to+\infty}\dfrac{(\sqrt{x^3-x}-\sqrt{x^3-2x^2})(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}{(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}=$

$$=\lim_{x\to+\infty}\frac{(\sqrt{x^3-x})^2-(\sqrt{x^3-2x^2})^2}{(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}=\lim_{x\to+\infty}\frac{x^3-x-(x^3-2x^2)}{(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}=\lim_{x\to+\infty}\frac{x^3-x-x^3+2x^2}{(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}=$$

$$=\lim_{x\to+\infty}\frac{2x^2-x}{(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}=\frac{+\infty}{+\infty}(\mathrm{I})=$$

$$=\begin{cases}\lim\limits_{x\to+\infty}\dfrac{\frac{2x^2}{x^2}-\frac{x}{x^2}}{\sqrt{\frac{x^3}{x^4}-\frac{x}{x^4}}+\sqrt{\frac{x^3}{x^4}-\frac{2x^2}{x^4}}}=\lim\limits_{x\to+\infty}\dfrac{2-\frac{1}{x}}{\sqrt{\frac{1}{x}-\frac{1}{x^3}}+\sqrt{\frac{1}{x}-\frac{2}{x^2}}}=\dfrac{2-0}{\sqrt{0-0}+\sqrt{0+0}}=\dfrac{2}{0}=+\infty\\ \lim\limits_{x\to+\infty}\dfrac{2x^2}{\sqrt{x^3}+\sqrt{x^3}}=\lim\limits_{x\to+\infty}\dfrac{2x^2}{2\sqrt{x^3}}=\lim\limits_{x\to+\infty}\dfrac{x^2}{\sqrt{x^3}}=\lim\limits_{x\to+\infty}\dfrac{x^2}{x^{\frac{3}{2}}}=\lim\limits_{x\to+\infty}x^{\frac{1}{2}}=\lim\limits_{x\to+\infty}\sqrt{x}=+\infty\end{cases}$$

20) $\lim\limits_{x\to+\infty}\dfrac{3^x}{2^x}=\lim\limits_{x\to+\infty}\left(\dfrac{3}{2}\right)^{x}=\left(\dfrac{3}{2}\right)^{+\infty}=+\infty$

21) $\lim\limits_{x\to+\infty}\dfrac{2^x}{3^x}=\lim\limits_{x\to+\infty}\left(\dfrac{2}{3}\right)^{x}=\left(\dfrac{2}{3}\right)^{+\infty}=0$

22) $\lim\limits_{x\to+\infty}\left(\dfrac{2x+5}{2x}\right)^{\frac{3x^2-1}{x+2}}=1^{\infty}(\mathrm{I})=e^{\lim\limits_{x\to+\infty}\left(\frac{2x+5}{2x}-1\right)\cdot\frac{3x^2-1}{x+2}}\underset{(*)}{=}e^{\frac{15}{2}}$

$$(*)\lim_{x\to+\infty}\left(\frac{2x+5}{2x}-1\right)\cdot\frac{3x^2-1}{x+2}=\lim_{x\to+\infty}\left(\frac{2x+5-2x}{2x}\right)\cdot\frac{3x^2-1}{x+2}=\lim_{x\to+\infty}\left(\frac{5}{2x}\right)\cdot\frac{3x^2-1}{x+2}=\lim_{x\to+\infty}\frac{15x^2-5}{2x^2+4x}=\frac{+\infty}{+\infty}=$$

$$=\begin{cases}\lim\limits_{x\to+\infty}\dfrac{15x^2-5}{2x^2+4x}=\lim\limits_{x\to+\infty}\dfrac{\frac{15x^2}{x^2}-\frac{5}{x^2}}{\frac{2x^2}{x^2}+\frac{4x}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{15-\frac{5}{x^2}}{2+\frac{4}{x}}=\dfrac{15-0}{2+0}=\dfrac{15}{2}\\[2ex] \lim\limits_{x\to+\infty}\dfrac{15x^2-5}{2x^2+4x}=\lim\limits_{x\to+\infty}\dfrac{15x^2}{2x^2}=\lim\limits_{x\to+\infty}\dfrac{15}{2}=\dfrac{15}{2}\end{cases}$$

23) $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x+a}+\sqrt{x+b}}{\sqrt{x}}=\dfrac{+\infty}{+\infty}(\mathrm{I})=$

$$\begin{cases}\circ\lim\limits_{x\to+\infty}\dfrac{\sqrt{x+a}+\sqrt{x+b}}{\sqrt{x}}=\lim\limits_{x\to+\infty}\dfrac{\sqrt{\frac{x}{x}+\frac{a}{x}}+\sqrt{\frac{x}{x}+\frac{b}{x}}}{\sqrt{\frac{x}{x}}}==\lim\limits_{x\to+\infty}\dfrac{\sqrt{1+\frac{a}{x}}+\sqrt{1+\frac{b}{x}}}{\sqrt{1}}=\\[2ex] =\dfrac{\sqrt{1+0}+\sqrt{1+0}}{\sqrt{1}}=\dfrac{1+1}{1}=2\\[2ex] \circ\lim\limits_{x\to+\infty}\dfrac{\sqrt{x}+\sqrt{x}}{\sqrt{x}}=\lim\limits_{x\to+\infty}\dfrac{2\sqrt{x}}{\sqrt{x}}=\lim\limits_{x\to+\infty}2=2\end{cases}$$

24) $\lim\limits_{x\to+\infty}\left(\sqrt[x-3]{\dfrac{3x+1}{3x+5}}\right)^{x^2-2x}=\lim\limits_{x\to+\infty}\left(\left(\dfrac{3x+1}{3x+5}\right)^{\frac{1}{x-3}}\right)^{x^2-2x}=\lim\limits_{x\to+\infty}\left(\dfrac{3x+1}{3x+5}\right)^{\frac{x^2-2x}{x-3}}=1^{\infty}(\mathrm{I})=e^{\lim\limits_{x\to+\infty}\left(\frac{3x+1}{3x+5}-1\right)\cdot\frac{x^2-2x}{x-3}}\underset{(*)}{=}e^{-\frac{4}{3}}$

$$(*)\lim_{x\to+\infty}\left(\frac{3x+1}{3x+5}-1\right)\cdot\frac{x^2-2x}{x-3}=\lim_{x\to+\infty}\left(\frac{3x+1-3x-5}{3x+5}\right)\cdot\frac{x^2-2x}{x-3}=\lim_{x\to+\infty}\left(\frac{-4}{3x+5}\right)\cdot\frac{x^2-2x}{x-3}=$$

$$=\lim_{x\to+\infty}\left(\frac{-4x^2+8x}{3x^2-4x-15}\right)=\frac{-\infty}{+\infty}=\begin{cases}\lim\limits_{x\to+\infty}\dfrac{-4x^2+8x}{3x^2-4x-15}=\lim\limits_{x\to+\infty}\dfrac{\frac{-4x^2}{x^2}+\frac{8x}{x^2}}{\frac{3x^2}{x^2}-\frac{4x}{x^2}-\frac{15}{x^2}}==\lim\limits_{x\to+\infty}\dfrac{-4+\frac{8}{x}}{3-\frac{4}{x}-\frac{15}{x^2}}=\dfrac{-4+0}{3-0-0}=-\dfrac{4}{3}\\[2ex] \lim\limits_{x\to+\infty}\dfrac{-4x^2}{3x^2}=\lim\limits_{x\to+\infty}\dfrac{-4}{3}=-\dfrac{4}{3}\end{cases}$$

25) $\lim\limits_{x\to+\infty}\dfrac{2x^2-3x-4}{\sqrt{x^4+1}}=\dfrac{+\infty}{+\infty}=$

$$\begin{cases}\lim\limits_{x\to+\infty}\dfrac{2x^2-3x-4}{\sqrt{x^4+1}}=\lim\limits_{x\to+\infty}\dfrac{\frac{2x^2}{x^2}-\frac{3x}{x^2}-\frac{4}{x^2}}{\sqrt{\frac{x^4}{x^4}+\frac{1}{x^4}}}=\lim\limits_{x\to+\infty}\dfrac{2-\frac{3}{x}-\frac{4}{x^2}}{\sqrt{1+\frac{1}{x^4}}}=\dfrac{2-0-0}{\sqrt{1+0}}=\dfrac{2}{1}=2\\[2ex] \lim\limits_{x\to+\infty}\dfrac{2x^2}{\sqrt{x^4}}=\lim\limits_{x\to+\infty}\dfrac{2x^2}{x^2}=\lim\limits_{x\to+\infty}2=2\end{cases}$$

26) $$\lim_{x\to+\infty}\frac{x^2}{10+x\sqrt{x}}=\lim_{x\to+\infty}\frac{x^2}{10+\sqrt{x^3}}=\frac{+\infty}{+\infty}(\mathrm{I})=\begin{cases}\lim\limits_{x\to+\infty}\dfrac{x^2}{10+\sqrt{x^3}}=\lim\limits_{x\to+\infty}\dfrac{\frac{x^2}{x^2}}{\frac{10}{x^2}+\sqrt{\frac{x^3}{x^4}}}=\lim\limits_{x\to+\infty}\dfrac{1}{\frac{10}{x^2}+\sqrt{\frac{1}{x}}}=\dfrac{1}{0}=+\infty\\ \lim\limits_{x\to+\infty}\dfrac{x^2}{\sqrt{x^3}}=\lim\limits_{x\to+\infty}\dfrac{x^2}{x^{\frac{3}{2}}}=\lim\limits_{x\to+\infty}x^{\frac{1}{2}}=\lim\limits_{x\to+\infty}\sqrt{x}=+\infty\end{cases}$$

27) $$\lim_{x\to+\infty}\frac{\sqrt[3]{x^2+1}}{x+1}=\frac{+\infty}{+\infty}(\mathrm{I})=\begin{cases}\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{x^2+1}}{x+1}=\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{\frac{x^2}{x^3}+\frac{1}{x^3}}}{\frac{x}{x}+\frac{1}{x}}=\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{\frac{1}{x}+\frac{1}{x^3}}}{1+\frac{1}{x}}=\dfrac{\sqrt[3]{0+0}}{1+0}=\dfrac{0}{1}=0\\ \lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{x^2}}{x}=\lim\limits_{x\to+\infty}\dfrac{x^{\frac{2}{3}}}{x}=\lim\limits_{x\to+\infty}x^{-\frac{1}{3}}=\lim\limits_{x\to+\infty}\dfrac{1}{x^{\frac{1}{3}}}=\lim\limits_{x\to+\infty}\dfrac{1}{\sqrt[3]{x}}=\dfrac{1}{+\infty}=0\end{cases}$$

28) $$\lim_{x\to a}\frac{x^2-(a+1)x+a}{x^3-a^3}=\frac{0}{0}(\mathrm{I})=\lim_{x\to a}\frac{(x-a)(x-1)}{(x-a)(x^2+ax+a^2)}=\lim_{x\to a}\frac{(x-1)}{(x^2+ax+a^2)}=\frac{a-1}{a^2+a^2+a^2}=\frac{a-1}{3a^2}$$

$$\begin{array}{c|ccc} & 1 & -(a+1) & +a \\ a & & +a & -a \\ \hline & 1 & -1 & \boxed{0} \end{array}\Rightarrow x^2-(a+1)x+a=(x-a)(x-1)$$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -a^3 \\ a & & +a & +a^2 & +a^3 \\ \hline & 1 & +a & +a^2 & \boxed{0} \end{array}\Rightarrow x^3-a^3=(x-a)(x^2+ax+a^2)$$

29) $$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}=\frac{0}{0}(\mathrm{I})=\lim_{x\to1}\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)}=\lim_{x\to1}\frac{(\sqrt{x})^2-(1)^2}{(x-1)(\sqrt{x}+1)}=\lim_{x\to1}\frac{x-1}{(x-1)(\sqrt{x}+1)}=\lim_{x\to1}\frac{1}{(\sqrt{x}+1)}=\frac{1}{\sqrt{1}+1}=\frac{1}{2}$$

30) $$\lim_{x\to+\infty}\frac{\sqrt[3]{x^2-2x+3}}{x-5}=\frac{+\infty}{+\infty}(\mathrm{I})=\begin{cases}\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{x^2-2x+3}}{x-5}=\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{\frac{x^2}{x^3}-\frac{2x}{x^3}+\frac{3}{x^3}}}{\frac{x}{x}-\frac{5}{x}}=\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{\frac{1}{x}-\frac{2}{x^2}+\frac{3}{x^3}}}{1-\frac{5}{x}}=\dfrac{\sqrt[3]{0-0+0}}{1-0}=0\\ \lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{x^2}}{x}=\lim\limits_{x\to+\infty}\dfrac{x^{\frac{2}{3}}}{x}=\lim\limits_{x\to+\infty}\dfrac{1}{x^{\frac{1}{3}}}=\lim\limits_{x\to+\infty}\dfrac{1}{\sqrt[3]{x}}=\dfrac{1}{+\infty}=0\end{cases}$$

31) $\lim\limits_{x\to-\infty}\frac{\sqrt{3x+x^4}}{2+5x}=\lim\limits_{x\to+\infty}\frac{\sqrt{3(-x)+(-x)^4}}{2+5(-x)}=\lim\limits_{x\to+\infty}\frac{\sqrt{-3x+x^4}}{2-5x}=\frac{+\infty}{-\infty}(\mathrm{I})=$

$$\begin{cases}\lim\limits_{x\to+\infty}\frac{\sqrt{\frac{-3x}{x^4}+\frac{x^4}{x^4}}}{\frac{2}{x^2}-\frac{5x}{x^2}}=\lim\limits_{x\to+\infty}\frac{\sqrt{\frac{-3}{x^3}+1}}{\frac{2}{x^2}-\frac{5}{x}}=\frac{\sqrt{0+1}}{0-0}=\frac{1}{0}=-\infty\\ \lim\limits_{x\to+\infty}\frac{\sqrt{x^4}}{-5x}=\lim\limits_{x\to+\infty}\frac{x^2}{-5x}=\lim\limits_{x\to+\infty}\frac{x}{-5}=\frac{+\infty}{-5}=-\infty\end{cases}$$

32) $\lim\limits_{x\to+\infty}\frac{\sqrt{4x+x^2}}{2+5x^2}=\frac{+\infty}{+\infty}(\mathrm{I})=\begin{cases}\lim\limits_{x\to+\infty}\frac{\sqrt{\frac{4x}{x^4}+\frac{x^2}{x^4}}}{\frac{2}{x^2}+\frac{5x^2}{x^2}}=\lim\limits_{x\to+\infty}\frac{\sqrt{\frac{4}{x^3}+\frac{1}{x^2}}}{\frac{2}{x^2}+5}=\frac{\sqrt{0+0}}{0+5}=\frac{0}{5}=0\\ \lim\limits_{x\to+\infty}\frac{\sqrt{x^2}}{5x^2}=\lim\limits_{x\to+\infty}\frac{x}{5x^2}=\lim\limits_{x\to+\infty}\frac{2}{5x}=\frac{2}{+\infty}=0\end{cases}$

33) $\lim\limits_{x\to-\infty}\frac{5x+1}{\sqrt[3]{x^2-3x-1}}=\lim\limits_{x\to+\infty}\frac{5(-x)+1}{\sqrt[3]{(-x)^2-3(-x)-1}}=\lim\limits_{x\to+\infty}\frac{-5x+1}{\sqrt[3]{x^2+3x-1}}=\frac{-\infty}{+\infty}(\mathrm{I})=$

$$\begin{cases}\lim\limits_{x\to+\infty}\frac{-5x+1}{\sqrt[3]{x^2+3x-1}}=\lim\limits_{x\to+\infty}\frac{\frac{-5x}{x}+\frac{1}{x}}{\sqrt[3]{\frac{x^2}{x^3}+\frac{3x}{x^3}-\frac{1}{x^3}}}=\lim\limits_{x\to+\infty}\frac{-5+\frac{1}{x}}{\sqrt[3]{\frac{1}{x}+\frac{3}{x^2}-\frac{1}{x^3}}}=\frac{-5+0}{\sqrt[3]{0+0-0}}=\frac{-5}{0}=-\infty\\ \lim\limits_{x\to+\infty}\frac{-5x+1}{\sqrt[3]{x^2+3x-1}}=\lim\limits_{x\to+\infty}\frac{-5x}{\sqrt[3]{x^2}}=\lim\limits_{x\to+\infty}\frac{-5x}{x^{\frac{2}{3}}}=\lim\limits_{x\to+\infty}(-5x^{\frac{1}{3}})=\lim\limits_{x\to+\infty}(-5\sqrt[3]{x})=-\infty\end{cases}$$

34) $\lim\limits_{x\to7}\frac{2-\sqrt{x-3}}{x^2-49}=\frac{0}{0}(\mathrm{I})=\lim\limits_{x\to7}\frac{(2-\sqrt{x-3})(2+\sqrt{x-3})}{(x^2-49)(2+\sqrt{x-3})}=\lim\limits_{x\to7}\frac{(2)^2-(\sqrt{x-3})^2}{(x^2-49)(2+\sqrt{x-3})}=\lim\limits_{x\to7}\frac{4-(x-3)}{(x^2-49)(2+\sqrt{x-3})}=$

$=\lim\limits_{x\to7}\frac{4-x+3}{(x^2-49)(2+\sqrt{x-3})}=\lim\limits_{x\to7}\frac{7-x}{(x^2-49)(2+\sqrt{x-3})}=\lim\limits_{x\to7}\frac{-(x-7)}{(x-7)(x+7)(2+\sqrt{x-3})}=$

$=\lim\limits_{x\to7}\frac{-1}{(x+7)(2+\sqrt{x-3})}=\frac{-1}{14\cdot(2+2)}=-\frac{1}{56}$

35) $\lim\limits_{x\to+\infty}\frac{3^x}{10x^2+5x-2}=\frac{+\infty}{+\infty}(\mathrm{I})=+\infty$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de *x*.

36) $\lim\limits_{x\to+\infty}\frac{\sqrt{x^5-3}}{10x^2-9}=\frac{+\infty}{+\infty}(\mathrm{I})=\lim\limits_{x\to+\infty}\frac{\sqrt{x^5}}{10x^2}=\lim\limits_{x\to+\infty}\frac{x^{\frac{5}{2}}}{10x^2}=\lim\limits_{x\to+\infty}\frac{x^{\frac{1}{2}}}{10}=\lim\limits_{x\to+\infty}\frac{\sqrt{x}}{10}=+\infty$

37) $\lim_{x\to-\infty}\frac{\sqrt{x^5-3}}{10x^2-9}=\lim_{x\to+\infty}\frac{\sqrt{(-x)^5-3}}{10(-x)^2-9}=\lim_{x\to+\infty}\frac{\sqrt{-x^5-3}}{10x^2-9}=\frac{-\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\sqrt{-x^5}}{10x^2}=\lim_{x\to+\infty}\frac{-x^{\frac{5}{2}}}{10x^2}=\lim_{x\to+\infty}\frac{-x^{\frac{1}{2}}}{10}=$

$=\lim_{x\to+\infty}\frac{-\sqrt{x}}{10}=-\infty$

38) $\lim_{x\to+\infty}\frac{\log(x^3+1)}{10x^2+1}=\frac{+\infty}{+\infty}(\mathrm{I})=0$

Las potencias de *x* son infinitos de orden superior a cualquier función logarítmica.

39) $\lim_{x\to+\infty}\frac{5^x}{\log(x^3+1)}=\frac{+\infty}{+\infty}(\mathrm{I})=+\infty$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier función logarítmica.

40) $\lim_{x\to+\infty}(2^x-\sqrt{x^5-1})=\infty-\infty(\mathrm{I})=+\infty$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de *x*.

41) $\lim_{x\to+\infty}(10x^2-\sqrt{x^5-1})=\infty-\infty(\mathrm{I})=-\infty$

Dadas dos potencias de *x* la de mayor exponente es un infinito de orden superior

42) $\lim_{x\to+\infty}[\log(x^3)-10x^2]=\infty-\infty(\mathrm{I})=-\infty$

Las potencias de *x* son infinitos de orden superior a cualquier función logarítmica.

43) $\lim_{x\to+\infty}(e^x-x^3)=\infty-\infty(\mathrm{I})=+\infty$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de *x*.

44) $\lim_{x\to+\infty}\frac{x^2+1}{e^x}=\frac{+\infty}{+\infty}(\mathrm{I})=0$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de *x*.

45) $\lim_{x\to+\infty}(\sqrt{x^2+x}-\sqrt{x+7})=\infty-\infty(\mathrm{I})=+\infty$

Dadas dos potencias de *x* la de mayor exponente es un infinito de orden superior

46) $\lim_{x\to+\infty}\frac{\ln(x^2+1)}{x}=\frac{+\infty}{+\infty}=0$

Las potencias de *x* son infinitos de orden superior a cualquier función logarítmica.

47) $\lim_{x\to-\infty}(0{,}5^x+1)=\lim_{x\to+\infty}(0{,}5^{-x}+1)=0{,}5^{-\infty}+1=+\infty+1=+\infty$

48) $\lim_{x\to-\infty}(2^{x+1}-5)=2^{-\infty}-5=0-5=-5$

49) $\lim\limits_{x\to+\infty}\left(1{,}2^x-\dfrac{3x^2}{x+1}\right)=\infty-\infty(\mathrm{I})=+\infty$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de *x*.

50) $\lim\limits_{x\leftarrow-\infty}\left(\dfrac{2x+7}{x}\right)^{1+3x}=\left[\lim\limits_{x\leftarrow-\infty}\left(\dfrac{2x+7}{x}\right)\right]^{\lim\limits_{x\leftarrow-\infty}(1+3x)}=2^{-\infty}=0$

∘ $\lim\limits_{x\leftarrow-\infty}\left(\dfrac{2x+7}{x}\right)=\dfrac{-\infty}{-\infty}(\mathrm{I})=\lim\limits_{x\leftarrow-\infty}\left(\dfrac{2x}{x}\right)=\lim\limits_{x\leftarrow-\infty}2=2$

∘ $\lim\limits_{x\leftarrow-\infty}(1+3x)=-\infty$

51) $\lim\limits_{x\to+\infty}\dfrac{x+\log x}{\log x}=\dfrac{+\infty}{+\infty}(\mathrm{I})=\lim\limits_{x\to+\infty}\left(\dfrac{x}{\log x}+\dfrac{\log x}{\log x}\right)=\lim\limits_{x\to+\infty}\left(\dfrac{x}{\log x}+1\right)=+\infty$

Las potencias de *x* son infinitos de orden superior a cualquier función logarítmica.

52) $\lim\limits_{x\to+\infty}\dfrac{3\cdot 2^x}{2^x+1}=\dfrac{+\infty}{+\infty}(\mathrm{I})=\lim\limits_{x\to+\infty}\dfrac{3\cdot 2^x}{2^x}=\lim\limits_{x\to+\infty}3=3$