

INTEGRACION POR SUSTITUCION

A veces es conveniente hacer un cambio de variable, para transformar la integral dada en otra, de forma conocida. La técnica en cuestión recibe el nombre de método de sustitución.

EJERCICIOS DESARROLLADOS

2.1.-Encontrar: $\int \frac{e^{\ell\eta x} dx}{x^2 + 7}$

Solución.- Como: $e^{\ell\eta x} = x$, se tiene: $\int \frac{e^{\ell\eta x} dx}{x^2 + 7} = \int \frac{x dx}{x^2 + 7}$

Sea la sustitución: $u = x^2 + 7$, donde: $du = 2x dx$, Dado que: $\int \frac{x dx}{x^2 + 7} = \frac{1}{2} \int \frac{2x dx}{x^2 + 7}$,

Se tiene: $\frac{1}{2} \int \frac{2x dx}{x^2 + 7} = \frac{1}{2} \int \frac{du}{u}$, integral que es inmediata.

Luego: $= \frac{1}{2} \int \frac{du}{u} \frac{1}{2} \ell\eta |u| + c = \frac{1}{2} \ell\eta |x^2 + 7| + c$

Respuesta: $\int \frac{e^{\ell\eta x} dx}{x^2 + 7} = \frac{1}{2} \ell\eta |x^2 + 7| + c$

2.2.-Encontrar: $\int \frac{e^{\ell\eta x^2} dx}{x^3 + 8}$

Solución.- Como: $e^{\ell\eta x^2} = x^2$, se tiene: $\int \frac{e^{\ell\eta x^2} dx}{x^3 + 8} = \int \frac{x^2 dx}{x^3 + 8}$

Sea la sustitución: $w = x^3 + 8$, donde: $dw = 3x^2 dx$, Dado que: $\int \frac{x^2 dx}{x^3 + 8} = \frac{1}{3} \int \frac{3x^2 dx}{x^3 + 8}$,

Se tiene: $\frac{1}{3} \int \frac{3x^2 dx}{x^3 + 8} = \frac{1}{3} \int \frac{dw}{w}$ integral que es inmediata.

Luego: $\frac{1}{3} \int \frac{dw}{w} = \frac{1}{3} \ell\eta |w| + c = \frac{1}{3} \ell\eta |x^3 + 8| + c$

Respuesta: $\int \frac{e^{\ell\eta x^2} dx}{x^3 + 8} = \frac{1}{3} \ell\eta |x^3 + 8| + c$

2.3.-Encontrar: $\int (x+2) \sin(x^2 + 4x - 6) dx$

Solución.- Sea la sustitución: $u = x^2 + 4x - 6$, donde: $du = (2x+4)dx$

Dado que: $\int (x+2) \sin(x^2 + 4x - 6) dx = \frac{1}{2} \int (2x+4) \sin(u) du$, se tiene:

$$= \frac{1}{2} \int (2x+4) \sin(x^2 + 4x - 6) dx = \frac{1}{2} \int \sin u du, \text{ integral que es inmediata.}$$

$$\text{Luego: } \frac{1}{2} \int \sin u du = \frac{1}{2} (-\cos u) + c = -\frac{1}{2} \cos u + c = -\frac{1}{2} \cos(x^2 + 4x - 6) + c$$

$$\text{Respuesta: } \int (x+2) \sin(x^2 + 4x - 6) dx = -\frac{1}{2} \cos(x^2 + 4x - 6) + c$$

$$\text{2.4.-Encontrar: } \int x \sin(1-x^2) dx$$

Solución.-Sea la sustitución: $w = 1 - x^2$, donde: $dw = -2x dx$

$$\text{Dado que: } \int x \sin(1-x^2) dx = -\frac{1}{2} \int (-2x) \sin(1-x^2) dx$$

$$\text{Se tiene que: } -\frac{1}{2} \int (-2x) \sin(1-x^2) dx = -\frac{1}{2} \sin w dw, \text{ integral que es inmediata.}$$

$$\text{Luego: } -\frac{1}{2} \int \sin w dw = -\frac{1}{2} (-\cos w) + c = \frac{1}{2} \cos w + c = \frac{1}{2} \cos(1-x^2) + c$$

$$\text{Respuesta: } \int x \sin(1-x^2) dx = \frac{1}{2} \cos(1-x^2) + c$$

$$\text{2.5.-Encontrar: } \int x \cot g(x^2 + 1) dx$$

Solución.-Sea la sustitución: $u = x^2 + 1$, donde: $du = 2x dx$

$$\text{Dado que: } \int x \cot g(x^2 + 1) dx = \frac{1}{2} \int 2x \cot g(x^2 + 1) dx$$

$$\text{Se tiene que: } \frac{1}{2} \int 2x \cot g(x^2 + 1) dx = \frac{1}{2} \int \cot g u du, \text{ integral que es inmediata.}$$

$$\text{Luego: } \frac{1}{2} \int \cot g u du = \frac{1}{2} \ell \eta |\sin u| + c = \frac{1}{2} \ell \eta |\sin(x^2 + 1)| + c$$

$$\text{Respuesta: } \int x \cot g(x^2 + 1) dx = \frac{1}{2} \ell \eta |\sin(x^2 + 1)| + c$$

$$\text{2.6.-Encontrar: } \int \sqrt{1+y^4} y^3 dy$$

Solución.-Sea la sustitución: $w = 1 + y^4$, donde: $dw = 4y^3 dy$

$$\text{Dado que: } \int \sqrt{1+y^4} y^3 dy = \frac{1}{4} \int (1+y^4)^{\frac{1}{2}} 4y^3 dy$$

$$\text{Se tiene que: } \frac{1}{4} \int (1+y^4)^{\frac{1}{2}} 4y^3 dy = \frac{1}{4} \int w^{\frac{1}{2}} dw, \text{ integral que es inmediata.}$$

$$\text{Luego: } \frac{1}{4} \int w^{\frac{1}{2}} dw = \frac{1}{4} \frac{w^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{1}{6} w^{\frac{3}{2}} + c = \frac{1}{6} (1+y^4)^{\frac{3}{2}} + c$$

$$\text{Respuesta: } \int \sqrt{1+y^4} y^3 dy = \frac{1}{6} (1+y^4)^{\frac{3}{2}} + c$$

$$\text{2.7.-Encontrar: } \int \frac{3tdt}{\sqrt[3]{t^2 + 3}}$$

Solución.-Sea la sustitución: $u = t^2 + 3$, donde: $du = 2tdt$

Dado que: $\int \frac{3tdt}{\sqrt[3]{t^2+3}} = \frac{3}{2} \int \frac{2tdt}{(t^2+3)^{\frac{1}{3}}}$

Se tiene que: $\frac{3}{2} \int \frac{2tdt}{(t^2+3)^{\frac{1}{3}}} = \frac{3}{2} \int \frac{du}{u^{\frac{1}{3}}}$, integral que es inmediata

Luego: $\frac{3}{2} \int \frac{du}{u^{\frac{1}{3}}} = \frac{3}{2} \int u^{-\frac{1}{3}} du = \frac{3}{2} \frac{u^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{9}{4} u^{\frac{2}{3}} + c = \frac{9}{4} (t^2+3)^{\frac{2}{3}} + c$

Respuesta: $\int \frac{3tdt}{\sqrt[3]{t^2+3}} = \frac{9}{4} (t^2+3)^{\frac{2}{3}} + c$

2.8.-Encontrar: $\int \frac{dx}{(a+bx)^{\frac{1}{3}}}$, a y b constantes.

Solución.- Sea: $w = a+bx$, donde: $dw = bdx$

Luego: $\int \frac{dx}{(a+bx)^{\frac{1}{3}}} = \frac{1}{b} \int \frac{bdx}{(a+bx)^{\frac{1}{3}}} = \frac{1}{b} \int \frac{dw}{w^{\frac{1}{3}}} = \frac{1}{b} \int w^{-\frac{1}{3}} dw = \frac{1}{b} \frac{w^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{3}{2b} w^{\frac{2}{3}} + c = \frac{3}{2b} (a+bx)^{\frac{2}{3}} + c$

Respuesta: $\int \frac{dx}{(a+bx)^{\frac{1}{3}}} = \frac{3}{2b} (a+bx)^{\frac{2}{3}} + c$

2.9.-Encontrar: $\int \frac{\sqrt{\arcsen x}}{1-x^2} dx$

Solución.- $\int \frac{\sqrt{\arcsen x}}{1-x^2} dx = \int \sqrt{\arcsen x} \frac{dx}{\sqrt{1-x^2}}$,

Sea: $u = \arcsen x$, donde: $du = \frac{dx}{\sqrt{1-x^2}}$

Luego: $\int \sqrt{\arcsen x} \frac{dx}{\sqrt{1-x^2}} = \int u^{\frac{1}{2}} du = \frac{2}{3} u^{\frac{3}{2}} + c = \frac{2}{3} \sqrt{(\arcsen x)^3} + c$

Respuesta: $\int \frac{\sqrt{\arcsen x}}{1-x^2} dx = \frac{2}{3} \sqrt{(\arcsen x)^3} + c$

2.10.-Encontrar: $\int \frac{\operatorname{arc tg} \frac{x}{2}}{4+x^2} dx$

Solución.- Sea: $w = \operatorname{arc tg} \frac{x}{2}$, donde: $dw = \frac{1}{1+(\frac{x}{2})^2} (\frac{1}{2}) dx = \frac{2dx}{4+x^2}$

Luego: $\int \frac{\operatorname{arc tg} \frac{x}{2}}{4+x^2} dx = \frac{1}{2} \int \operatorname{arc tg} \left(\frac{x}{2} \right) \frac{2dx}{4+x^2} = \frac{1}{2} \int w dw = \frac{1}{4} w^2 + c = \frac{1}{4} \left(\operatorname{arc tg} \frac{x}{2} \right)^2 + c$

Respuesta: $\int \frac{\operatorname{arc tg} \frac{x}{2}}{4+x^2} dx = \frac{1}{4} \left(\operatorname{arc tg} \frac{x}{2} \right)^2 + c$

2.11.-Encontrar: $\int \frac{x - \operatorname{arc} \tau g 2x}{1+4x^2} dx$

Solución.- $\int \frac{x - \operatorname{arc} \tau g 2x}{1+4x^2} dx = \int \frac{xdx}{1+4x^2} - \int \frac{\sqrt{\operatorname{arc} \tau g 2x}}{1+4x^2}$

Sea: $u = 1+4x^2$, donde: $du = 8xdx$; $w = \operatorname{arc} \tau g 2x$, donde: $dw = \frac{2dx}{1+4x^2}$

Luego: $\int \frac{xdx}{1+4x^2} - \int \frac{\sqrt{\operatorname{arc} \tau g 2x}}{1+4x^2} = \frac{1}{8} \int \frac{8xdx}{1+4x^2} - \frac{1}{2} \int \sqrt{\operatorname{arc} \tau g 2x} \frac{2dx}{1+4x^2}$
 $= \frac{1}{8} \int \frac{du}{u} - \frac{1}{2} \int w^{\frac{1}{2}} dw = \frac{1}{8} \ell \eta |u| - \frac{1}{3} w^{\frac{3}{2}} + c = \frac{1}{8} \ell \eta |1+4x^2| - \frac{1}{3} (\operatorname{arc} \tau g 2x)^{\frac{3}{2}} + c$

Respuesta: $\int \frac{x - \operatorname{arc} \tau g 2x}{1+4x^2} dx = \frac{1}{8} \ell \eta |1+4x^2| - \frac{1}{3} (\operatorname{arc} \tau g 2x)^{\frac{3}{2}} + c$

2.12.-Encontrar: $\int \frac{dx}{\sqrt{(1+x^2)\ell \eta |x+\sqrt{1+x^2}|}}$

Solución.- $\int \frac{dx}{\sqrt{(1+x^2)\ell \eta |x+\sqrt{1+x^2}|}} = \int \frac{dx}{\sqrt{1+x^2} \sqrt{\ell \eta |x+\sqrt{1+x^2}|}}$

Sea: $u = \ell \eta |x+\sqrt{1+x^2}|$, donde: $du = \frac{1}{x+\sqrt{1+x^2}} (1 + \frac{2x}{2\sqrt{1+x^2}}) \Rightarrow du = \frac{dx}{\sqrt{1+x^2}}$

Luego: $\int \frac{dx}{\sqrt{1+x^2} \sqrt{\ell \eta |x+\sqrt{1+x^2}|}} = \int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + c = 2\sqrt{\ell \eta |x+\sqrt{1+x^2}|} + c$

Respuesta: $\int \frac{dx}{\sqrt{(1+x^2)\ell \eta |x+\sqrt{1+x^2}|}} = 2\sqrt{\ell \eta |x+\sqrt{1+x^2}|} + c$

2.13.-Encontrar: $\int \frac{\operatorname{co} \tau g(\ell \eta x)}{x} dx$

Solución.- Sea: $w = \ell \eta x$, donde: $dw = \frac{dx}{x}$

Luego: $\int \frac{\operatorname{co} \tau g(\ell \eta x)}{x} dx = \int \operatorname{co} \tau g w dw = \ell \eta |\operatorname{sen} w| + c = \ell \eta |\operatorname{sen}(\ell \eta x)| + c$

Respuesta: $\int \frac{\operatorname{co} \tau g(\ell \eta x)}{x} dx = \ell \eta |\operatorname{sen}(\ell \eta x)| + c$

2.14.-Encontrar: $\int \frac{dx}{x(\ell \eta x)^3}$

Solución.- Sea: $u = \ell \eta x$, donde: $du = \frac{dx}{x}$

Luego: $\int \frac{dx}{x(\ell \eta x)^3} = \int \frac{du}{u^3} = \int u^{-3} du = \frac{u^{-2}}{2} + c = \frac{1}{2u^2} + c = \frac{1}{2(\ell \eta x)^2} + c$

Respuesta: $\int \frac{dx}{x(\ell\eta x)^3} = \frac{1}{2(\ell\eta x)^2} + c$

2.15.-Encontrar: $\int \frac{e^{\frac{1}{x^2}}}{x^3} dx$

Solución.- Sea: $w = \frac{1}{x^2}$, donde: $dw = -\frac{2}{x^3} dx$

Luego: $\int \frac{e^{\frac{1}{x^2}}}{x^3} dx = -\frac{1}{2} \int e^{\frac{1}{x^2}} \frac{-2dx}{x^3} = -\frac{1}{2} \int e^w dw = -\frac{1}{2} e^w + c = -\frac{1}{2} e^{\frac{1}{x^2}} + c$

Respuesta: $\int \frac{e^{\frac{1}{x^2}}}{x^3} dx = -\frac{1}{2} e^{\frac{1}{x^2}} + c$

2.16.-Encontrar: $\int e^{-x^2+2} x dx$

Solución.- Sea: $u = -x^2 + 2$, donde: $du = -2x dx$

Luego: $\int e^{-x^2+2} x dx = -\frac{1}{2} \int e^{-x^2+2} (-2x dx) = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + c = -\frac{1}{2} e^{-x^2+2} + c$

Respuesta: $\int e^{-x^2+2} x dx = -\frac{1}{2} e^{-x^2+2} + c$

2.17.-Encontrar: $\int x^2 e^{x^3} dx$

Solución.- Sea: $w = x^3$, donde: $dw = 3x^2 dx$

Luego: $\int x^2 e^{x^3} dx = \frac{1}{3} \int 3x^2 e^{x^3} dx = \frac{1}{3} \int e^w dw = \frac{1}{3} e^{x^3} + c$

Respuesta: $\int x^2 e^{x^3} dx = \frac{1}{3} e^{x^3} + c$

2.18.-Encontrar: $\int (e^x + 1)^2 e^x dx$

Solución.- Sea: $u = e^x + 1$, donde: $du = e^x dx$

Luego: $\int (e^x + 1)^2 e^x dx = \int u^2 du = \frac{u^3}{3} + c = \frac{(e^x + 1)^3}{3} + c$

Respuesta: $\int (e^x + 1)^2 e^x dx = \frac{(e^x + 1)^3}{3} + c$

2.19.-Encontrar: $\int \frac{e^x - 1}{e^x + 1} dx$

Solución.- $\int \frac{e^x - 1}{e^x + 1} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{1}{e^x + 1} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{e^x e^{-x}}{e^x + 1} dx$
 $= \int \frac{e^x}{e^x + 1} dx - \int \frac{e^{-x}}{e^{-x}(e^x + 1)} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{e^{-x}}{1 + e^x} dx$

Sea: $u = e^x + 1$, donde: $du = e^x dx$; $w = 1 + e^{-x}$, donde: $dw = -e^{-x} dx$

Luego: $\int \frac{e^x}{e^x + 1} dx - \int \frac{e^{-x}}{1 + e^x} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{-e^{-x}}{1 + e^{-x}} dx = \int \frac{du}{u} + \int \frac{dw}{w}$

$$= \ell \eta |u| + c_1 + \ell \eta |w| + c_2 = \ell \eta |e^x + 1| + \ell \eta |1 + e^{-x}| + C = \ell \eta [|e^x + 1| |1 + e^{-x}|] + c$$

Respuesta: $\int \frac{e^x - 1}{e^x + 1} dx = \ell \eta [(e^x + 1)(1 + e^{-x})] + c$, otra respuesta seria:

$$\int \frac{e^x - 1}{e^x + 1} dx = \ell \eta |e^x + 1|^2 - x + c$$

2.20.-Encontrar: $\int \frac{e^{2x} - 1}{e^{2x} + 3} dx$

$$\begin{aligned} \text{Solución.- } & \int \frac{e^{2x} - 1}{e^{2x} + 3} dx = \int \frac{e^{2x}}{e^{2x} + 3} dx - \int \frac{e^0}{e^{2x} + 3} dx \\ & = \int \frac{e^{2x}}{e^{2x} + 3} dx - \int \frac{e^{2x} e^{-2x}}{e^{2x} + 3} dx = \int \frac{e^{2x}}{e^{2x} + 3} dx - \int \frac{e^{-2x}}{e^{-2x}(e^{2x} + 3)} dx = \int \frac{e^{2x}}{e^{2x} + 3} dx - \int \frac{e^{-2x}}{1 + 3e^{-2x}} dx \end{aligned}$$

Sea: $u = e^{2x} + 3$, donde: $du = 2e^{2x} dx$; $w = 1 + 3e^{-2x}$, donde: $dw = -6e^{-2x} dx$

$$\begin{aligned} \text{Luego: } & \int \frac{e^{2x}}{e^{2x} + 3} dx - \int \frac{e^{-2x}}{1 + 3e^{-2x}} dx = \frac{1}{2} \int \frac{2e^{2x}}{e^{2x} + 3} dx + \frac{1}{6} \int \frac{-6e^{-2x}}{1 + 3e^{-2x}} dx = \frac{1}{2} \int \frac{du}{u} + \frac{1}{6} \int \frac{dw}{w} \\ & \frac{1}{2} \ell \eta |u| + \frac{1}{6} \ell \eta |w| + c = \frac{1}{2} \ell \eta |e^{2x} + 3| + \frac{1}{6} \ell \eta |1 + 3e^{-2x}| + c = \frac{1}{2} \ell \eta |e^{2x} + 3| + \frac{1}{6} \ell \eta \left| 1 + \frac{3}{e^{2x}} \right| + c \\ & = \frac{1}{2} \ell \eta |e^{2x} + 3| + \frac{1}{6} \ell \eta \left| \frac{e^{2x} + 3}{e^{2x}} \right| + c = \frac{1}{2} \ell \eta |e^{2x} + 3| + \frac{1}{6} \ell \eta |e^{2x} + 3| - \frac{1}{6} \ell \eta e^{2x} + c \\ & = \ell \eta (e^{2x} + 3)^{1/2} + \ell \eta (e^{2x} + 3)^{1/6} - \frac{1}{6} 2x + c = \ell \eta \left[(e^{2x} + 3)^{1/2} (e^{2x} + 3)^{1/6} \right] - \frac{x}{3} + c \\ & = \ell \eta (e^{2x} + 3)^{2/3} - \frac{x}{3} + c \end{aligned}$$

Respuesta: $\int \frac{e^{2x} - 1}{e^{2x} + 3} dx = \ell \eta (e^{2x} + 3)^{2/3} - \frac{x}{3} + c$

2.22.-Encontrar: $\int \frac{x^2 + 1}{x - 1} dx$

Solución.- Cuando el grado del polinomio dividendo es MAYOR o IGUAL que el grado del polinomio divisor, es necesario efectuar previamente la división de polinomios. El resultado de la división dada es:

$$\frac{x^2 + 1}{x - 1} = (x + 1) + \frac{2}{x - 1}, \text{ Luego: } \int \frac{x^2 + 1}{x - 1} dx = \int \left(x + 1 + \frac{2}{x - 1} \right) dx = \int x dx + \int dx + 2 \int \frac{dx}{x - 1}$$

Sea $u = x - 1$, donde $du = dx$

$$\text{Luego: } \int x dx + \int dx + 2 \int \frac{dx}{x - 1} = \int x dx + \int dx + 2 \int \frac{du}{u} = \frac{x^2}{2} + x + \ell \eta |x - 1| + c$$

$$\text{Respuesta: } \int \frac{x^2 + 1}{x - 1} dx = \frac{x^2}{2} + x + \ell \eta |x - 1| + c$$

2.23.-Encontrar: $\int \frac{x+2}{x+1} dx$

Solución.- $\frac{x+2}{x+1} = 1 + \frac{1}{x+1}$, Luego: $\int \frac{x+2}{x+1} dx = \int \left(1 + \frac{1}{x+1}\right) dx = \int dx + \int \frac{dx}{x+1}$

Sea $u = x+1$, donde $du = dx$

$$\int dx + \int \frac{du}{u} = x + \ell \eta |u| + c = x + \ell \eta |x+1| + c$$

Respuesta: $\int \frac{x+2}{x+1} dx = x + \ell \eta |x+1| + c$

2.24.-Encontrar: $\int \tau g^5 x \sec^2 x dx$

Solución.- Sea: $w = \tau g x$, donde: $dw = \sec^2 x$

$$\text{Luego: } \int \tau g^5 x \sec^2 x dx = \int (\tau g x)^5 \sec^2 x dx = \int w^5 dw = \frac{w^6}{6} + c = \frac{(\tau g x)^6}{6} + c = \frac{\tau g^6 x}{6} + c$$

Respuesta: $\int \tau g^5 x \sec^2 x dx = \frac{\tau g^6 x}{6} + c$

2.25.-Encontrar: $\int \sin x \sec^2 x dx$

Solución.- $\int \sin x \sec^2 x dx = \int \sin x \frac{1}{\cos^2 x} dx = \int \frac{\sin x}{\cos^2 x} dx$

Sea: $u = \cos x$, donde: $du = -\sin x$

$$\text{Luego: } \int \frac{\sin x}{\cos^2 x} dx = - \int \frac{-\sin x dx}{\cos^2 x} = - \int \frac{du}{u} = - \int u^{-2} du = - \frac{u^{-1}}{-1} + c = \frac{1}{u} + c = \frac{1}{\cos x} + c$$

Respuesta: $\int \sin x \sec^2 x dx = \sec x + c$

2.26.-Encontrar: $\int \frac{\sec^2 3x dx}{1 + \tau g 3x}$

Solución.- Sea: $u = 1 + \tau g 3x$, donde: $du = 3 \sec^2 3x dx$

$$\text{Luego: } \int \frac{\sec^2 3x dx}{1 + \tau g 3x} = \frac{1}{3} \int \frac{3 \sec^2 3x dx}{1 + \tau g 3x} = \frac{1}{3} \int \frac{du}{u} = \frac{1}{3} \ell \eta |u| + c = \frac{1}{3} \ell \eta |1 + \tau g 3x| + c$$

Respuesta: $\int \frac{\sec^2 3x dx}{1 + \tau g 3x} = \frac{1}{3} \ell \eta |1 + \tau g 3x| + c$

2.27.-Encontrar: $\int \sin^3 x \cos x dx$

Solución.- Sea: $w = \sin x$, donde: $dw = \cos x dx$

$$\text{Luego: } \int \sin^3 x \cos x dx = \int (\sin x)^3 \cos x dx = \int w^3 dw = \int \frac{w^4}{4} + c = \int \frac{\sin^4 x}{4} + c$$

Respuesta: $\int \sin^3 x \cos x dx = \int \frac{\sin^4 x}{4} + c$

2.28.-Encontrar: $\int \cos^4 x \sin x dx$

Solución.- Sea: $u = \cos x$, donde: $du = -\sin x$

$$\text{Luego: } \int \cos^4 x \sin x dx = \int (\cos x)^4 \sin x dx = - \int (\cos x)^4 (-\sin x) dx = - \int u^4 du$$

$$= -\frac{u^5}{5} + c = -\frac{\cos x^5}{5} + c = -\frac{\cos^5 x}{5} + c$$

Respuesta: $\int \cos^4 x \sin x dx = -\frac{\cos^5 x}{5} + c$

2.29.-Encontrar: $\int \frac{\sec^5}{\cos ecx} dx$

Solución.- $\int \frac{\sec^5}{\cos ecx} dx = \int \frac{\frac{1}{\cos^5 x}}{\frac{1}{\sin x}} dx = \int \frac{\sin x}{(\cos x)^5} dx$

Sea: $w = \cos x$, donde: $dw = -\sin x dx$

Luego: $\int \frac{\sin x}{(\cos x)^5} dx = -\int \frac{dw}{w^5} = -\int w^{-5} dw = -\frac{w^{-4}}{-4} + c = \frac{1}{4} \frac{1}{w^4} + c = \frac{1}{4 \cos^4 x} + c$

$$= \frac{\sec^4 x}{4} + c$$

Respuesta: $\int \frac{\sec^5}{\cos ecx} dx = \frac{\sec^4 x}{4} + c$

2.30.-Encontrar: $\int e^{\tau g 2x} \sec^2 2x dx$

Solución.- Sea: $u = \tau g 2x$, donde: $du = 2 \sec^2 2x dx$

Luego: $\int e^{\tau g 2x} \sec^2 2x dx = \frac{1}{2} \int e^{\tau g 2x} (2 \sec^2 2x dx) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + c = \frac{1}{2} e^{\tau g 2x} + c$

Respuesta: $\int e^{\tau g 2x} \sec^2 2x dx = \frac{1}{2} e^{\tau g 2x} + c$

2.31.-Encontrar: $\int \frac{2x-5}{3x^2-2} dx$

Solución.- Sea: $w = 3x^2 - 2$, donde: $dw = 6x dx$

Luego: $\int \frac{2x-5}{3x^2-2} dx = \frac{1}{3} \int \frac{3(2x-5)}{3x^2-2} dx = \frac{1}{3} \int \frac{6x-15}{3x^2-2} dx = \frac{1}{3} \int \frac{6xdx}{3x^2-2} - \frac{15}{3} \int \frac{dx}{3x^2-2}$
 $= \frac{1}{3} \int \frac{6xdx}{3x^2-2} - 5 \int \frac{dx}{3(x^2-\frac{2}{3})} = \frac{1}{3} \int \frac{6xdx}{3x^2-2} - \frac{5}{3} \int \frac{dx}{(x^2-\frac{2}{3})} = \frac{1}{3} \int \frac{6xdx}{3x^2-2} - \frac{5}{3} \int \frac{dx}{x^2-(\sqrt{\frac{2}{3}})^2}$
 $\frac{1}{3} \int \frac{dw}{w} - \frac{5}{3} \int \frac{dx}{x^2-(\sqrt{\frac{2}{3}})^2} = \frac{1}{3} \ell \eta |w| + c_1 - \frac{5}{3} \int \frac{dx}{x^2-(\sqrt{\frac{2}{3}})^2}$; Sea: $v = x$, donde: $dv = dx$

Además: $a = \sqrt{\frac{2}{3}}$; se tiene: $\frac{1}{3} \ell \eta |w| + c_1 - \frac{5}{3} \int \frac{dv}{v^2-a^2}$

$$= \frac{1}{3} \ell \eta |3x^2-2| + c_1 - \frac{5}{3} \frac{1}{2a} \ell \eta \left| \frac{v-a}{v+a} \right| + c_2 = \frac{1}{3} \ell \eta |3x^2-2| - \frac{5}{3} \left[\frac{1}{2\sqrt{\frac{2}{3}}} \ell \eta \left| \frac{x-\sqrt{\frac{2}{3}}}{x+\sqrt{\frac{2}{3}}} \right| \right] + C$$

$$= \frac{1}{3} \ell \eta |3x^2-2| - \frac{5}{\sqrt{32}\sqrt{2}} \ell \eta \left| \frac{\sqrt{3}x-\sqrt{2}}{\sqrt{3}x+\sqrt{2}} \right| + C = \frac{1}{3} \ell \eta |3x^2-2| - \frac{5}{2\sqrt{6}} \ell \eta \left| \frac{\sqrt{3}x-\sqrt{2}}{\sqrt{3}x+\sqrt{2}} \right| + C$$

Respuesta: $\int \frac{2x-5}{3x^2-2} dx = \frac{1}{3} \ell \eta |3x^2-2| - \frac{5}{2\sqrt{6}} \ell \eta \left| \frac{\sqrt{3}x-\sqrt{2}}{\sqrt{3x+\sqrt{2}}} \right| + C$

2.32.-Encontrar: $\int \frac{dx}{x\sqrt{4-9\ell\eta^2x}}$

Solución.- $\int \frac{dx}{x\sqrt{4-9\ell\eta^2x}} = \int \frac{dx}{x\sqrt{2^2-(3\ell\eta x)^2}}$

Sea: $u = 3\ell\eta x$, donde: $du = \frac{3dx}{x}$

Luego: $\int \frac{dx}{x\sqrt{2^2-(3\ell\eta x)^2}} = \frac{1}{3} \int \frac{3dx}{x\sqrt{2^2-(3\ell\eta x)^2}} = \frac{1}{3} \int \frac{du}{\sqrt{2^2-(u)^2}} = \frac{1}{3} \arcsen \frac{u}{2} + c$
 $= \frac{1}{3} \arcsen \frac{3\ell\eta x}{2} + c = \frac{1}{3} \arcsen \ell\eta |x|^{\frac{1}{2}} + c$

Respuesta: $\int \frac{dx}{x\sqrt{4-9\ell\eta^2x}} = \frac{1}{3} \arcsen \ell\eta |x|^{\frac{1}{2}} + c$

2.33.-Encontrar: $\int \frac{dx}{\sqrt{e^x-1}}$

Solución.- Sea: $u = \sqrt{e^x-1}$, donde: $du = \frac{e^x dx}{2\sqrt{e^x-1}}$; Tal que: $e^x = u^2 + 1$

Luego: $\int \frac{dx}{\sqrt{e^x-1}} = \int \frac{2du}{u^2+1} = 2 \int \frac{du}{u^2+1} = 2 \operatorname{arc tg} u + c = 2 \operatorname{arc tg} \sqrt{e^x+1} + c$

Respuesta: $\int \frac{dx}{\sqrt{e^x-1}} = 2 \operatorname{arc tg} \sqrt{e^x+1} + c$

2.34.-Encontrar: $\int \frac{x^2+2x+2}{x+1} dx$

Solución.- $\int \frac{x^2+2x+2}{x+1} dx = \int \frac{(x^2+2x+1)+1}{x+1} dx = \int \frac{(x+1)^2+1}{x+1} dx = \int \frac{(x+1)^2+1}{x+1} dx$
 $= \int (x+1 + \frac{1}{x+1}) dx = \int x dx + \int dx + \int \frac{dx}{x+1}$, Sea: $w = x+1$, donde: $dw = dx$

Luego: $\int x dx + \int dx + \int \frac{dx}{x+1} = \int x dx + \int dx + \int \frac{dw}{w} = \frac{x^2}{2} + x + \ell\eta |w| + c$

$= \frac{x^2}{2} + x + \ell\eta |x+1| + c$

Respuesta: $\int \frac{x^2+2x+2}{x+1} dx = \frac{x^2}{2} + x + \ell\eta |x+1| + c$

2.35.-Encontrar: $\int \frac{e^{2x}}{\sqrt{e^x+1}} dx$

Solución.- Sea: $u = e^x + 1$, donde: $du = e^x dx$

$$\begin{aligned} \text{Luego: } & \int \frac{e^{2x}}{\sqrt{e^x + 1}} dx = \int \frac{u - 1}{u^{\frac{1}{2}}} du = \int (u^{\frac{1}{2}} - u^{-\frac{1}{2}}) du = \int u^{\frac{1}{2}} du - \int u^{-\frac{1}{2}} du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{-\frac{1}{2}}}{\frac{1}{2}} + c \\ & = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{-\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{3}u^{\frac{3}{2}} - \frac{1}{2}u^{-\frac{1}{2}} + c = \frac{2}{3}\sqrt{(e^x + 1)^3} - 2\sqrt{(e^x + 1)} + c \end{aligned}$$

Respuesta: $\int \frac{e^{2x}}{\sqrt{e^x + 1}} dx = \frac{2}{3}\sqrt{(e^x + 1)^3} - 2\sqrt{(e^x + 1)} + c$

2.36.-Encontrar: $\int \frac{\ell \eta 2x}{\ell \eta 4x} \frac{dx}{x}$

Solución.- Sea: $u = \ell \eta 4x$, donde: $du = \frac{dx}{x}$; además: $\ell \eta 4x = (2 \times 2x) = \ell \eta 2 + \ell \eta 2x$
 $\Rightarrow u = \ell \eta 2 + \ell \eta 2x \Rightarrow \ell \eta 2x = u - \ell \eta 2$

$$\begin{aligned} \text{Luego: } & \int \frac{\ell \eta 2x}{\ell \eta 4x} \frac{dx}{x} = \int \frac{u - \ell \eta 2}{u} du = \int du - \int \frac{\ell \eta 2}{u} du = \int du - \ell \eta 2 \int \frac{du}{u} = u - \ell \eta 2 |u| + c \\ & = \ell \eta 4x - \ell \eta 2 [\ell \eta (\ell \eta 4x)] + c \end{aligned}$$

Respuesta: $\int \frac{\ell \eta 2x}{\ell \eta 4x} \frac{dx}{x} = \ell \eta 4x - \ell \eta 2 [\ell \eta (\ell \eta 4x)] + c$

2.37.-Encontrar: $\int x(3x+1)^7 dx$

Solución.- Sea: $w = 3x+1$, donde: $dw = 3dx$; además: $w-1 = 3x \Rightarrow x = \frac{w-1}{3}$

$$\begin{aligned} \text{Luego: } & \int x(3x+1)^7 dx = \int \frac{w-1}{3} w^7 \frac{dw}{3} = \frac{1}{9} \int (w-1) w^7 dw = \frac{1}{9} \int (w^8 - w^7) dw \\ & = \frac{1}{9} \int w^8 dw - \frac{1}{9} \int w^7 dw = \frac{1}{9} \frac{w^9}{9} - \frac{1}{9} \frac{w^8}{8} + c = \frac{1}{81} w^9 - \frac{1}{72} w^8 + c \\ & = \frac{1}{81} (3x+1)^9 - \frac{1}{72} (3x+1)^8 + c \end{aligned}$$

Respuesta: $\int x(3x+1)^7 dx = \frac{(3x+1)^9}{81} - \frac{(3x+1)^8}{72} + c$

2.38.-Encontrar: $\int \frac{x^2 - 5x + 6}{x^2 + 4} dx$

Solución.- $\frac{x^2 - 5x + 6}{x^2 + 4} dx = 1 + \frac{2 - 5x}{x^2 + 4}$

$$\text{Luego: } \int \frac{x^2 - 5x + 6}{x^2 + 4} dx = \int \left(1 + \frac{2 - 5x}{x^2 + 4}\right) dx = \int dx + 2 \int \frac{dx}{x^2 + 4} - 5 \int \frac{xdx}{x^2 + 4}$$

Sea: $u = x^2 + 4$, donde: $du = 2xdx$; Entonces:

$$= x + \arctan \frac{x}{2} - \frac{5}{2} \int \frac{du}{u} = x + \arctan \frac{x}{2} - \frac{5}{2} \ell \eta |u| + c = x + \arctan \frac{x}{2} - \frac{5}{2} \ell \eta |x^2 + 4| + c$$

Respuesta: $\int \frac{x^2 - 5x + 6}{x^2 + 4} dx = x + \arctan \frac{x}{2} - \frac{5}{2} \ell \eta |x^2 + 4| + c$

EJERCICIOS PROPUESTOS

Usando Esencialmente la técnica de integración por sustitución, encontrar las siguientes integrales:

$$2.39.- \int 3^x e^x dx$$

$$2.40.- \int \frac{adx}{a-x}$$

$$2.41.- \int \frac{4t+6}{2t+1} dt$$

$$2.42.- \int \frac{1-3x}{3+2x} dx$$

$$2.43.- \int \frac{xdx}{a+bx}$$

$$2.44.- \int \frac{ax-b}{\alpha x+\beta} dx$$

$$2.45.- \int \frac{3t^2+3}{t-1} dt$$

$$2.46.- \int \frac{x^2+5x+7}{x+3} dx$$

$$2.47.- \int \frac{x^4+x^2+1}{x-1} dx$$

$$2.48.- \int \left(a + \frac{b}{x-a} \right)^2 dx$$

$$2.49.- \int \frac{x}{(x+1)^2} dx$$

$$2.50.- \int \frac{bdy}{\sqrt{1-y}}$$

$$2.51.- \int \sqrt{a-bx} dx$$

$$2.52.- \int \frac{xdx}{\sqrt{x^2+1}}$$

$$2.53.- \int \frac{\sqrt{x+\ell}\eta x}{x} dx$$

$$2.54.- \int \frac{dx}{3x^2+5}$$

$$2.55.- \int \frac{x^3 dx}{a^2 - x^2}$$

$$2.56.- \int \frac{y^2 - 5y + 6}{y^2 + 4} dy$$

$$2.57.- \int \frac{6t-15}{3t^2-2} dt$$

$$2.58.- \int \frac{3-2x}{5x^2+7} dx$$

$$2.59.- \int \frac{3x+1}{\sqrt{5x^2+1}} dx$$

$$2.60.- \int \frac{xdx}{x^2-5}$$

$$2.61.- \int \frac{xdx}{2x^2+3}$$

$$2.62.- \int \frac{ax+b}{a^2x^2+b^2} dx$$

$$2.63.- \int \frac{xdx}{\sqrt{a^4-x^4}}$$

$$2.64.- \int \frac{x^2 dx}{1+x^6}$$

$$2.65.- \int \frac{x^2 dx}{\sqrt{x^6-1}}$$

$$2.66.- \int \frac{x-\sqrt{\arctan t} 3x}{1+9x^2} dx$$

$$2.67.- \int \sqrt{\frac{\arcsen t}{4-4t^2}} dt$$

$$2.68.- \int \frac{\arctan(\frac{x}{3})}{9+x^2} dx$$

$$2.69.- \int \frac{dt}{\sqrt{(9+9t^2)\ell\eta|t+\sqrt{1+t^2}|}}$$

$$2.70.- \int ae^{-mx} dx$$

$$2.71.- \int 4^{2-3x} dx$$

$$2.72.- \int (e^t - e^{-t}) dt$$

$$2.73.- \int e^{-(x^2+1)} x dx$$

$$2.74.- \int (e^{\frac{x}{a}} - e^{-\frac{x}{a}})^2 dx$$

$$2.75.- \int \frac{a^{2x}-1}{\sqrt{a^x}} dx$$

$$2.76.- \int \frac{e^{\frac{x}{a}}}{x^2} dx$$

$$2.77.- \int 5^{\sqrt{x}} \frac{dx}{\sqrt{x}}$$

$$2.78.- \int x 7^{x^2} dx$$

$$2.79.- \int \frac{e^t dt}{e^t - 1}$$

$$2.80.- \int e^x \sqrt{a-be^x} dx$$

$$2.81.- \int (e^{\frac{x}{a}} + 1)^{\frac{1}{3}} e^{\frac{x}{a}} dx$$

$$2.82.- \int \frac{dx}{2^x + 3}$$

$$2.83.- \int \frac{a^x dx}{1+a^{2x}}; a > 0$$

$$2.84.- \int \frac{e^{-bx}}{1-e^{-2bx}} dx$$

$$2.85.- \int \frac{e^t dt}{\sqrt{1-e^{2t}}}$$

$$2.86.- \int \cos \frac{x}{\sqrt{2}} dx$$

$$2.87.- \int \sin(a+bx) dx$$

$$2.88.- \int \cos \sqrt{x} \frac{dx}{\sqrt{x}}$$

$$2.89.- \int \sin(\ell\eta x) \frac{dx}{x}$$

$$2.90.- \int (\cos ax + \sin ax)^2 dx$$

$$2.91.- \int \sin^2 x dx$$

$$2.92.- \int \cos^2 x dx$$

$$2.93.- \int \sec^2(ax+b)dx$$

$$2.96.- \int \frac{dx}{3\cos(5x-\frac{\pi}{4})}$$

$$2.99.- \int \operatorname{co} \tau g \frac{x}{a-b} dx$$

$$2.102.- \int \left(\frac{1}{\operatorname{sen} x \sqrt{2}} - 1 \right)^2 dx$$

$$2.105.- \int t \operatorname{sen}(1-2t^2)dt$$

$$2.108.- \int \frac{\operatorname{sen} x \cos x}{\sqrt{\cos^2 x - \operatorname{sen}^2 x}} dx$$

$$2.111.- \int t \operatorname{co} \tau g(2t^2 - 3)dt$$

$$2.114.- \int \sqrt{1+3\cos^2 x} \operatorname{sen} 2x dx$$

$$2.117.- \int \frac{(\cos ax + \operatorname{sen} ax)^2}{\operatorname{sen} ax} dx$$

$$2.120.- \int \frac{x^3 - 1}{x^4 - 4x + 1} dx$$

$$2.123.- \int \frac{\tau g 3x - \operatorname{co} \tau g 3x}{\operatorname{sen} 3x} dx$$

$$2.126.- \int \frac{\sec^2 x dx}{\sqrt{\tau g^2 x - 2}}$$

$$2.129.- \int \frac{x^2}{\sqrt{x^3 + 1}} dx$$

$$2.132.- \int \frac{\sec^2 x dx}{\sqrt{4 - \tau g^2 x}}$$

$$2.135.- \int \tau g \sqrt{x-1} \frac{dx}{\sqrt{x-1}}$$

$$2.138.- \int \frac{e^{\operatorname{arc} \tau g x} + x \ell \eta (1+x^2) + 1}{1+x^2}$$

$$2.141.- \int \frac{(1 - \operatorname{sen} \frac{x}{\sqrt{2}})^2}{\operatorname{sen} \frac{x}{\sqrt{2}}} dx$$

$$2.144.- \int \frac{d\theta}{\operatorname{sen} a\theta \cos a\theta}$$

$$2.94.- \int \cos \tau g^2 a x dx$$

$$2.97.- \int \frac{dx}{\operatorname{sen}(ax+b)}$$

$$2.100.- \int \tau g \sqrt{x} \frac{dx}{\sqrt{x}}$$

$$2.103.- \int \frac{dx}{\operatorname{sen} x \cos x}$$

$$2.106.- \int \frac{\operatorname{sen} 3x}{3 + \cos 3x} dx$$

$$2.109.- \int \frac{\sqrt{\tau g x}}{\cos^2 x} dx$$

$$2.95.- \int \frac{dx}{\operatorname{sen} \frac{x}{a}}$$

$$2.98.- \int \frac{xdx}{\cos^2 x^2}$$

$$2.101.- \int \frac{dx}{\tau g \frac{x}{5}}$$

$$2.104.- \int \frac{\cos ax}{\operatorname{sen}^5 ax} dx$$

$$2.107.- \int \tau g^3 \frac{x}{3} \sec^2 \frac{x}{3} dx$$

$$2.110.- \int \cos \frac{x}{a} \operatorname{sen} \frac{x}{a} dx$$

$$2.112.- \int \frac{x^3 dx}{x^8 + 5}$$

$$2.115.- \int x \sqrt[5]{5-x^2} dx$$

$$2.118.- \int \frac{x^3 - 1}{x+1} dx$$

$$2.121.- \int x e^{-x^2} dx$$

$$2.124.- \int \frac{dx}{\sqrt{e^x}}$$

$$2.127.- \int \frac{dx}{x \ell \eta^2 x}$$

$$2.130.- \int \frac{xdx}{\sqrt{1-x^4}}$$

$$2.133.- \int \frac{dx}{\cos \frac{x}{a}}$$

$$2.113.- \int \operatorname{sen}^3 6x \cos 6x dx$$

$$2.116.- \int \frac{1 + \operatorname{sen} 3x}{\cos^2 3x} dx$$

$$2.119.- \int \frac{\operatorname{cos} ec^2 3x dx}{b - a \operatorname{co} \tau g 3x}$$

$$2.122.- \int \frac{3 - \sqrt{2 + 3x^2}}{2 + 3x^2} dx$$

$$2.125.- \int \frac{1 + \operatorname{sen} x}{x + \cos x} dx$$

$$2.128.- \int a^{\operatorname{sen} x} \cos x dx$$

$$2.131.- \int \tau g^2 a x dx$$

$$2.134.- \int \frac{\sqrt[3]{1 + \ell \eta x}}{x} dx$$

$$2.137.- \int \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x} dx$$

$$2.140.- \int e^{\operatorname{sen}^2 x} \operatorname{sen} 2x dx$$

$$2.143.- \int \frac{ds}{e^s + 1}$$

$$2.146.- \int \operatorname{sen}(\frac{2\pi t}{T} + \varphi_0) dt$$

$$2.145.- \int \frac{e^s}{\sqrt{e^{2s} - 2}} ds$$

$$2.147.- \int \frac{\arccos \frac{x}{2}}{\sqrt{4-x^2}} dx$$

$$2.148.- \int \frac{dx}{x(4-\ell\eta^2 x)}$$

$$2.149.- \int e^{-\tau gx} \sec^2 x dx$$

$$2.150.- \int \frac{\operatorname{sen} x \cos x}{\sqrt{2-\operatorname{sen}^4 x}} dx$$

$$2.151.- \int \frac{\sec x \tau g x}{\sqrt{\sec^2 x + 1}} dx$$

$$2.152.- \int \frac{dt}{\operatorname{sen}^2 t \cos^2 t}$$

$$2.153.- \int \frac{\operatorname{arc sen} x + x}{\sqrt{1-x^2}} dx$$

$$2.154.- \int \frac{xdx}{\sqrt{x+1}}$$

$$2.155.- \int x(5x^2 - 3)^7 dx$$

$$2.156.- \int \sqrt{\frac{\ell\eta(x+\sqrt{x^2+1})}{x^2+1}} dx$$

$$2.157.- \int \frac{\operatorname{sen}^3 x}{\sqrt{\cos x}} dx$$

$$2.158.- \int \frac{\cos x dx}{\sqrt{1+\operatorname{sen}^2 x}}$$

$$2.159.- \int \frac{(\operatorname{arc sen} x)^2}{\sqrt{1-x^2}} dx$$

$$2.150.- \int e^{x+e^x} dx$$

$$2.161.- \int t(4t+1)^7 dt$$

$$2.162.- \int \frac{2t^2-10t+12}{t^2+4} dt$$

$$2.163.- \int \frac{e^t - e^{-t}}{e^t + e^{-t}} dt$$

RESPUESTAS

$$2.39.- \int 3^x e^x dx ,$$

Sea: $u = x, du = dx, a = 3e$

$$\int (3e)^x dx = \int (a)^u du = \frac{a^u}{\ell\eta a} + c = \frac{(3e)^x}{\ell\eta(3e)} + c = \frac{(3e)^x}{\ell\eta 3\ell\eta e} + c = \frac{3^x e^x}{\ell\eta 3 + \ell\eta e} + c = \frac{3^x e^x}{\ell\eta 3 + 1} + c$$

$$2.40.- \int \frac{adx}{a-x} ,$$

Sea: $u = a-x, du = -dx$

$$\int \frac{adx}{a-x} = -a \int \frac{du}{u} = -a\ell\eta|u| + c = -a\ell\eta|a-x| + c$$

$$2.41.- \int \frac{4t+6}{2t+1} dt ,$$

Sea: $u = 2t+1, du = 2dt; \quad \frac{2t+3}{2t+1} = 1 + \frac{2}{2t+1}$

$$\begin{aligned} \int \frac{4t+6}{2t+1} dt &= 2 \int \left(1 + \frac{2}{2t+1}\right) dt = 2 \int dt + 2 \int \frac{2}{2t+1} dt = 2 \int dt + 2 \int \frac{du}{u} = 2t + 2\ell\eta|u| + c \\ &= 2t + 2\ell\eta|2t+1| + c \end{aligned}$$

$$2.42.- \int \frac{1-3x}{3+2x} dx ,$$

Sea: $u = 3+2x, du = 2dx; \quad \frac{1-3x}{3+2x} = -\frac{3}{2} + \frac{11/2}{2x+3}$

$$\int \frac{1-3x}{3+2x} dx = \int \left(-\frac{3}{2} + \frac{11/2}{2x+3}\right) dx = -\frac{3}{2} \int dx + \frac{11}{4} \int \frac{dx}{2x+3} = -\frac{3}{2} \int dx + \frac{11}{4} \int \frac{du}{u}$$

$$-\frac{3}{2}x + \frac{11}{4}\ell\eta|2x+3| + c$$

$$2.43.- \int \frac{xdx}{a+bx} ,$$

Sea: $u = a+bx, du = bdx; \quad \frac{x}{a+bx} = \frac{1}{b} - \frac{a/b}{a+bx}$

$$\int \frac{xdx}{a+bx} = \frac{1}{b} \int dx - \frac{a}{b} \int \frac{dx}{a+bx} = \frac{1}{b} \int dx - \frac{a}{b^2} \int \frac{du}{u} = \frac{1}{b}x - \frac{a}{b^2}\ell\eta|u| + c = \frac{x}{b} - \frac{a}{b^2}\ell\eta|a+bx| + c$$

$$\textbf{2.44.-} \int \frac{ax-b}{\alpha x+\beta} dx, \quad \text{Sea: } u = \alpha x + \beta, du = \alpha dx; \quad \frac{ax-b}{\alpha x+\beta} = \frac{a}{\alpha} - \frac{\frac{\alpha\beta}{\alpha}+b}{\alpha x}$$

$$\begin{aligned} \int \frac{ax-b}{\alpha x+\beta} dx &= \int \left(\frac{a}{\alpha} - \frac{\frac{\alpha\beta}{\alpha}+b}{\alpha x} \right) dx = \int \frac{a}{\alpha} dx - \int \frac{\frac{\alpha\beta}{\alpha}+ab}{\alpha x+\beta} dx = \frac{a}{\alpha} \int dx - \frac{\alpha\beta+ab}{\alpha} \int \frac{dx}{a\beta+\alpha b} \\ &= \frac{a}{\alpha} \int dx - \frac{\alpha\beta+ab}{\alpha^2} \int \frac{du}{u} = \frac{a}{\alpha} x - \frac{\alpha\beta+ab}{\alpha^2} \ell \eta |u| + c = \frac{a}{\alpha} x - \frac{\alpha\beta+ab}{\alpha^2} \ell \eta |\ell x + \beta| + c \end{aligned}$$

$$\textbf{2.45.-} \int \frac{3t^2+3}{t-1} dt, \quad \text{Sea: } u = t-1, du = dt; \quad \frac{t^2+1}{t-1} = t+1 + \frac{2}{t-1}$$

$$\begin{aligned} \int \frac{3t^2+3}{t-1} dt &= 3 \int \left(t+1 + \frac{2}{t-1} \right) dt = 3 \int t dt + 3 \int dt + 3 \int \frac{2}{t-1} dt = \frac{3}{2} t^2 + 3t + 6 \ell \eta |u| + c \\ &= \frac{3}{2} t^2 + 3t + 6 \ell \eta |t-1| + c \end{aligned}$$

$$\textbf{2.46.-} \int \frac{x^2+5x+7}{x+3} dx, \quad \text{Sea: } u = t-1, du = t+1; \quad \frac{x^2+5x+7}{x+3} = x+2 + \frac{1}{x+3}$$

$$\begin{aligned} \int \frac{x^2+5x+7}{x+3} dx &= \int \left(x+2 + \frac{1}{x+3} \right) dx = \int x dx + 2 \int dx + \int \frac{1}{x+3} dx = \frac{x^2}{2} + 2x + \ell \eta |u| + c \\ &= \frac{x^2}{2} + 2x + \ell \eta |u| + c = \frac{x^2}{2} + 2x + \ell \eta |x+3| + c \end{aligned}$$

$$\textbf{2.47.-} \int \frac{x^4+x^2+1}{x-1} dx, \quad \text{Sea: } u = x-1, du = dx;$$

$$\begin{aligned} \int \frac{x^4+x^2+1}{x-1} dx &= \int \left(x^3 + x^2 + 2x + 2 + \frac{3}{x-1} \right) dx = \int x^3 dx + \int x^2 dx + 2 \int dx + 3 \int \frac{dx}{x-1} \\ &= \frac{x^4}{4} + \frac{x^3}{3} + x^2 + 2 + 3 \ell \eta |u| + c = \frac{x^4}{4} + \frac{x^3}{3} + x^2 + 2x + 3 \ell \eta |x-1| + c \end{aligned}$$

$$\textbf{2.48.-} \int \left(a + \frac{b}{x-a} \right)^2 dx, \quad \text{Sea: } u = x-a, du = dx$$

$$\begin{aligned} \int \left(a + \frac{b}{x-a} \right)^2 dx &= \int \left(a^2 + \frac{2ab}{x-a} + \frac{b^2}{(x-a)^2} \right) dx = a^2 \int dx + 2ab \int \frac{dx}{x-a} + b^2 \int \frac{dx}{(x-a)^2} \\ &= a^2 \int dx + 2ab \int \frac{du}{u} + b^2 \int \frac{du}{u^2} = a^2 x + 2ab \ell \eta |u| + b^2 \frac{u^{-1}}{-1} + c = a^2 x + 2ab \ell \eta |x-a| - \frac{b^2}{x-a} + c \quad \textbf{2.} \end{aligned}$$

$$\textbf{49.-} \int \frac{x}{(x+1)^2} dx, \quad \text{Sea: } u = x+1, du = dx$$

$$\int \frac{x}{(x+1)^2} dx = \int \frac{(x+1)-1}{(x+1)^2} dx = \int \frac{x+1}{(x+1)^2} dx - \int \frac{dx}{(x+1)^2} = \int \frac{dx}{u} - \int \frac{dx}{u^2} = \ell \eta |u| - \frac{u^{-1}}{-1} + c$$

$$= \ell \eta |x+1| + \frac{1}{x+1} + c$$

2.50.- $\int \frac{bdy}{\sqrt{1-y}}, \quad \text{Sea: } u = 1-y, du = -dy$

$$\int \frac{bdy}{\sqrt{1-y}} = -b \int \frac{du}{\sqrt{u}} = -b \int u^{-\frac{1}{2}} du = -2bu^{\frac{1}{2}} + c = -2b(1-y)^{\frac{1}{2}} + c$$

2.51.- $\int \sqrt{a-bx} dx, \quad \text{Sea: } u = a-bx, du = -b dx$

$$\int \sqrt{a-bx} dx = -\frac{1}{b} \int u^{\frac{1}{2}} du = -\frac{1}{b} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{2}{3b} u^{\frac{3}{2}} + c = -\frac{3}{2b} (a-bx)^{\frac{3}{2}} + c$$

2.52.- $\int \frac{xdx}{\sqrt{x^2+1}}, \quad \text{Sea: } u = x^2+1, du = 2x dx$

$$\int \frac{xdx}{\sqrt{x^2+1}} = \frac{1}{2} \int \frac{du}{\sqrt{u}} = \frac{1}{2} \int u^{-\frac{1}{2}} du = \frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = (x^2+1)^{\frac{1}{2}} + c$$

2.53.- $\int \frac{\sqrt{x+\ell\eta x}}{x} dx, \quad \text{Sea: } u = \ell\eta x, du = \frac{dx}{x}$

$$\int \frac{\sqrt{x+\ell\eta x}}{x} dx = \int x^{-1/2} dx + \int \frac{\ell\eta x}{x} dx = \int x^{-1/2} dx + \int u du = \frac{x^{1/2}}{1/2} + \frac{u^2}{2} + c$$

$$= 2\sqrt{x} + \frac{\ell\eta^2 x}{2} + c$$

2.54.- $\int \frac{dx}{3x^2+5}, \quad \text{Sea: } u^2 = 3x^2, u = \sqrt{3}x, du = \sqrt{3}dx; a^2 = 5; a = \sqrt{5}$

$$\int \frac{dx}{3x^2+5} = \frac{1}{\sqrt{3}} \int \frac{du}{u^2+a^2} = \frac{1}{\sqrt{3}} \frac{1}{a} \operatorname{arctg} \frac{u}{a} + c = \frac{1}{\sqrt{3}} \frac{1}{\sqrt{5}} \operatorname{arctg} \frac{\sqrt{3}x}{\sqrt{5}} + c = \frac{\sqrt{15}}{15} \operatorname{arctg} \sqrt{\frac{3x}{5}} + c$$

2.55.- $\int \frac{x^3 dx}{a^2-x^2}, \quad \text{Sea: } u = x^2-a^2, du = 2x dx$

$$\begin{aligned} \int \frac{x^3 dx}{a^2-x^2} &= -\int x dx - \int \frac{a^2 x dx}{x^2-a^2} = -\int x dx - a^2 \int \frac{xdx}{x^2-a^2} = -\int x dx - \frac{a^2}{2} \int \frac{du}{u} \\ &= -\frac{x^2}{2} - \frac{a^2}{2} \ell \eta |u| + c = -\frac{x^2}{2} - \frac{a^2}{2} \ell \eta |x^2-a^2| + c \end{aligned}$$

2.56.- $\int \frac{y^2-5y+6}{y^2+4} dy, \quad \text{Sea: } u = y^2+4, du = 2y dy$

$$\begin{aligned} \int \frac{y^2-5y+6}{y^2+4} dy &= \int (1 + \frac{-5y+2}{y^2+4}) dy = \int dy + \int \frac{-5y+2}{y^2+4} dy = \int dy - 5 \int \frac{y dy}{y^2+4} + 2 \int \frac{dy}{y^2+2^2} \\ &= y - \frac{5}{2} \ell \eta |u| + \frac{1}{2} \operatorname{arc tg} \frac{y}{2} + c = y - \frac{5}{2} \ell \eta |y^2+4| + \operatorname{arc tg} \frac{y}{2} + c \end{aligned}$$

2.57.- $\int \frac{6t-15}{3t^2-2} dt, \quad \text{Sea: } u = 3t^2-2, du = 6tdt; w = \sqrt{3}t, dw = \sqrt{3}dt$

$$\begin{aligned}
\int \frac{6t-15}{3t^2-2} dt &= 6 \int \frac{tdt}{3t^2-2} - 15 \int \frac{dt}{3t^2-2} = 6 \int \frac{tdt}{3t^2-2} - 15 \int \frac{dt}{(\sqrt{3t})^2 - (\sqrt{2})^2} \\
&= \int \frac{du}{u} - \frac{15}{\sqrt{3}} \int \frac{dw}{w^2 - (\sqrt{2})^2} = \ell \eta |u| - \frac{15\sqrt{3}}{3} \frac{1}{2\sqrt{2}} \ell \eta \left| \frac{w-\sqrt{2}}{w+\sqrt{2}} \right| + c \\
&= \ell \eta |3t^2-2| - \frac{5\sqrt{6}}{4} \ell \eta \left| \frac{t\sqrt{3}-\sqrt{2}}{t\sqrt{3}+\sqrt{2}} \right| + c
\end{aligned}$$

2.58.- $\int \frac{3-2x}{5x^2+7} dx$, Sea: $u = 5x^2 + 7, du = 10x dx; w = \sqrt{5}x, dw = \sqrt{5}dx$

$$\begin{aligned}
\int \frac{3-2x}{5x^2+7} dx &= 3 \int \frac{dx}{5x^2+7} - 2 \int \frac{dx}{5x^2+7} = 3 \int \frac{dx}{(\sqrt{5x})^2 + (\sqrt{7})^2} - \frac{2}{10} \int \frac{du}{u} \\
&= \frac{3}{\sqrt{5}} \int \frac{dw}{w^2 + (\sqrt{7})^2} - \frac{1}{5} \int \frac{du}{u} = \frac{3}{\sqrt{5}} \frac{1}{\sqrt{7}} \arctan \tau g \frac{x\sqrt{5}}{\sqrt{7}} - \frac{1}{5} \ell \eta |u| + c \\
&= \frac{3\sqrt{35}}{35} \arctan \tau g x \sqrt{\frac{5}{7}} - \frac{1}{5} \ell \eta |5x^2 + 7| + c
\end{aligned}$$

2.59.- $\int \frac{3x+1}{\sqrt{5x^2+1}} dx$, Sea: $u = 5x^2 + 1, du = 10x dx; w = x\sqrt{5}, dw = \sqrt{5}dx$

$$\begin{aligned}
\int \frac{3x+1}{\sqrt{5x^2+1}} dx &= 3 \int \frac{xdx}{\sqrt{5x^2+1}} + \int \frac{dx}{\sqrt{(x\sqrt{5})^2 + 1^2}} = 3 \int \frac{xdx}{\sqrt{5x^2+1}} + \int \frac{dx}{\sqrt{(x\sqrt{5})^2 + 1^2}} \\
&= \frac{3}{10} \int \frac{du}{\sqrt{u}} + \frac{1}{\sqrt{5}} \int \frac{dw}{\sqrt{w^2 + 1^2}} = \frac{3}{10} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + \frac{1}{\sqrt{5}} \ell \eta \left| w + \sqrt{w^2 + 1} \right| + c \\
&= \frac{3}{5} \sqrt{5x^2+1} + \frac{1}{\sqrt{5}} \ell \eta \left| x\sqrt{5} + \sqrt{5x^2+1} \right| + c
\end{aligned}$$

2.60.- $\int \frac{xdx}{x^2-5}$, Sea: $u = x^2 + 5, du = 2x dx$

$$\int \frac{xdx}{x^2-5} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ell \eta |u| + c = \frac{1}{2} \ell \eta |x^2 - 5| + c$$

2.61.- $\int \frac{xdx}{2x^2+3}$, Sea: $u = 2x^2 + 3, du = 4x dx$

$$\int \frac{xdx}{2x^2+3} = \frac{1}{4} \int \frac{du}{u} = \frac{1}{4} \ell \eta |u| + c = \frac{1}{4} \ell \eta |2x^2 + 3| + c$$

2.62.- $\int \frac{ax+b}{a^2x^2+b^2} dx$, Sea: $u = a^2x^2 + b^2, du = 2a^2x dx; w = ax, dw = adx$

$$\begin{aligned}
\int \frac{ax+b}{a^2x^2+b^2} dx &= a \int \frac{xdx}{a^2x^2+b^2} + b \int \frac{dx}{a^2x^2+b^2} = \frac{a}{2a^2} \int \frac{du}{u} + \frac{b}{a} \int \frac{dw}{w^2+b^2} \\
&= \frac{1}{2} \ell \eta |u| + \frac{b}{a} \frac{1}{\sqrt{b^2/a}} \arctan \tau g \frac{w}{b} + c = \frac{1}{2} \ell \eta |a^2x^2 + b^2| + \frac{1}{a} \arctan \tau g \frac{ax}{b} + c
\end{aligned}$$

$$\text{2.63.-} \int \frac{x dx}{\sqrt{a^4 - x^4}}, \quad \text{Sea: } u = x^2, du = 2x dx$$

$$\begin{aligned} \int \frac{x dx}{\sqrt{a^4 - x^4}} &= \int \frac{x dx}{\sqrt{(\sqrt{a^2})^2 - (\sqrt{x^2})^2}} = \frac{1}{2} \int \frac{du}{\sqrt{(\sqrt{a^2})^2 - u^2}} = \frac{1}{2} \arcsen \frac{u}{a^2} + c \\ &= \frac{1}{2} \arcsen \frac{x^2}{a^2} + c \end{aligned}$$

$$\text{2.64.-} \int \frac{x^2 dx}{1+x^6}, \quad \text{Sea: } u = x^3, du = 3x^2 dx$$

$$\int \frac{x^2 dx}{1+x^6} = \int \frac{x^2 dx}{1+(x^3)^2} = \frac{1}{3} \int \frac{du}{1+u^2} = \frac{1}{3} \operatorname{arc \tau g} |u| + c = \frac{1}{3} \operatorname{arc \tau g} x^3 + c$$

$$\text{2.65.-} \int \frac{x^2 dx}{\sqrt{x^6 - 1}}, \quad \text{Sea: } u = x^3, du = 3x^2 dx$$

$$\int \frac{x^2 dx}{\sqrt{x^6 - 1}} = \int \frac{x^2 dx}{\sqrt{(x^3)^2 - 1}} = \frac{1}{3} \int \frac{du}{\sqrt{u^2 - 1}} = \frac{1}{3} \ell \eta |u + \sqrt{u^2 - 1}| + c = \frac{1}{3} \ell \eta |x^3 + \sqrt{x^6 - 1}| + c$$

$$\text{2.66.-} \int \frac{x - \sqrt{\operatorname{arc \tau g} 3x}}{1+9x^2} dx, \quad \text{Sea: } u = 1+9x^2, du = 18x dx; w = \operatorname{arc \tau g} 3x, dw = \frac{3dx}{1+9x^2}$$

$$\begin{aligned} \int \frac{x - \sqrt{\operatorname{arc \tau g} 3x}}{1+9x^2} dx &= \int \frac{xdx}{1+9x^2} - \int \frac{\sqrt{\operatorname{arc \tau g} 3x}}{1+9x^2} dx = \frac{1}{18} \int \frac{du}{u} - \frac{1}{3} \int w^{\frac{1}{2}} dw \\ &= \frac{1}{18} \ell \eta |u| - \frac{1}{3} \frac{w^{\frac{3}{2}}}{3/2} + c = \frac{1}{18} \ell \eta |1+9x^2| - \frac{2(\operatorname{arc \tau g} 3x)^{\frac{3}{2}}}{9} + c \end{aligned}$$

$$\text{2.67.-} \int \sqrt{\frac{\operatorname{arc \sen t}}{4-4t^2}} dt, \quad \text{Sea: } u = \operatorname{arc \sen t}, du = \frac{dt}{\sqrt{1-t^2}}$$

$$\int \sqrt{\frac{\operatorname{arc \sen t}}{4-4t^2}} dt = \frac{1}{2} \int \sqrt{\frac{\operatorname{arc \sen t}}{1-t^2}} dt = \frac{1}{2} \int \frac{\sqrt{\operatorname{arc \sen t}}}{\sqrt{1-t^2}} dt = \frac{1}{2} \int \sqrt{u} du = \frac{1}{2} \frac{u^{\frac{3}{2}}}{3/2} + c = \frac{1}{3} u^{\frac{3}{2}} + c$$

$$= \frac{1}{3} \sqrt{(\operatorname{arc \sen t})^3} + c$$

$$\text{2.68.-} \int \frac{\operatorname{arc \tau g}(\frac{x}{3})}{9+x^2} dx, \quad \text{Sea: } u = \operatorname{arc \tau g} \frac{x}{3}, du = \frac{3dx}{9+x^2}$$

$$\int \frac{\operatorname{arc \tau g}(\frac{x}{3})}{9+x^2} dx = \frac{1}{3} \int u du = \frac{1}{3} \frac{u^2}{2} + c = \frac{1}{6} u^2 + c = \frac{\operatorname{arc \tau g}(\frac{x}{3})^2}{6} + c$$

$$\text{2.69.-} \int \frac{dt}{\sqrt{(9+9t^2)\ell \eta |t+\sqrt{1+t^2}|}}, \quad \text{Sea: } u = \ell \eta |t+\sqrt{1+t^2}|, du = \frac{dt}{\sqrt{1+t^2}}$$

$$= \frac{1}{3} \int \frac{dt}{\sqrt{(1+t^2)} \sqrt{\ell \eta |t+\sqrt{1+t^2}|}} = \frac{1}{3} \int \frac{du}{\sqrt{u}} = \frac{1}{3} \frac{u^{\frac{1}{2}}}{1/2} + c = \frac{2}{3} \sqrt{u} + c = \frac{2}{3} \sqrt{\ell \eta |t+\sqrt{1+t^2}|} + c$$

2.70.- $\int ae^{-mx} dx,$ **Sea:** $u = -mx, du = -mdx$

$$\int ae^{-mx} dx = a \int e^{-mx} dx = -\frac{a}{m} \int e^u du = -\frac{a}{m} e^u + c = -\frac{a}{m} e^{-mx} + c$$

2.71.- $\int 4^{2-3x} dx,$ **Sea:** $u = 2 - 3x, du = -3dx; a = 4$

$$\int 4^{2-3x} dx = -\frac{1}{3} \int a^u du = -\frac{1}{3} \frac{a^u}{\ell \eta a} + c = -\frac{4^{2-3x}}{3\ell \eta 4} + c$$

2.72.- $\int (e^t - e^{-t}) dt,$ **Sea:** $u = -t, du = -dt$

$$\int (e^t - e^{-t}) dt = \int e^t dt - \int e^{-t} dt = \int e^t dt - \int e^u dt = e^t + e^u + c = e^t + e^{-t} + c$$

2.73.- $\int e^{-(x^2+1)} x dx,$ **Sea:** $u = -x^2 - 1, du = -2x dx$

$$\int e^{-(x^2+1)} x dx = \int e^{-x^2-1} x dx = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + c = -\frac{1}{2} e^{-(x^2+1)} + c = -\frac{1}{2e^{x^2+1}} + c$$

2.74.- $\int (e^{\sqrt{a}} - e^{-\sqrt{a}})^2 dx,$ **Sea:** $u = \frac{2x}{a}, du = \frac{2dx}{a}; w = -\frac{2x}{a}, dw = -\frac{2dx}{a}$

$$\int (e^{\sqrt{a}} - e^{-\sqrt{a}})^2 dx = \int (e^{\frac{2\sqrt{a}}{a}} + 2e^{\frac{\sqrt{a}}{a}} e^{-\sqrt{a}} + e^{-\frac{2\sqrt{a}}{a}}) dx = \int e^{\frac{2\sqrt{a}}{a}} dx + 2 \int dx + \int e^{-\frac{2\sqrt{a}}{a}} dx$$

$$= \frac{a}{2} \int e^u du + 2 \int dx - \frac{a}{2} \int e^w dw = \frac{a}{2} e^u + 2x - \frac{a}{2} e^w + c = \frac{a}{2} e^{\frac{2x}{a}} + 2x - \frac{a}{2} e^{-\frac{2x}{a}} + c$$

2.75.- $\int \frac{a^{2x}-1}{\sqrt{a^x}} dx,$ **Sea:** $u = -\frac{x}{2}, du = -\frac{dx}{2}; w = \frac{3x}{2}, dw = \frac{3dx}{2}$

$$\int \frac{a^{2x}-1}{\sqrt{a^x}} dx = \int \frac{a^{2x} dx}{\sqrt{a^x}} - \int \frac{dx}{\sqrt{a^x}} = \int a^{2x-\frac{1}{2}} dx - \int a^{-\frac{1}{2}} dx = \int a^{\frac{3}{2}} dx - \int a^{-\frac{1}{2}} dx$$

$$= \frac{2}{3} \int a^w dw + 2 \int a^u du = \frac{2}{3} \frac{a^w}{\ell \eta a} + 2 \frac{a^u}{\ell \eta a} + c = \frac{2}{3} \frac{a^{\frac{3}{2}}}{\ell \eta a} + 2 \frac{a^{-\frac{1}{2}}}{\ell \eta a} + c = \frac{2}{\ell \eta a} \left(\frac{a^{\frac{3}{2}}}{3} + a^{-\frac{1}{2}} \right) + c$$

2.76.- $\int \frac{e^{\sqrt{x}}}{x^2} dx,$ **Sea:** $u = \frac{1}{x}, du = -\frac{dx}{x^2}$

$$\int \frac{e^{\sqrt{x}}}{x^2} dx = - \int e^u du = -e^u + c = -e^{\sqrt{x}} + c = -\sqrt[x]{e} + c$$

2.77.- $\int 5^{\sqrt{x}} \frac{dx}{\sqrt{x}},$ **Sea:** $u = \sqrt{x}, du = \frac{dx}{2\sqrt{x}}$

$$\int 5^{\sqrt{x}} \frac{dx}{\sqrt{x}} = 2 \int 5^u du = \frac{2 \times 5^u}{\ell \eta 5} + c = \frac{2 \times 5^{\sqrt{x}}}{\ell \eta 5} + c$$

2.78.- $\int x 7^{x^2} dx,$ **Sea:** $u = x^2, du = 2x dx$

$$\int x 7^{x^2} dx = \frac{1}{2} \int 7^u du = \frac{1}{2} \frac{7^u}{\ell \eta 7} + c = \frac{1}{2} \frac{7^{x^2}}{\ell \eta 7} + c$$

2.79.- $\int \frac{e^t dt}{e^t - 1},$ **Sea:** $u = e^t - 1, du = e^t dt$

$$\int \frac{e^t dt}{e^t - 1} = \int \frac{du}{u} = \ell \eta |u| + c = \ell \eta |e^t - 1| + c$$

2.80.- $\int e^x \sqrt{a - be^x} dx$, **Sea:** $u = a - be^x, du = -be^x dx$

$$\int e^x \sqrt{a - be^x} dx = -\frac{1}{b} \int \sqrt{u} du = -\frac{1}{b} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{2}{3b} u^{\frac{3}{2}} + c = -\frac{2}{3b} (a - be^x)^{\frac{3}{2}} + c$$

2.81.- $\int (e^{\frac{x}{a}} + 1)^{\frac{1}{3}} e^{\frac{x}{a}} dx$, **Sea:** $u = e^{\frac{x}{a}+1}, du = \frac{e^{\frac{x}{a}}}{a} dx$

$$\int (e^{\frac{x}{a}} + 1)^{\frac{1}{3}} e^{\frac{x}{a}} dx = \int \sqrt[3]{e^{\frac{x}{a}} + 1} e^{\frac{x}{a}} dx = a \int u^{\frac{1}{3}} du = \frac{au^{\frac{4}{3}}}{\frac{4}{3}} + c = \frac{3a(e^{\frac{x}{a}} + 1)^{\frac{4}{3}}}{4} + c$$

2.82.- $\int \frac{dx}{2^x + 3}$, **Sea:** $u = 2^x + 3, du = 2^x \ell \eta 2 dx$

$$\begin{aligned} \int \frac{dx}{2^x + 3} &= \frac{1}{3} \int \frac{3dx}{2^x + 3} = \frac{1}{3} \int \frac{2^x + 3 - 2^x}{2^x + 3} dx = \frac{1}{3} \int \frac{2^x + 3}{2^x + 3} dx - \frac{1}{3} \int \frac{2^x}{2^x + 3} dx = \frac{1}{3} \int dx - \frac{1}{3} \int \frac{du}{u} \\ &= \frac{1}{3} x - \frac{1}{3} \ell \eta |u| + c = \frac{1}{3} x - \frac{1}{3 \ell \eta 2} \ell \eta |u| + c = \frac{1}{3} x - \frac{\ell \eta |2^x + 3|}{3 \ell \eta 2} + c \end{aligned}$$

2.83.- $\int \frac{a^x dx}{1 + a^{2x}}$, **Sea:** $u = a^x, du = a^x \ell \eta a dx; a > 0$

$$\int \frac{a^x dx}{1 + a^{2x}} = \int \frac{a^x dx}{1 + (a^x)^2} = \frac{1}{\ell \eta a} \int \frac{du}{1 + u^2} = \frac{1}{\ell \eta a} \arctan gu + c = \frac{1}{\ell \eta a} \arctan ga^x + c$$

2.84.- $\int \frac{e^{-bx}}{1 - e^{-2bx}} dx$, **Sea:** $u = e^{-bx}, du = -be^{-bx} dx$

$$\begin{aligned} \int \frac{e^{-bx}}{1 - e^{-2bx}} dx &= \int \frac{e^{-bx}}{1 - (e^{-bx})^2} dx = -\frac{1}{b} \int \frac{du}{1 - u^2} = -\frac{1}{b} \int \frac{du}{(-1)(u^2 - 1)} = \frac{1}{2b} \ell \eta \left| \frac{u-1}{u+1} \right| + c \\ &= \frac{1}{2b} \ell \eta \left| \frac{e^{-bx} - 1}{e^{-bx} + 1} \right| + c. \end{aligned}$$

2.85.- $\int \frac{e^t dt}{\sqrt{1 - e^{2t}}}$, **Sea:** $u = e^t, du = e^t dt$

$$\int \frac{e^t dt}{\sqrt{1 - e^{2t}}} = \int \frac{e^t dt}{\sqrt{1 - (e^t)^2}} = \int \frac{du}{\sqrt{1 - u^2}} = \arcsen u + c = \arcsen e^t + c$$

2.86.- $\int \cos \frac{x}{\sqrt{2}} dx$, **Sea:** $u = \frac{x}{\sqrt{2}}, du = \frac{dx}{\sqrt{2}}$

$$\int \cos \frac{x}{\sqrt{2}} dx = \sqrt{2} \int \cos u du = \sqrt{2} \sen u + c = \sqrt{2} \sen \frac{x}{\sqrt{2}} + c$$

2.87.- $\int \sen(a + bx) dx$, **Sea:** $u = a + bx, du = bdx$

$$\int \sen(a + bx) dx = \frac{1}{b} \int \sen u du = -\frac{1}{b} \cos u + c = -\frac{1}{b} \cos(a + bx) + c$$

2.88.- $\int \cos \sqrt{x} \frac{dx}{\sqrt{x}},$ Sea: $u = \sqrt{x}, du = \frac{dx}{2\sqrt{x}}$

$$\int \cos \sqrt{x} \frac{dx}{\sqrt{x}} = 2 \int \cos u du = 2 \sin u + c = 2 \sin \sqrt{x} + c$$

2.89.- $\int \operatorname{sen}(\ell \eta x) \frac{dx}{x},$ Sea: $u = \ell \eta x, du = \frac{dx}{x}$

$$\int \operatorname{sen}(\ell \eta x) \frac{dx}{x} = \int \operatorname{sen} u du = -\cos u + c = -\cos \ell \eta x + c$$

2.90.- $\int (\cos ax + \operatorname{sen} ax)^2 dx,$ Sea: $u = 2ax, du = 2adx$

$$\int (\cos ax + \operatorname{sen} ax)^2 dx = \int (\cos^2 ax + 2 \cos ax \operatorname{sen} ax + \operatorname{sen}^2 ax) dx$$

$$= \int (1 + 2 \cos ax \operatorname{sen} ax) dx = \int dx + 2 \int \cos ax \operatorname{sen} ax dx = \int dx + \int \operatorname{sen} 2ax dx$$

$$= x - \frac{1}{2a} \cos 2ax + c$$

2.91.- $\int \operatorname{sen}^2 x dx,$ Sea: $u = 2x, du = 2dx$

$$\int \operatorname{sen}^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x dx = \frac{1}{2} \int dx - \frac{1}{4} \int \cos u du = \frac{1}{2} x - \frac{1}{4} \operatorname{sen} u + c$$

$$= \frac{1}{2} x - \frac{1}{4} \operatorname{sen} 2x + c$$

2.92.- $\int \cos^2 x dx,$ Sea: $u = 2x, du = 2dx$

$$\int \cos^2 x dx = \int \frac{1 + \cos 2x}{2} dx = \frac{1}{2} \int dx + \frac{1}{2} \int \cos 2x dx = \frac{1}{2} \int dx + \frac{1}{4} \int \cos u du = \frac{1}{2} x + \frac{1}{4} \operatorname{sen} u + c$$

$$= \frac{1}{2} x + \frac{1}{4} \operatorname{sen} 2x + c$$

2.93.- $\int \sec^2(ax + b) dx,$ Sea: $u = ax + b, du = adx$

$$\int \sec^2(ax + b) dx = \frac{1}{a} \int \sec^2 u du = \frac{1}{a} \tau g u + c = \frac{1}{a} \tau g(ax + b) = +c$$

2.94.- $\int \cot g^2 a x dx,$ Sea: $u = ax, du = adx$

$$\int \cot g^2 a x dx = \frac{1}{a} \int \cot g^2 u du = \frac{1}{a} \int (\operatorname{cosec}^2 u - 1) du = \frac{1}{a} \int \operatorname{cosec}^2 u du - \frac{1}{a} \int du$$

$$= -\frac{\cot g u}{a} - \frac{u}{a} + c = -\frac{\cot g a x}{a} - \frac{a x}{a} + c = -\frac{\cot g a x}{a} - x + c$$

2.95.- $\int \frac{dx}{\operatorname{sen} \frac{x}{a}},$ Sea: $u = \frac{x}{a}, du = \frac{dx}{a}$

$$\int \frac{dx}{\operatorname{sen} \frac{x}{a}} = \int \operatorname{cosec} \frac{x}{a} dx = a \int \operatorname{cosec} u du = a \ell \eta |\operatorname{cosec} u - \cot g u| + c$$

$$= a \ell \eta |\operatorname{cosec} \frac{x}{a} - \cot g \frac{x}{a}| + c$$

2.96.- $\int \frac{dx}{3\cos(5x - \frac{\pi}{4})}$, **Sea:** $u = 5x - \frac{\pi}{4}$, $du = 5dx$

$$\begin{aligned} \int \frac{dx}{3\cos(5x - \frac{\pi}{4})} &= \frac{1}{3} \int \sec(5x - \frac{\pi}{4}) dx = \frac{1}{15} \int \sec u du = \frac{1}{15} \ell \eta |\sec u + \tau g u| + c \\ &= \frac{1}{15} \ell \eta |\sec(5x - \frac{\pi}{4}) + \tau g(5x - \frac{\pi}{4})| + c \end{aligned}$$

2.97.- $\int \frac{dx}{\sin(ax+b)}$, **Sea:** $u = ax + b$, $du = adx$

$$\begin{aligned} \int \frac{dx}{\sin(ax+b)} &= \int \csc ec(ax+b) dx = \frac{1}{a} \int \csc ec u du = \frac{1}{a} \ell \eta |\csc ec u - \co \tau g u| + c \\ &= \frac{1}{a} \ell \eta |\csc ec(ax+b) - \co \tau g(ax+b)| + c \end{aligned}$$

2.98.- $\int \frac{x dx}{\cos^2 x^2}$, **Sea:** $u = x^2$, $du = 2x dx$

$$\int \frac{x dx}{\cos^2 x^2} = \int x \sec^2 x^2 dx = \frac{1}{2} \int \sec^2 u du = \frac{1}{2} \tau g u + c = \frac{1}{2} \tau g x^2 + c$$

2.99.- $\int \co \tau g \frac{x}{a-b} dx$, **Sea:** $u = \frac{x}{a-b}$, $du = \frac{dx}{a-b}$

$$\int \co \tau g \frac{x}{a-b} dx = (a-b) \int \co \tau g u du = (a-b) \ell \eta |\sin u| + c = (a-b) \ell \eta \left| \sin \frac{x}{a-b} \right| + c$$

2.100.- $\int \tau g \sqrt{x} \frac{dx}{\sqrt{x}}$, **Sea:** $u = \sqrt{x}$, $du = \frac{dx}{2\sqrt{x}}$

$$\int \tau g \sqrt{x} \frac{dx}{\sqrt{x}} = 2 \int \tau g u du = 2 \ell \eta |\sec u| + c = 2 \ell \eta |\sec \sqrt{x}| + c$$

2.101.- $\int \frac{dx}{\tau g \frac{x}{5}}$, **Sea:** $u = \frac{x}{5}$, $du = \frac{dx}{5}$

$$\int \frac{dx}{\tau g \frac{x}{5}} = \int \co \tau g \frac{x}{5} dx = 5 \int \co \tau g u du = 5 \ell \eta |\sin u| + c = 5 \ell \eta \left| \sin \frac{x}{5} \right| + c$$

2.102.- $\int \left(\frac{1}{\sin x \sqrt{2}} - 1 \right)^2 dx$, **Sea:** $u = x \sqrt{2}$, $du = \sqrt{2} dx$

$$\begin{aligned} \int \left(\frac{1}{\sin x \sqrt{2}} - 1 \right)^2 dx &= \int (\cos ec x \sqrt{2} - 1)^2 dx = \int (\cos ec^2 x \sqrt{2} - 2 \cos ec x \sqrt{2} + 1) dx \\ &= \int \cos ec^2 x \sqrt{2} dx - 2 \int \cos ec x \sqrt{2} dx + \int dx = \frac{1}{\sqrt{2}} \int \cos ec^2 u du - \frac{2}{\sqrt{2}} \int \cos ec u du + \int dx \\ &= -\frac{1}{\sqrt{2}} \co \tau g u - \sqrt{2} \ell \eta |\csc ec u - \co \tau g u| + x + c \\ &= -\frac{1}{\sqrt{2}} \co \tau g x \sqrt{2} - \sqrt{2} \ell \eta \left| \csc ec x \sqrt{2} - \co \tau g x \sqrt{2} \right| + x + c \end{aligned}$$

- 2.103.-** $\int \frac{dx}{\sin x \cos x}$, Sea: $u = 2x, du = 2dx$
- $$\begin{aligned} \int \frac{dx}{\sin x \cos x} &= \int \frac{dx}{\frac{1}{2} \sin 2x} = 2 \int \csc 2x dx = \int \csc u du = \ell \eta |\csc u - \cot u| + c \\ &= \ell \eta |\csc 2x - \cot 2x| + c \end{aligned}$$
- 2.104.-** $\int \frac{\cos ax}{\sin^5 ax} dx$, Sea: $u = \sin ax, du = a \cos ax dx$
- $$\int \frac{\cos ax}{\sin^5 ax} dx = \frac{1}{a} \int \frac{du}{u^5} = \frac{1}{a} \frac{u^{-4}}{-4} + c = -\frac{u^{-4}}{4a} + c = -\frac{\sin^{-4} ax}{4a} + c = -\frac{1}{4a \sin^4 ax} + c$$
- 2.105.-** $\int t \sin(1-2t^2) dt$, Sea: $u = 1-2t^2, du = -4t dt$
- $$\int t \sin(1-2t^2) dt = -\frac{1}{4} \int \sin u du = \frac{1}{4} \cos u + c = \frac{1}{4} \cos(1-2t^2) + c$$
- 2.106.-** $\int \frac{\sin 3x}{3+\cos 3x} dx$, Sea: $u = 3+\cos 3x, du = -3 \sin 3x dx$
- $$\int \frac{\sin 3x}{3+\cos 3x} dx = -\frac{1}{3} \int \frac{du}{u} = -\frac{1}{3} \ell \eta |u| + c = -\frac{1}{3} \ell \eta |3+\cos 3x| + c$$
- 2.107.-** $\int \tau g^3 \frac{x}{3} \sec^2 \frac{x}{3} dx$, Sea: $u = \tau g(\frac{x}{3}), du = \frac{1}{3} \sec^2(\frac{x}{3}) dx$
- $$\int \tau g^3 \frac{x}{3} \sec^2 \frac{x}{3} dx = 3 \int u^3 du = \frac{3u^4}{4} + c = \frac{3\tau g^4(\frac{x}{3})}{4} + c$$
- 2.108.-** $\int \frac{\sin x \cos x}{\sqrt{\cos^2 x - \sin^2 x}} dx$, Sea: $u = \cos 2x, du = 2 \sin 2x dx$
- $$\begin{aligned} \int \frac{\sin x \cos x}{\sqrt{\cos^2 x - \sin^2 x}} dx &= \int \frac{\sin x \cos x}{\sqrt{\cos 2x}} dx = \frac{1}{4} \int \frac{\sin 2x}{\sqrt{\cos 2x}} dx = \frac{1}{4} \int \frac{du}{\sqrt{u}} = \frac{1}{4} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{u^{\frac{1}{2}}}{2} + c \\ &= \frac{\sqrt{\cos 2x}}{2} + c \end{aligned}$$
- 2.109.-** $\int \frac{\sqrt{\tau gx}}{\cos^2 x} dx$, Sea: $u = \tau gx, du = \sec^2 x dx$
- $$\int \frac{\sqrt{\tau gx}}{\cos^2 x} dx = \int \sqrt{\tau gx} \sec^2 x dx = \int u^{\frac{1}{2}} du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3} u^{\frac{3}{2}} + c = \frac{2}{3} \tau g^{\frac{3}{2}} x + c$$
- 2.110.-** $\int \cos \frac{x}{a} \sin \frac{x}{a} dx$, Sea: $u = 2x/a, du = 2dx$
- $$\int \cos \frac{x}{a} \sin \frac{x}{a} dx = \frac{1}{2} \int \sin \frac{2x}{a} dx = \frac{a}{4} \int \sin u du = -\frac{a}{4} \cos u + c = -\frac{a}{4} \cos \frac{2x}{a} + c$$
- 2.111.-** $\int t \cot g(2t^2 - 3) dt$, Sea: $u = 2t^2 - 3, du = 4t dt$
- $$\int t \cot g(2t^2 - 3) dt = \frac{1}{4} \int \cot g u du = \frac{1}{4} \ell \eta |\sin u| + c = \frac{1}{4} \ell \eta |\sin(2t^2 - 3)| + c$$

$$\mathbf{2.112.-} \int \frac{x^3 dx}{x^8 + 5},$$

Sea: $u = x^4, du = 4x^3 dx$

$$\int \frac{x^3 dx}{x^8 + 5} = \int \frac{x^3 dx}{(x^4)^2 + (\sqrt{5})^2} = \frac{1}{4} \int \frac{du}{u^2 + (\sqrt{5})^2} = \frac{1}{4} \frac{1}{\sqrt{5}} \operatorname{arc \tau g} \frac{u}{\sqrt{5}} + c = \frac{\sqrt{5}}{20} \operatorname{arc \tau g} \frac{x^4}{\sqrt{5}} + c$$

$$\mathbf{2.113.-} \int \sin^3 6x \cos 6x dx, \quad \text{Sea: } u = \sin 6x, du = 6 \cos 6x dx$$

$$\int \sin^3 6x \cos 6x dx = \frac{1}{6} \int u^3 du = \frac{1}{6} \frac{u^4}{4} + c = \frac{u^4}{24} + c = \frac{\sin^4 6x}{24} + c$$

$$\mathbf{2.114.-} \int \sqrt{1+3\cos^2 x} \sin 2x dx, \quad \text{Sea: } u = \frac{5+3\cos 2x}{2}, du = -3 \sin 2x dx$$

$$\int \sqrt{1+3\cos^2 x} \sin 2x dx = \int \sqrt{1+3(\frac{1+\cos 2x}{2})} \sin 2x dx = \int \sqrt{1+\frac{3+3\cos 2x}{2}} \sin 2x dx$$

$$= \int \sqrt{\frac{5+3\cos 2x}{2}} \sin 2x dx = -\frac{1}{3} \int u^{\frac{1}{2}} du = -\frac{1}{3} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{2}{9} u^{\frac{3}{2}} + c$$

$$\mathbf{2.115.-} \int x^5 \sqrt{5-x^2} dx, \quad \text{Sea: } u = 5-x^2, du = -2x dx$$

$$\int x^5 \sqrt{5-x^2} dx = -\frac{1}{2} \int u^{\frac{5}{2}} du = -\frac{1}{2} \frac{u^{\frac{7}{2}}}{\frac{7}{2}} + c = -\frac{5}{12} u^{\frac{7}{2}} + c = -\frac{5(5-x^2)^{\frac{7}{2}}}{12} + c$$

$$\mathbf{2.116.-} \int \frac{1+\sin 3x}{\cos^2 3x} dx,$$

Sea: $u = \sin 3x, du = 3dx; w = \cos u, dw = -\sin u du$

$$\int \frac{1+\sin 3x}{\cos^2 3x} dx = \int \frac{dx}{\cos^2 3x} + \int \frac{\sin 3x}{\cos^2 3x} dx = \frac{1}{3} \int \sec^2 u du + \frac{1}{3} \int \frac{\sin u}{\cos^2 u} du$$

$$= \frac{1}{3} \int \sec^2 u du - \frac{1}{3} \int \frac{dw}{w^2} = \frac{1}{3} \tau g u + \frac{1}{3w} + c = \frac{1}{3} \tau g u + \frac{1}{3 \cos u} + c = \frac{1}{3} \tau g 3x + \frac{1}{3 \cos 3x} + c$$

$$\mathbf{2.117.-} \int \frac{(\cos ax + \sin ax)^2}{\sin ax} dx, \quad \text{Sea: } u = ax, du = adx$$

$$\int \frac{(\cos ax + \sin ax)^2}{\sin ax} dx = \int \frac{\cos^2 ax + 2 \cos ax \sin ax + \sin^2 ax}{\sin ax} dx$$

$$= \int \frac{\cos^2 ax}{\sin ax} dx + 2 \int \frac{\cos ax \sin ax}{\sin ax} dx + \int \frac{\sin^2 ax}{\sin ax} dx$$

$$= \int \frac{1 - \sin^2 ax}{\sin ax} dx + 2 \int \cos ax dx + \int \sin ax dx$$

$$= \int \frac{dx}{\sin ax} + 2 \int \cos ax dx$$

$$= \int \cos ec ax dx + 2 \int \cos ax dx = \frac{1}{a} \int \cos ec u du + \frac{2}{a} \int \cos u du$$

$$= \frac{1}{a} \ell \eta |\cos ec u - \cos \tau g u| + \frac{2}{a} \sin u + c = \frac{1}{a} \ell \eta |\cos ec ax - \cos \tau g ax| + \frac{2}{a} \sin ax + c$$

2.118. - $\int \frac{x^3 - 1}{x+1} dx,$ Sea: $u = x+1, du = dx$

$$\begin{aligned} \int \frac{x^3 - 1}{x+1} dx &= \int (x^2 - x + 1 - \frac{2}{x+1}) dx = \int x^2 dx - \int x dx + \int dx - \int \frac{2}{x+1} dx \\ &= \int x^2 dx - \int x dx + \int dx - 2 \int \frac{du}{u} = \frac{x^3}{3} - \frac{x^2}{2} + x - 2 \ell \eta |x+1| + c \end{aligned}$$

2.119. - $\int \frac{\cos ec^2 3x dx}{b - a \cos \tau g 3x},$ Sea: $u = b - a \cos \tau g 3x, du = 3a \cos ec^2 3x dx$

$$\int \frac{\cos ec^2 3x dx}{b - a \cos \tau g 3x} = \frac{1}{3a} \int \frac{du}{u} = \frac{1}{3a} \ell \eta |u| + c = \frac{1}{3a} \ell \eta |b - a \cos \tau g 3x| + c$$

2.120. - $\int \frac{x^3 - 1}{x^4 - 4x + 1} dx,$ Sea: $u = x^4 - 4x + 1, du = (4x^3 - 4)dx$

$$\int \frac{x^3 - 1}{x^4 - 4x + 1} dx = \frac{1}{4} \int \frac{(4x^3 - 4)dx}{x^4 - 4x + 1} = \frac{1}{4} \int \frac{du}{u} = \frac{1}{4} \ell \eta |u| + c = \frac{1}{4} \ell \eta |x^4 - 4x + 1| + c$$

2.121. - $\int x e^{-x^2} dx,$ Sea: $u = -x^2, du = -2x dx$

$$\int x e^{-x^2} dx = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + c = -\frac{1}{2} e^{-x^2} + c$$

2.122. - $\int \frac{3 - \sqrt{2+3x^2}}{2+3x^2} dx,$ Sea: $u = x\sqrt{3}, du = \sqrt{3}dx; a = \sqrt{2}$

$$\begin{aligned} \int \frac{3 - \sqrt{2+3x^2}}{2+3x^2} dx &= 3 \int \frac{dx}{(\sqrt{2})^2 + (\sqrt{3}x)^2} - \int \frac{(2+3x^2)^{\frac{1}{2}}}{2+3x^2} dx \\ &= \frac{3}{\sqrt{3}} \int \frac{\sqrt{3}dx}{(\sqrt{2})^2 + (\sqrt{3}x)^2} - \int \frac{(2+3x^2)^{\frac{1}{2}}}{2+3x^2} dx = \frac{3}{\sqrt{3}} \int \frac{\sqrt{3}dx}{(\sqrt{2})^2 + (\sqrt{3}x)^2} - \int (2+3x^2)^{-\frac{1}{2}} dx \end{aligned}$$

$$\begin{aligned} &= \frac{3}{\sqrt{3}} \int \frac{du}{(a)^2 + (u)^2} - \int (2+3x^2)^{-\frac{1}{2}} dx = \sqrt{3} \int \frac{du}{(a)^2 + (u)^2} - \int \frac{dx}{\sqrt{(\sqrt{2})^2 + (x\sqrt{3})^2}} \\ &= \sqrt{3} \int \frac{du}{(a)^2 + (u)^2} - \frac{1}{\sqrt{3}} \int \frac{du}{\sqrt{a^2 + u^2}} = \frac{\sqrt{3}}{a} \arctan \frac{u}{a} - \frac{1}{\sqrt{3}} \ell \eta |u + \sqrt{a^2 + u^2}| + c \end{aligned}$$

$$\begin{aligned} &= \frac{\sqrt{3}}{\sqrt{2}} \arctan \frac{x\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{3}}{3} \ell \eta |x\sqrt{3} + \sqrt{2+3+x^2}| + c \end{aligned}$$

2.123. - $\int \frac{\tau g 3x - \cos \tau g 3x}{\sin 3x} dx,$ Sea: $u = 3x, du = 3dx; w = \sin u, dw = \cos u du$

$$\int \frac{\tau g 3x - \cos \tau g 3x}{\sin 3x} dx = \int \frac{\sin 3x - \cos 3x}{\cos 3x - \sin 3x} dx = \int \frac{dx}{\cos 3x} - \int \frac{\cos 3x}{\sin^2 3x} dx$$

$$= \int \sec 3x dx - \int \frac{\cos 3x}{\sin^2 3x} dx = \frac{1}{3} \int \sec u du - \frac{1}{3} \int \frac{\cos u}{\sin^2 u} du = \frac{1}{3} \int \sec u du - \frac{1}{3} \int \frac{dw}{w^2}$$

$$= \frac{1}{3} \ell \eta |\sec u + \tau g u| - \frac{1}{3} \frac{w^{-1}}{-1} + c = \frac{1}{3} \ell \eta |\sec 3x + \tau g 3x| + \frac{1}{3 \sin 3x} + c$$

2.124. - $\int \frac{dx}{\sqrt{e^x}}, \quad \text{Sea: } u = -\frac{x}{2}, du = -\frac{dx}{2}$

$$\int \frac{dx}{\sqrt{e^x}} = \int \frac{dx}{(e^x)^{\frac{1}{2}}} = \int e^{-\frac{x}{2}} dx = -2 \int e^u du = -2e^u + c = -2e^{-\frac{x}{2}} + c = \frac{-2}{e^{\frac{x}{2}}} + c$$

2.125. - $\int \frac{1 + \sin x}{x + \cos x} dx, \quad \text{Sea: } u = x + \cos x, du = (1 - \sin x) dx$

$$\int \frac{1 + \sin x}{x + \cos x} dx = \int \frac{du}{u} = \ell \eta |u| + c = \ell \eta |x + \cos x| + c$$

2.126. - $\int \frac{\sec^2 x dx}{\sqrt{\tau g^2 x - 2}}, \quad \text{Sea: } u = \tau g x, du = \sec^2 x dx$

$$\int \frac{\sec^2 x dx}{\sqrt{\tau g^2 x - 2}} = \int \frac{du}{\sqrt{u^2 - 2}} = \ell \eta |u + \sqrt{u^2 - 2}| + c = \ell \eta |\tau g x + \sqrt{\tau g x^2 - 2}| + c$$

2.127. - $\int \frac{dx}{x \ell \eta^2 x}, \quad \text{Sea: } u = \ell \eta x, du = \frac{dx}{2}$

$$\int \frac{dx}{x \ell \eta^2 x} = \int \frac{dx}{x (\ell \eta x)^2} = \int \frac{du}{u^2} = \frac{u^{-1}}{-1} + c = -\frac{1}{u} + c = -\frac{1}{\ell \eta |x|} + c$$

2.128. - $\int a^{\sin x} \cos x dx, \quad \text{Sea: } u = \sin x, du = \cos x dx$

$$\int a^{\sin x} \cos x dx = \int a^u du = \frac{a^u}{\ell \eta a} + c = \frac{a^{\sin x}}{\ell \eta a} + c$$

2.129. - $\int \frac{x^2}{\sqrt{x^3 + 1}} dx, \quad \text{Sea: } u = x^3 + 1, du = 3x^2 dx$

$$\int \frac{x^2 dx}{\sqrt{x^3 + 1}} = \int \frac{x^2 dx}{(x^3 + 1)^{\frac{1}{3}}} = \frac{1}{3} \int \frac{du}{u^{\frac{1}{3}}} = \frac{1}{3} \cancel{\frac{u^{\frac{2}{3}}}{2}} + c = \frac{u^{\frac{2}{3}}}{2} + c = \frac{(x^2 + 1)^{\frac{2}{3}}}{2} + c = \frac{\sqrt[3]{(x^2 + 1)^2}}{2} + c$$

2.130. - $\int \frac{xdx}{\sqrt{1-x^4}}, \quad \text{Sea: } u = x^2, du = 2x dx$

$$\int \frac{xdx}{\sqrt{1-x^4}} = \int \frac{xdx}{\sqrt{1-(x^2)^2}} = \frac{1}{2} \int \frac{2xdx}{\sqrt{1-(x^2)^2}} = \frac{1}{2} \int \frac{2xdx}{\sqrt{1-(u)^2}} = \frac{1}{2} \arcsen u + c$$

$$= \frac{1}{2} \arcsen x^2 + c$$

2.131. - $\int \tau g^2 a x dx, \quad \text{Sea: } u = ax, du = adx$

$$\begin{aligned}\int \tau g^2 ax dx &= \int (\sec^2 ax - 1) dx = \int \sec^2 ax dx - \int dx = \frac{1}{a} \int \sec^2 u du - \int dx = \frac{1}{a} \tau g u - x + c \\ &= \frac{1}{a} \tau g a x - x + c\end{aligned}$$

2.132. - $\int \frac{\sec^2 x dx}{\sqrt{4 - \tau g^2 x}}, \quad \text{Sea: } u = \tau g x, du = \sec^2 x dx$

$$\int \frac{\sec^2 x dx}{\sqrt{4 - \tau g^2 x}} = \int \frac{du}{\sqrt{2^2 - u^2}} = \arcsen \frac{u}{2} + c = \arcsen \frac{\tau g x}{2} + c$$

2.133. - $\int \frac{dx}{\cos \frac{x}{a}}, \quad \text{Sea: } u = \frac{x}{a}, du = \frac{dx}{a}$

$$\int \frac{dx}{\cos \frac{x}{a}} = \int \sec \frac{x}{a} dx = a \int \sec u du = a \ell \eta |\sec u + \tau g u| + c = a \ell \eta |\sec \frac{x}{a} + \tau g \frac{x}{a}| + c$$

2.134. - $\int \frac{\sqrt[3]{1 + \ell \eta x}}{x} dx, \quad \text{Sea: } u = 1 + \ell \eta x, du = \frac{dx}{x}$

$$\int \frac{\sqrt[3]{1 + \ell \eta x}}{x} dx = \int u^{\frac{1}{3}} du = \frac{u^{\frac{4}{3}}}{4/3} + c = \frac{3u^{\frac{4}{3}}}{4} + c = \frac{3(1 + \ell \eta x)^{\frac{4}{3}}}{4} + c$$

2.135. - $\int \tau g \sqrt{x-1} \frac{dx}{\sqrt{x-1}}, \quad \text{Sea: } u = \sqrt{x-1}, du = \frac{dx}{2\sqrt{x-1}}$

$$\int \tau g \sqrt{x-1} \frac{dx}{\sqrt{x-1}} = 2 \int \tau g u \frac{du}{u} = 2 \ell \eta |\sec \sqrt{x-1}| + c = -2 \ell \eta |\cos \sqrt{x-1}| + c$$

2.136. - $\int \frac{xdx}{\operatorname{sen} x^2}, \quad \text{Sea: } u = x^2, du = 2x dx$

$$\begin{aligned}\int \frac{xdx}{\operatorname{sen} x^2} &= \frac{1}{2} \int \frac{du}{\operatorname{sen} u} = \frac{1}{2} \int \cos ec u du = \frac{1}{2} \ell \eta |\cos ec u - \operatorname{co} \tau g u| + c \\ &= \frac{1}{2} \ell \eta |\cos ec x^2 - \operatorname{co} \tau g x^2| + c\end{aligned}$$

2.137. - $\int \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x} dx, \quad \text{Sea: } u = \operatorname{sen} x + \cos x, du = (\cos x - \operatorname{sen} x) dx$

$$\int \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x} dx = - \int \frac{du}{u} = -\ell \eta |\operatorname{sen} x + \cos x| + c$$

2.138. - $\int \frac{e^{\operatorname{arc} \tau g x} + x \ell \eta (1+x^2) + 1}{1+x^2}, \quad \text{Sea: } u = \operatorname{arc} \tau g x, du = \frac{dx}{1+x^2}; w = \ell \eta (1+x^2) d, dw = \frac{2x dx}{1+x^2}$

$$\begin{aligned}\int \frac{e^{\operatorname{arc} \tau g x} + x \ell \eta (1+x^2) + 1}{1+x^2} &= \int \frac{e^{\operatorname{arc} \tau g x} dx}{1+x^2} + \int \frac{x \ell \eta (1+x^2) dx}{1+x^2} + \int \frac{dx}{1+x^2} \\ &= \int e^u du + \frac{1}{2} \int w dw + \int \frac{dx}{1+x^2} = e^u + \frac{1}{2} \frac{w^2}{2} + \operatorname{arc} \tau g x + c = e^u + \frac{\ell \eta^2 (1+x^2)}{4} + \operatorname{arc} \tau g x + c\end{aligned}$$

2.139. - $\int \frac{x^2 dx}{x^2 - 2},$

$$\int \frac{x^2 dx}{x^2 - 2} = \int \left(1 + \frac{2}{x^2 - 2}\right) dx = \int dx + 2 \int \frac{dx}{x^2 - 2} = x + 2 \frac{1}{2\sqrt{2}} \ell \eta \left| \frac{x - \sqrt{2}}{x + \sqrt{2}} \right| + c$$

$$= x + \frac{\sqrt{2}}{2} \ell \eta \left| \frac{x - \sqrt{2}}{x + \sqrt{2}} \right| + c$$

2.140. - $\int e^{\operatorname{sen}^2 x} \operatorname{sen} 2x dx,$ **Sea:** $u = \frac{1 - \cos 2x}{2}, du = \operatorname{sen} 2x dx$

$$\int e^{\operatorname{sen}^2 x} \operatorname{sen} 2x dx = \int e^{\frac{1 - \cos 2x}{2}} \operatorname{sen} 2x dx = \int e^u du = e^u + c = e^{\operatorname{sen}^2 x} + c$$

2.141. - $\int \frac{(1 - \operatorname{sen} \frac{x}{\sqrt{2}})^2}{\operatorname{sen} \frac{x}{\sqrt{2}}} dx,$ **Sea:** $u = \frac{x}{\sqrt{2}}, du = \frac{dx}{\sqrt{2}}$

$$\int \frac{(1 - \operatorname{sen} \frac{x}{\sqrt{2}})^2}{\operatorname{sen} \frac{x}{\sqrt{2}}} dx = \int \left(\frac{1 - 2 \operatorname{sen} \frac{x}{\sqrt{2}} + \operatorname{sen}^2 \frac{x}{\sqrt{2}}}{\operatorname{sen} \frac{x}{\sqrt{2}}} \right) dx = \int \operatorname{cosec} \frac{x}{\sqrt{2}} dx - 2 \int dx + \int \operatorname{sen} \frac{x}{\sqrt{2}} dx$$

$$= \sqrt{2} \int \operatorname{cosec} u du - 2 \int dx + \sqrt{2} \int \operatorname{sen} u du = \sqrt{2} \ell \eta |\operatorname{cosec} u - \operatorname{cotan} u| - 2x - \sqrt{2} \operatorname{cos} u + c$$

$$= \sqrt{2} \ell \eta |\operatorname{cosec} \frac{x}{\sqrt{2}} - \operatorname{cotan} \frac{x}{\sqrt{2}}| - 2x - \sqrt{2} \operatorname{cos} \frac{x}{\sqrt{2}} + c$$

2.142. - $\int \frac{5 - 3x}{\sqrt{4 - 3x^2}} dx,$ **Sea:** $u = x\sqrt{3}, du = \sqrt{3}dx; w = 4 - 3x^2, dw = -6xdx$

$$\int \frac{5 - 3x}{\sqrt{4 - 3x^2}} dx = 5 \int \frac{dx}{\sqrt{4 - 3x^2}} - 3 \int \frac{xdx}{\sqrt{4 - 3x^2}} = 5 \int \frac{dx}{\sqrt{4 - (x\sqrt{3})^2}} - 3 \int \frac{xdx}{\sqrt{4 - 3x^2}}$$

$$= \frac{5}{\sqrt{3}} \int \frac{du}{\sqrt{2^2 - u^2}} + \frac{3}{6} \int \frac{dw}{\sqrt{w}} = \frac{5}{\sqrt{3}} \arcsen \frac{u}{2} + \frac{1}{2} \frac{w^{1/2}}{1/2} + c = \frac{5\sqrt{3}}{3} \arcsen \frac{x\sqrt{3}}{2} + \sqrt{4 - 3x^2} + c$$

2.143. - $\int \frac{ds}{e^s + 1},$ **Sea:** $u = 1 + e^{-s}, du = -e^{-s} ds$

$$\int \frac{ds}{e^s + 1} = \int \frac{e^{-s} ds}{e^{-s} + 1} = - \int \frac{du}{u} = -\ell \eta |u| + c = -\ell \eta |e^{-s} + 1| + c$$

2.144. - $\int \frac{d\theta}{\operatorname{sen} a\theta \cos a\theta},$ **Sea:** $u = 2a\theta, du = 2ad\theta$

$$\int \frac{d\theta}{\operatorname{sen} a\theta \cos a\theta} = \int \frac{d\theta}{\frac{1}{2} \operatorname{sen} 2a\theta} = 2 \int \operatorname{cosec} 2a\theta d\theta = \frac{2}{2a} \int \operatorname{cosec} u du$$

$$= \frac{1}{a} \ell \eta |\operatorname{cosec} u - \operatorname{cotan} u| + c = \frac{1}{a} \ell \eta |\operatorname{cosec} 2a\theta - \operatorname{cotan} 2a\theta| + c$$

2.145. - $\int \frac{e^s}{\sqrt{e^{2s} - 2}} ds,$ **Sea:** $u = e^s, du = e^s ds$

$$\int \frac{e^s}{\sqrt{e^{2s} - 2}} ds = \int \frac{e^s}{\sqrt{(e^s)^2 - 2}} ds = - \int \frac{du}{\sqrt{u^2 - 2}} = \ell \eta |u + \sqrt{u^2 - 2}| + c$$

$$= \ell \eta |e^s + \sqrt{(e^s)^2 - 2}| + c = \ell \eta |e^s + \sqrt{e^{2s} - 2}| + c$$

$$\mathbf{2.146.-} \int \sin\left(\frac{2\pi t}{T} + \varphi_0\right) dt, \quad \text{Sea: } u = \frac{2\pi t}{T} + \varphi_0, du = \frac{2\pi}{T} dt$$

$$\int \sin\left(\frac{2\pi t}{T} + \varphi_0\right) dt = \frac{T}{2\pi} \int \sin u du = -\frac{T}{2\pi} \cos u + c = -\frac{T}{2\pi} \cos\left(\frac{2\pi t}{T} + \varphi_0\right) + c$$

$$\mathbf{2.147.-} \int \frac{\arccos \frac{x}{2}}{\sqrt{4-x^2}} dx, \quad \text{Sea: } u = \arccos \frac{x}{2}, du = -\frac{dx}{\sqrt{4-x^2}}$$

$$\int \frac{\arccos \frac{x}{2}}{\sqrt{4-x^2}} dx = - \int u du = -\frac{u^2}{2} + c = -\frac{(\arccos \frac{x}{2})^2}{2} + c$$

$$\mathbf{2.148.-} \int \frac{dx}{x(4-\ell\eta^2 x)}, \quad \text{Sea: } u = \ell\eta x, du = \frac{dx}{x}$$

$$\int \frac{dx}{x(4-\ell\eta^2 x)} = \int \frac{dx}{x[2^2 - (\ell\eta x)^2]} = \int \frac{du}{2^2 - u^2} = \frac{1}{4} \ell\eta \left| \frac{2+u}{2-u} \right| + c = \frac{1}{4} \ell\eta \left| \frac{2+\ell\eta x}{2-\ell\eta x} \right| + c$$

$$\mathbf{2.149.-} \int e^{-\tau gx} \sec^2 x dx, \quad \text{Sea: } u = -\tau gx, du = -\sec^2 x dx$$

$$\int e^{-\tau gx} \sec^2 x dx = - \int e^u du = -e^u + c = -e^{-\tau gx} + c$$

$$\mathbf{2.150.-} \int \frac{\sin x \cos x}{\sqrt{2-\sin^4 x}} dx, \quad \text{Sea: } u = \sin^2 x, du = 2 \sin x \cos x dx$$

$$\begin{aligned} \int \frac{\sin x \cos x}{\sqrt{2-\sin^4 x}} dx &= \int \frac{\sin x \cos x}{\sqrt{2-(\sin^2 x)^2}} dx = \frac{1}{2} \int \frac{du}{\sqrt{2-u^2}} = \frac{1}{2} \arcsin \frac{u}{\sqrt{2}} + c \\ &= \frac{1}{2} \arcsin \frac{(\sin^2 x)}{\sqrt{2}} + c \end{aligned}$$

$$\mathbf{2.151.-} \int \frac{\sec x \tau g x}{\sqrt{\sec^2 x + 1}} dx, \quad \text{Sea: } u = \sec x, du = \sec x \tau g x dx$$

$$\int \frac{\sec x \tau g x}{\sqrt{\sec^2 x + 1}} dx = \int \frac{du}{\sqrt{u^2 + 1}} = \ell\eta \left| u + \sqrt{u^2 + 1} \right| + c = \ell\eta \left| \sec x + \sqrt{\sec^2 x + 1} \right| + c$$

$$\mathbf{2.152.-} \int \frac{dt}{\sin^2 t \cos^2 t}, \quad \text{Sea: } u = 2t, du = 2dt$$

$$\begin{aligned} \int \frac{dt}{\sin^2 t \cos^2 t} &= \int \frac{dt}{(\sin t \cos t)^2} = \int \frac{dt}{(\frac{1}{2} \sin 2t)^2} = 4 \int \frac{dt}{\sin^2 2t} = 4 \int \cos ec^2 2t dt \\ &= 2 \int \cos ec^2 u du = -2 \cot g u + c = -2 \cot g 2t + c \end{aligned}$$

$$\mathbf{2.153.-} \int \frac{\arcsen x + x}{\sqrt{1-x^2}} dx,$$

$$\text{Sea: } u = \arcsen x, du = \frac{dx}{\sqrt{1-x^2}}; w = 1-x^2, dw = -2x dx$$

$$\int \frac{\arcsen x + x}{\sqrt{1-x^2}} dx = \int \frac{\arcsen x}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx = \int u du - \frac{1}{2} \int \frac{dw}{\sqrt{w}} = \int u du - \frac{1}{2} \int w^{-\frac{1}{2}} dw$$

$$= \frac{u^2}{2} - \frac{1}{2} \frac{w^{\frac{1}{2}}}{\sqrt{2}} + c = \frac{(\arcsen x)^2}{2} - \sqrt{1-x^2} + c$$

2.154. - $\int \frac{xdx}{\sqrt{x+1}}, \quad \text{Sea: } t = \sqrt{x+1} \Rightarrow x = t^2 - 1; dx = 2tdt$

$$\int \frac{xdx}{\sqrt{x+1}} = \int \frac{(t^2-1)2tdt}{t} = 2 \int (t^2-1)dt = 2\left(\frac{t^3}{3}-t\right) + c = \frac{2\sqrt{(x+1)^3}}{3} - 2\sqrt{x+1} + c$$

2.155. - $\int x(5x^2-3)^7 dx, \quad \text{Sea: } u = 5x^2 - 3, du = 10xdx$

$$\int x(5x^2-3)^7 dx = \frac{1}{10} \int u^7 du = \frac{1}{10} \frac{u^8}{8} + c = \frac{u^8}{80} + c = \frac{(5x^2-3)^8}{80} + c$$

2.156. - $\int \sqrt{\frac{\ell\eta(x+\sqrt{x^2+1})}{x^2+1}} dx, \quad \text{Sea: } u = \ell\eta(x+\sqrt{x^2+1}), du = \frac{dx}{\sqrt{x^2+1}}$

$$\int \sqrt{\frac{\ell\eta(x+\sqrt{x^2+1})}{x^2+1}} dx = \int \frac{\sqrt{\ell\eta(x+\sqrt{x^2+1})}}{\sqrt{x^2+1}} dx = \int \sqrt{u} du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2\sqrt{\left[\ell\eta(x+\sqrt{x^2+1})\right]^3}}{3} + c$$

2.157. - $\int \frac{\sen^3 x}{\cos x} dx, \quad \text{Sea: } u = \cos x, du = -\sen x dx$

$$\int \frac{\sen^3 x}{\cos x} dx = \int \frac{\sen^2 x \sen x dx}{\cos x} = \int \frac{(1-\cos^2 x)\sen x dx}{\cos x} = \int \frac{\sen x dx}{\cos x} - \int \frac{\cos^2 x \sen x dx}{\cos x}$$

$$= \int \cos^{-\frac{1}{2}} x \sen x dx - \int \cos^{\frac{1}{2}} x \sen x dx = - \int u^{\frac{1}{2}} du + \int u^{\frac{3}{2}} du = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= -\frac{2u^{\frac{3}{2}}}{3} + \frac{2u^{\frac{5}{2}}}{5} + c = -\frac{2\cos x^{\frac{3}{2}}}{3} + \frac{2\cos x^{\frac{5}{2}}}{5} + c = -\frac{2\sqrt{\cos^3 x}}{3} + \frac{2\sqrt{\cos^5 x}}{5} + c$$

2.158. - $\int \frac{\cos x dx}{\sqrt{1+\sen^2 x}},$

Sea: $t = \sqrt{1+\sen^2 x} \Rightarrow \sen^2 x = t^2 - 1; 2\sen x \cos x dx = 2tdt$

$$\int \frac{\cos x dx}{\sqrt{1+\sen^2 x}} = \int \frac{\sqrt{t^2-1}}{t} dt = \int \frac{dt}{\sqrt{t^2-1}} = \ell\eta \left| \sqrt{1+\sen^2 x} + \sen x \right| + c$$

2.159. - $\int \frac{(\arcsen x)^2}{\sqrt{1-x^2}} dx, \quad \text{Sea: } u = \arcsen x, du = \frac{dx}{\sqrt{1-x^2}}$

$$\int \frac{(\arcsen x)^2}{\sqrt{1-x^2}} dx = \int u^2 du = \frac{u^3}{3} + c = \frac{(\arcsen x)^3}{3} + c$$

2.160. - $\int e^{x+e^x} dx, \quad \text{Sea: } u = e^{e^x}, du = e^{e^x} e^x dx$

$$\int e^{x+e^x} dx = \int e^x e^{e^x} dx = \int du = u + c = e^{e^x} + c$$

2.161. - $\int t(4t+1)^7 dt$, **Sea:** $u = 4t+1 \Rightarrow t = \frac{u-1}{4}, du = 4dt$

$$\begin{aligned}\int t(4t+1)^7 dt &= \int \frac{u-1}{4} u^7 \frac{du}{4} = \frac{1}{16} \int (u-1) u^7 du = \frac{1}{16} \int (u^8 - u^7) du = \frac{1}{16} \frac{u^9}{9} - \frac{1}{16} \frac{u^8}{8} + c \\ &= \frac{(4t+1)^9}{144} - \frac{(4t+1)^8}{128} + c\end{aligned}$$

2.162. - $\int \frac{2t^2 - 10t + 12}{t^2 + 4} dt$, **Sea:** $u = t^2 + 4, du = 2tdt$

$$\begin{aligned}\int \frac{2t^2 - 10t + 12}{t^2 + 4} dt &= 2 \int \frac{t^2 - 5t + 6}{t^2 + 4} dt = 2 \int \left(1 + \frac{2 - 5t}{t^2 + 4}\right) dt = 2 \int dt + 4 \int \frac{dt}{t^2 + 4} - 10 \int \frac{dt}{t^2 + 4} \\ &= 2 \int dt + 4 \int \frac{dt}{t^2 + 4} - 5 \int \frac{du}{u} = 2t + 2 \arctan \frac{t}{2} - 5 \ell \eta |u| + c = 2t + 2 \arctan \frac{t}{2} - 5 \ell \eta |t^2 + 4| + c\end{aligned}$$

2.163. - $\int \frac{e^t - e^{-t}}{e^t + e^{-t}} dt$,

Sea: $u = e^{2t} + 1, du = 2e^{2t} dt; w = 1 + e^{-2t}, dw = -2e^{-2t} dt$

$$\begin{aligned}\int \frac{e^t - e^{-t}}{e^t + e^{-t}} dt &= \int \frac{e^t dt}{e^t + e^{-t}} - \int \frac{e^{-t} dt}{e^t + e^{-t}} = \int \frac{e^{2t} dt}{e^{2t} + 1} - \int \frac{e^{-2t} dt}{1 + e^{-2t}} = \frac{1}{2} \int \frac{du}{u} + \frac{1}{2} \int \frac{dw}{w} \\ &= \frac{1}{2} (\ell \eta |u| + \ell \eta |w|) + c = \frac{1}{2} \ell \eta |uw| + c = \frac{1}{2} \ell \eta (e^{2t} + 1)(1 + e^{-2t}) + c\end{aligned}$$