INTEGRALES ELEMENTALES

El Propósito de este capitulo, antes de conocer y practicar las técnicas propiamente tales; es familiarizarse con aquellas integrales para las cuales basta una transformación algebraica elemental.

EJERCICIOS DESARROLLADOS

1.1.- Encontrar: $\int e^{\ell \eta x^2} x dx$

Solución.- Se sabe que: $e^{\ell \eta x^2} = x^2$

Por lo tanto: $\int e^{\ell \eta x^2} x dx = \int x^2 x dx = \int x^3 dx = \frac{x^4}{4} + c$

Respuesta: $\int e^{\ell \eta x^2} x dx = \frac{x^4}{4} + c$, Fórmula utilizada: $\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$

1.2 .- Encontrar: $\int 3a^7 x^6 dx$

Solución.-

$$\int 3a^7 x^6 dx = 3a^7 \int x^6 dx = 3a^7 \frac{x^7}{7} + c$$

Respuesta: $\int 3a^7 x^6 dx = 3a^7 \frac{x^7}{7} + c$, Fórmula utilizada: del ejercicio anterior.

1.3.- Encontrar: $\int (3x^2 + 2x + 1) dx$

Solución.-

$$\int (3x^2 + 2x + 1) dx = \int (3x^2 + 2x + 1) dx = \int 3x^2 dx + \int 2x dx + \int dx$$

$$= 3\int x^2 dx + 2\int x dx + \int dx = \cancel{3} \frac{x^3}{\cancel{3}} + \cancel{2} \frac{x^2}{\cancel{2}} + x + c = x^3 + x^2 + x + c$$

Respuesta: $\int (3x^2 + 2x + 1) dx = x^3 + x^2 + x + c$

1.4.- Encontrar: $\int x(x+a)(x+b) dx$

Solución.-

$$\int x(x+a)(x+b) dx = \int x\left[x^2 + (a+b)x + ab\right] dx = \int \left[x^3 + \left(a+b\right)x^2 + abx\right] dx$$

$$= \int x^3 dx + \int (a+b)x^2 dx + \int abx dx = \int x^3 dx + (a+b)\int x^2 dx + ab\int x dx$$

$$= \frac{x^4}{4} + (a+b)\frac{x^3}{3} + ab\frac{x^2}{2} + c$$

Respuesta: $\int x(x+a)(x+b) dx = \frac{x^4}{4} + \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c$

1.5.- Encontrar: $\int (a + bx^3)^2 dx$

Solución.-

$$\int (a + bx^3)^2 dx = \int (a^2 + 2abx^3 + b^2 x^6) dx = \int a^2 dx + \int 2abx^3 dx + \int b^2 x^6 dx$$

$$= a^2 \int dx + 2ab\int x^3 dx + b^2 \int x^6 dx = a^2 x + 2ab\frac{x^4}{4} + b^2 \frac{x^7}{7} + c$$

Respuesta: $\int (a + bx^3)^2 dx = a^2 x + \frac{abx^4}{2} + \frac{b^2 x^7}{7} + c$

1.6.- Encontrar: $\int \sqrt{2px} dx$

Solución.-

$$\int \sqrt{2px}dx = \int \sqrt{2p}x^{\frac{1}{2}}dx = \sqrt{2p}\int x^{1/2}dx = \sqrt{2p}\,\frac{x^{3/2}}{3/2} + c = \frac{2\sqrt{2p}x^{3/2}}{3} + c$$

Respuesta: $\int \sqrt{2px}dx = \dfrac{2\sqrt{2p}x\sqrt{x}}{3} + c$

1.7.-Encontrar: $\int \dfrac{dx}{\sqrt[n]{x}}$

Solución.-

$$\int \frac{dx}{\sqrt[n]{x}} = \int x^{-1/n}dx = \frac{x^{\frac{-1}{n}+1}}{\frac{-1}{n}+1} + c = \frac{x^{\frac{-1+n}{n}}}{\frac{-1+n}{n}} + c = \frac{nx^{\frac{-1+n}{n}}}{n-1} + c$$

Respuesta: $\int \dfrac{dx}{\sqrt[n]{x}} = \dfrac{nx^{\frac{-1+n}{n}}}{n-1} + c$

1.8.- Encontrar: $\int (nx)^{\frac{1-n}{n}}dx$

Solución.-

$$\int (nx)^{\frac{1-n}{n}}dx = \int n^{\frac{1-n}{n}}x^{\frac{1-n}{n}}dx = n^{\frac{1-n}{n}}\int x^{\frac{1-n}{n}}dx = n^{\frac{1-n}{n}}\int x^{\frac{1}{n}-1}dx$$

$$= = n^{\frac{1-n}{n}}\frac{x^{\frac{1}{n}-1+1}}{\frac{1}{n}-1+1} + c = n^{\frac{1-n}{n}}\frac{x^{\frac{1}{n}}}{\frac{1}{n}} + c = n^{\frac{1-n}{n}}nx^{\frac{1}{n}} + c = n^{\frac{1-n}{n}+1}x^{\frac{1}{n}} + c = n^{\frac{1-n+n}{n}}x^{\frac{1}{n}} + c = n^{1/n}x^{1/n} + c$$

Respuesta: $\int (nx)^{\frac{1-n}{n}}dx = \sqrt[n]{nx} + c$

1.9.- Encontrar: $\int (a^{2/3} - x^{2/3})^3dx$

Solución.-

$$\int (a^{2/3} - x^{2/3})^3dx = \int\left[\left(a^{2/3}\right)^3 - 3\left(a^{2/2}\right)^2 x^{2/3} + 3a^{2/3}\left(x^{2/3}\right)^2 - \left(x^{2/3}\right)^3\right]dx$$

$$= \int (a^2 - 3a^{4/3}x^{2/3} + 3a^{2/3}x^{4/3} - x^2)dx = \int a^2dx - \int 3a^{4/3}x^{2/3}dx + \int 3a^{2/3}x^{4/3}dx - \int x^2dx$$

$$= a^2\int dx - 3a^{4/3}\int x^{2/3}dx + 3a^{2/3}\int x^{4/3}dx - \int x^2dx = a^2x - 3a^{4/3}\frac{x^{5/3}}{5/3} + 3a^{2/3}\frac{x^{7/3}}{7/3} - \frac{x^3}{3} + c$$

$$= a^2x - \frac{9a^{4/3}x^{5/3}}{5} + \frac{9a^{2/3}x^{7/3}}{7} - \frac{x^3}{3} + c$$

Respuesta: $\int (a^{2/3} - x^{2/3})^3dx = a^2x - \dfrac{9a^{4/3}x^{5/3}}{5} + \dfrac{9a^{2/3}x^{7/3}}{7} - \dfrac{x^3}{3} + c$

1.10.- Encontrar: $\int (\sqrt{x}+1)(x-\sqrt{x}+1)dx$

Solución.-

$$\int (\sqrt{x}+1)(x-\sqrt{x}+1)dx = (x\sqrt{x} - \cancel{(\sqrt{x})^2} + \cancel{\sqrt{x}} + \cancel{x} - \cancel{\sqrt{x}} + 1)dx$$

$$= \int (x\sqrt{x}+1)dx = \int (xx^{1/2}+1)dx = \int (x^{3/2}+1)dx = \int x^{3/2}dx + \int dx = \frac{x^{5/2}}{5/2} + x + c = \frac{2x^{5/2}}{5} + x + c$$

Respuesta: $\int (\sqrt{x}+1)(x-\sqrt{x}+1)dx = \dfrac{2x^{5/2}}{5} + x + c$

1.11.- Encontrar: $\int \dfrac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}}$

Solución.-

$$\int\frac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}}=\int\frac{(x^4-x^2-2)dx}{x^{2/3}}=\int\frac{x^4}{x^{2/3}}dx-\int\frac{x^2}{x^{2/3}}dx-\int\frac{2}{x^{2/3}}dx$$

$$=\int x^{10/3}dx-\int x^{4/3}dx-2\int x^{-2/3}dx=\frac{x^{\frac{10}{3}+1}}{\frac{10}{3}+1}-\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}-2\frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1}=\frac{x^{13/3}}{\frac{13}{3}}-\frac{x^{7/3}}{\frac{7}{3}}-2\frac{x^{1/3}}{1/3}+c$$

$$=3\frac{x^{13/3}}{13}-3\frac{x^{7/3}}{7}-6x^{1/3}+c=3\frac{\sqrt[3]{x^{13}}}{13}-3\frac{\sqrt[3]{x^7}}{7}-6\sqrt[3]{x}+c=3\frac{x^4\sqrt[3]{x}}{13}-3\frac{x^2\sqrt[3]{x}}{7}-6\sqrt[3]{x}+c$$

Respuesta: $\int\frac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}}=\left(\frac{3x^4}{13}-\frac{3x^2}{7}-6\right)\sqrt[3]{x}+c$

1.12.- Encontrar: $\int\frac{(x^m-x^n)^2}{\sqrt{x}}dx$

Solución.-

$$\int\frac{(x^m-x^n)^2}{\sqrt{x}}dx=\int\frac{(x^{2m}-2x^mx^n+x^{2n})}{\sqrt{x}}dx=\int\frac{(x^{2m}-2x^mx^n+x^{2n})}{x^{1/2}}dx$$

$$=\int(x^{2m-1/2}-2x^{m+n-1/2}+x^{2n-1/2})dx=\frac{x^{2m-1/2+1}}{2m-1/2+1}-\frac{2x^{m+n+1/2}}{m+n+1/2}+\frac{x^{2n+1/2}}{2n+1/2}+c$$

$$=\frac{x^{\frac{4m+1}{2}}}{\frac{4m+1}{2}}-\frac{2x^{\frac{2m+2n+1}{2}}}{\frac{2m+2n+1}{2}}+\frac{x^{\frac{4n+1}{2}}}{\frac{4n+1}{2}}+c=\frac{2x^{\frac{4m+1}{2}}}{4m+1}-\frac{4x^{\frac{2m+2n+1}{2}}}{2m+2n+1}+\frac{2x^{\frac{4n+1}{2}}}{4n+1}+c$$

$$=\frac{2x^{2m}\sqrt{x}}{4m+1}-\frac{4x^{m+n}\sqrt{x}}{2m+2n+1}+\frac{2x^{2n}\sqrt{x}}{4n+1}+c$$

Respuesta: $\int\frac{(x^m-x^n)^2}{\sqrt{x}}dx=\sqrt{x}\left(\frac{2x^{2m}}{4m+1}-\frac{4x^{m+n}}{2m+2n+1}+\frac{2x^{2n}}{4n+1}\right)+c$

1.13.- Encontrar: $\int\frac{(\sqrt{a}-\sqrt{x})^4}{\sqrt{ax}}dx$

Solución.-

$$\int\frac{(\sqrt{a}-\sqrt{x})^4}{\sqrt{ax}}dx=\int\frac{a^2-4a\sqrt{ax}+6xa-4x\sqrt{ax}+x^2}{\sqrt{ax}}dx$$

$$=\int\frac{a^2}{(ax)^{1/2}}dx-\int\frac{4a\cancel{\sqrt{ax}}}{\cancel{\sqrt{ax}}}dx+\int\frac{6ax}{(ax)^{1/2}}dx-\int\frac{4x\cancel{\sqrt{ax}}}{\cancel{\sqrt{ax}}}dx+\int\frac{x^2}{(ax)^{1/2}}dx$$

$$=\int a^2a^{-1/2}x^{-1/2}dx-\int 4adx+\int 6aa^{-1/2}xx^{-1/2}dx-\int 4xdx+\int a^{-1/2}x^2x^{-1/2}dx$$

$$=a^{3/2}\int x^{-1/2}dx-4a\int dx+6a^{1/2}\int x^{1/2}dx-4\int xdx+a^{-1/2}\int x^{3/2}dx$$

$$=a^{3/2}\frac{x^{-1/2+1}}{\frac{-1}{2}+1}-4ax+6a^{1/2}\frac{x^{1/2+1}}{\frac{1}{2}+1}-4\frac{x^{1+1}}{1+1}+a^{-1/2}\frac{x^{3/2+1}}{\frac{3}{2}+1}+c$$

$$=a^{3/2}\frac{x^{1/2}}{\frac{1}{2}}-4ax+6a^{1/2}\frac{x^{3/2}}{\frac{3}{2}}-4\frac{x^2}{2}+a^{-1/2}\frac{x^{5/2}}{\frac{5}{2}}+c$$

$$=2a^{3/2}x^{1/2}-4ax+4a^{1/2}x^{3/2}-2x^2+2a^{-1/2}\frac{x^{5/2}}{5}+c$$

Respuesta: $\int \frac{(\sqrt{a}-\sqrt{x})^4}{\sqrt{ax}}dx = 2a^{3/2}x^{1/2} - 4ax + 4a^{1/2}x^{3/2} - 2x^2 + \frac{2x^3}{5\sqrt{xa}} + c$

1.14.- Encontrar: $\int \frac{dx}{x^2 - 10}$

Solución.-

Sea: $a = \sqrt{10}$, Luego: $\int \frac{dx}{x^2-10} = \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ell\eta\left|\frac{x-a}{x+a}\right| + c$

$= \frac{1}{2\sqrt{10}}\ell\eta\left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right| + c = \frac{\sqrt{10}}{20}\ell\eta\left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right| + c$

Respuesta: $\int \frac{dx}{x^2-10} = \frac{\sqrt{10}}{20}\ell\eta\left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right| + c$

1.15.- Encontrar: $\int \frac{dx}{x^2+7}$

Solución.- Sea: a=$\sqrt{7}$, Luego: $\int \frac{dx}{x^2+7} = \int \frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{arc}\tau g\frac{x}{a} + c$

$\frac{1}{\sqrt{7}}\operatorname{arc}\tau g\frac{x}{\sqrt{7}} + c = \frac{\sqrt{7}}{7}\operatorname{arc}\tau g\frac{\sqrt{7}x}{a} + c$

Respuesta: $\int \frac{dx}{x^2+7} = \frac{\sqrt{7}}{7}\operatorname{arc}\tau g\frac{\sqrt{7}x}{a} + c$

1.16.- Encontrar: $\int \sqrt{\frac{dx}{4+x^2}}$

Solución.-

Sea: $a = 2$, Luego: $\int \frac{dx}{\sqrt{4+x^2}} = \int \frac{dx}{\sqrt{a^2+x^2}} = \ell\eta\left|x+\sqrt{a^2+x^2}\right| + c$

$= \ell\eta\left|x+\sqrt{4+x^2}\right| + c$

Respuesta: $\int \frac{dx}{\sqrt{4+x^2}} = \ell\eta\left|x+\sqrt{4+x^2}\right| + c$

1.17.- Encontrar: $\int \frac{dx}{\sqrt{8-x^2}}$

Solución.-

Sea: $a = \sqrt{8}$, Luego: $\int \frac{dx}{\sqrt{8-x^2}} = \int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{arcs}e\operatorname{n}\frac{x}{a} + c$

$= \operatorname{arcs}e\operatorname{n}\frac{x}{\sqrt{8}} + c = \operatorname{arcs}e\operatorname{n}\frac{x}{2\sqrt{2}} + c$

Respuesta: $\int \frac{dx}{\sqrt{8-x^2}} = \operatorname{arcs}e\operatorname{n}\frac{\sqrt{2}x}{4} + c$

1.18.- Encontrar: $\int \frac{dy}{x^2+9}$

Solución.-

La expresión: $\frac{1}{x^2+9}$ actúa como constante, luego:

$\int \frac{dy}{x^2+9} = \frac{1}{x^2+9}\int dy = \frac{1}{x^2+9}y + c = \frac{y}{x^2+9} + c$

Respuesta: $\int \frac{dy}{x^2+9} = \frac{y}{x^2+9} + c$

1.19.- Encontrar: $\int \frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}dx$

Solución.-

$$\int \frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}dx = \int \sqrt{\frac{2+x^2}{4-x^4}}dx - \int \sqrt{\frac{2-x^2}{4-x^4}}dx$$

$$= \int \sqrt{\frac{\cancel{2+x^2}}{(2-x^2)\cancel{(2+x^2)}}}dx - \int \sqrt{\frac{\cancel{2-x^2}}{\cancel{(2-x^2)}(2+x^2)}}dx = \int \frac{dx}{\sqrt{2-x^2}} - \int \frac{dx}{\sqrt{2+x^2}}$$

Sea: $a=\sqrt{2}$, Luego: $\int \frac{dx}{\sqrt{a^2-x^2}} - \int \frac{dx}{\sqrt{a^2+x^2}} = \operatorname{arcs}e\mathrm{n}\frac{x}{a} - \ell\eta\left|x+\sqrt{a^2+x^2}\right| + c$

$$= \operatorname{arcs}e\mathrm{n}\frac{x}{\sqrt{2}} - \ell\eta\left|x+\sqrt{(\sqrt{2})^2+x^2}\right| + c = \operatorname{arcs}e\mathrm{n}\frac{x}{\sqrt{2}} - \ell\eta\left|x+\sqrt{2+x^2}\right| + c$$

Respuesta: $\int \frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}dx = \operatorname{arcs}e\mathrm{n}\frac{x}{\sqrt{2}} - \ell\eta\left|x+\sqrt{2+x^2}\right| + c$

1.20.- Encontrar: $\int \tau g^2 x dx$

Solución.-

$$\int \tau g^2 x dx = \int (\sec^2 x - 1)dx = \int \sec^2 x dx - \int dx = \tau g x - x + c$$

Respuesta: $\int \tau g^2 x dx = \tau g x - x + c$

1.21.- Encontrar: $\int \mathrm{co}\,\tau g^2 x dx$

Solución.-

$$\int \mathrm{co}\,\tau g^2 x dx = \int (\cos ec^2 x - 1)dx = \int \cos ec^2 x dx - \int dx = -\mathrm{co}\,\tau g x - x + c$$

Respuesta: $\int \mathrm{co}\,\tau g^2 x dx = -\mathrm{co}\,\tau g x - x + c$

1.22.- Encontrar: $\int \frac{dx}{2x^2+4}$

Solución.-

$$\int \frac{dx}{2x^2+4} = \int \frac{dx}{2(x^2+2)} = \frac{1}{2}\int \frac{dx}{x^2+2} = \frac{1}{2}\frac{1}{\sqrt{2}}\operatorname{arc}\tau g\frac{x}{\sqrt{2}} + c \quad = \frac{\sqrt{2}}{4}\operatorname{arc}\tau g\frac{\sqrt{2}x}{2} + c$$

Respuesta: $\int \frac{dx}{2x^2+4} = \frac{\sqrt{2}}{4}\operatorname{arc}\tau g\frac{\sqrt{2}x}{2} + c$

1.23.- Encontrar: $\int \frac{dx}{7x^2-8}$

Solución.-

$$\int \frac{dx}{7x^2-8} = \int \frac{dx}{7(x^2-\frac{8}{7})} = \int \frac{dx}{7\left[(x^2-(\sqrt{\frac{8}{7}})^2\right]} = \frac{1}{7}\int \frac{dx}{\left[x^2-(\sqrt{\frac{8}{7}})^2\right]}$$

$$= \frac{1}{7}\frac{1}{2(\sqrt{\frac{8}{7}})}\ell\eta\left|\frac{x-\sqrt{\frac{8}{7}}}{x+\sqrt{\frac{8}{7}}}\right| + c = \frac{1}{14\frac{\sqrt{8}}{\sqrt{7}}}\ell\eta\left|\frac{x-\sqrt{\frac{8}{7}}}{x+\sqrt{\frac{8}{7}}}\right| + c = \frac{\sqrt{7}}{14\sqrt{8}}\ell\eta\left|\frac{\sqrt{7}x-\sqrt{8}}{\sqrt{7}x+\sqrt{8}}\right| + c$$

$$= \frac{1}{4\sqrt{14}} \ell\eta \left| \frac{\sqrt{7}x - 2\sqrt{2}}{\sqrt{7}x + 2\sqrt{2}} \right| + c = \frac{\sqrt{14}}{56} \ell\eta \left| \frac{\sqrt{7}x - 2\sqrt{2}}{\sqrt{7}x + 2\sqrt{2}} \right| + c$$

Respuesta: $\int \frac{dx}{7x^2 - 8} = \frac{\sqrt{14}}{56} \ell\eta \left| \frac{\sqrt{7}x - 2\sqrt{2}}{\sqrt{7}x + 2\sqrt{2}} \right| + c$

1.24.- Encontrar: $\int \frac{x^2 dx}{x^2 + 3}$

Solución.-

$$\int \frac{x^2 dx}{x^2+3} = \int (1 - \frac{3}{x^2+3}) dx = \int dx - 3\int \frac{dx}{x^2+3} = \int dx - 3\int \frac{dx}{x^2 + (\sqrt{3})^2}$$

$$= x - 3\frac{1}{\sqrt{3}} \operatorname{arc}\tau g \frac{x}{\sqrt{3}} + c == x - \sqrt{3} \operatorname{arc}\tau g \frac{\sqrt{3}x}{3} + c$$

Respuesta: $\int \frac{x^2 dx}{x^2+3} = x - \sqrt{3} \operatorname{arc}\tau g \frac{\sqrt{3}x}{3} + c$

1.25.- Encontrar: $\int \frac{dx}{\sqrt{7 + 8x^2}}$

Solución.-

$$\int \frac{dx}{\sqrt{7+8x^2}} = \int \frac{dx}{\sqrt{(\sqrt{8}x)^2 + (\sqrt{7})^2}} = \frac{1}{\sqrt{8}} \ell\eta \left| \sqrt{8}x + \sqrt{7+8x^2} \right| + c$$

Respuesta: $\int \frac{dx}{\sqrt{7+8x^2}} = \frac{\sqrt{2}}{4} \ell\eta \left| \sqrt{8}x + \sqrt{7+8x^2} \right| + c$

1.26.- Encontrar: $\int \frac{dx}{\sqrt{7 - 5x^2}}$

Solución.-

$$\int \frac{dx}{\sqrt{7-5x^2}} = \int \frac{dx}{\sqrt{(\sqrt{7})^2 - (\sqrt{5}x)^2}} = \frac{1}{\sqrt{5}} \operatorname{arcs} e \operatorname{n} x \frac{\sqrt{5}}{\sqrt{7}} + c$$

Respuesta: $\int \frac{dx}{\sqrt{7-5x^2}} = \frac{\sqrt{5}}{5} \operatorname{arcs} e \operatorname{n} \frac{\sqrt{35}x}{7} + c$

1.27.- Encontrar: $\int \frac{(a^x - b^x)^2 dx}{a^x b^x}$

Solución.-

$$\int \frac{(a^x - b^x)^2 dx}{a^x b^x} = \int \frac{(a^{2x} - 2a^x b^x + b^{2x})}{a^x b^x} dx = \int \frac{a^{2x}}{a^x b^x} dx - \int \frac{2 a^x b^x}{a^x b^x} dx + \int \frac{b^{2x}}{a^x b^x} dx$$

$$= \int \frac{a^x}{b^x} dx - \int 2dx + \int \frac{b^x}{a^x} dx = \int \left(\frac{a}{b}\right)^x dx - 2\int dx + \int \left(\frac{b}{a}\right)^x dx = \frac{(a/b)^x}{\ell\eta \frac{a}{b}} - 2x + \frac{(b/a)^x}{\ell\eta \frac{b}{a}} + c$$

$$= \frac{(a/b)^x}{\ell\eta a - \ell\eta b} - 2x + \frac{(b/a)^x}{\ell\eta b - \ell\eta a} + c = \frac{(a/b)^x}{\ell\eta a - \ell\eta b} - 2x - \frac{(b/a)^x}{\ell\eta a - \ell\eta b} + c$$

$$= \frac{\left(\frac{a^x}{b^x} - \frac{b^x}{a^x}\right)}{\ell\eta a - \ell\eta b} - 2x + c$$

Respuesta: $\int \frac{(a^x - b^x)^2 dx}{a^x b^x} = \frac{\left(\frac{a^{2x} - b^{2x}}{a^x b^x}\right)}{\ell\eta a - \ell\eta b} - 2x + c$

1.28.- Encontrar: $\int \operatorname{sen}^2 \frac{x}{2} dx$

Solución.-

$$\int \operatorname{sen}^2 \frac{x}{2} dx = \int \frac{1-\cos 2\frac{x}{2}}{2} dx = \int \frac{1-\cos x}{2} dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos x dx$$

$$= \frac{x}{2} - \frac{senx}{2} + c$$

Respuesta: $\int \operatorname{sen}^2 \frac{x}{2} dx = \frac{x}{2} - \frac{senx}{2} + c$

1.29.- Encontrar: $\int \frac{dx}{(a+b)+(a-b)x^2}; (0<b<a)$

Solución.-

Sea: $c^2 = a+b,\ \ d^2 = a-b,$; luego $\int \frac{dx}{(a+b)+(a-b)x^2} = \int \frac{dx}{c^2+d^2x^2}$

$$\int \frac{dx}{d^2\left(\frac{c^2}{d^2}+x^2\right)} = \frac{1}{d^2}\int \frac{dx}{\left(\frac{c}{d}\right)^2+x^2} = \frac{1}{d^2}\frac{1}{\frac{c}{d}} arctg \frac{x}{c/d} + c = \frac{1}{cd} arctg \frac{dx}{c} + c$$

$$= \frac{1}{\sqrt{a+b}\sqrt{a-b}} arctg \frac{\sqrt{a-b}x}{\sqrt{a+b}} + c = \frac{1}{\sqrt{a^2-b^2}} arctg \sqrt{\frac{a-b}{a+b}}x + c$$

Respuesta: $\int \frac{dx}{(a+b)+(a-b)x^2} = \frac{1}{\sqrt{a^2-b^2}} arctg \sqrt{\frac{a-b}{a+b}}x + c$

1.30.-Encontrar: $\int \frac{dx}{(a+b)-(a-b)x^2}; (0<b<a)$

Solución.-

Sea: $c^2 = a+b,\ d^2 = a-b,$ Luego: $\int \frac{dx}{(a+b)-(a-b)x^2} = \int \frac{dx}{c^2-d^2x^2}$

$$= \int \frac{dx}{d^2\left(\frac{c^2}{d^2}-x^2\right)} = \frac{1}{d^2}\int \frac{dx}{\left(\frac{c}{d}\right)^2-x^2} = -\frac{1}{d^2}\frac{1}{\frac{2c}{d}} \ell\eta \left|\frac{x-c/d}{x+c/d}\right| + c = -\frac{1}{2cd}\ell\eta\left|\frac{dx-c}{dx+c}\right| + c$$

$$= -\frac{1}{2\sqrt{a^2-b^2}} \ell\eta \left|\frac{\sqrt{a-b}x-\sqrt{a+b}}{\sqrt{a-b}x+\sqrt{a+b}}\right| + c$$

Respuesta: $\int \frac{dx}{(a+b)-(a-b)x^2} = -\frac{1}{2\sqrt{a^2-b^2}} \ell\eta \left|\frac{\sqrt{a-b}x-\sqrt{a+b}}{\sqrt{a-b}x+\sqrt{a+b}}\right| + c$

1.31.- Encontrar: $\int \left[\left(a^{2x}\right)^0 - 1\right] dx$

Solución.-

$$\int \left[\left(a^{2x}\right)^0 - 1\right] dx = \int (a^0-1)dx = \int (1-1)dx = \int dx - \int dx = \int 0dx = c$$

Respuesta: $\int \left[\left(a^{2x}\right)^0 - 1\right] dx = c$

EJERCICIOS PROPUESTOS

Mediante el uso del álgebra elemental, o algunas identidades trigonométricas, transformar en integrales de fácil solución, las integrales que se presentan a continuación.

1.32.- $\int 3x^5 dx$

1.33.- $\int (1+e)^x dx$

1.34.- $\int (1+\tau gx)dx$

1.35.- $\int \cos^2 \frac{x}{2} dx$

1.36.- $\int (1+\sqrt{x})^3 dx$

1.37.- $\int (1+\sqrt{x})^0 dx$

1.38.- $\int \frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}} dy$

1.39.- $\int \frac{dx}{\sqrt{5-x^2}}$

1.40.- $\int \frac{dx}{\sqrt{x^2-5}}$

1.41.- $\int \frac{dx}{\sqrt{x^2+5}}$

1.42.- $\int \frac{dx}{x^2+5}$

1.43.- $\int \frac{dx}{x^2-5}$

1.44.- $\int (\operatorname{sen}^2 x+\cos^2 x-1)dx$

1.45.- $\int \sqrt{x}(1-\sqrt{x})dx$

1.46.- $\int (\tau g^2 x+1)dx$

1.47.- $\int \frac{dx}{x^2-12}$

1.48.- $\int \frac{dx}{x^2+12}$

1.49.- $\int \frac{dx}{\sqrt{x^2-12}}$

1.50.- $\int \frac{dx}{\sqrt{x^2+12}}$

1.51.- $\int \frac{dx}{\sqrt{12-x^2}}$

1.52.- $\int \frac{dx}{x\sqrt{x^2-12}}$

1.53.- $\int \frac{dx}{x\sqrt{12-x^2}}$

1.54.- $\int \frac{dx}{x\sqrt{12+x^2}}$

1.55.- $\int \frac{dx}{\sqrt{8-2x^2}}$

1.56.- $\int \frac{dx}{\sqrt{2x^2-8}}$

1.57.- $\int \frac{dx}{\sqrt{2x^2+8}}$

1.58.- $\int \sqrt{x^2-10}dx$

1.59.- $\int \sqrt{x^2+10}dx$

1.60.- $\int \sqrt{10-x^2}dx$

1.61.- $\int \frac{1-\cos^2 x}{\operatorname{sen}^2 x} dx$

1.62.- $\int \sqrt{1-\operatorname{sen}^2 x}dx$

1.63.- $\int \sqrt{1-\cos^2 x}dx$

1.64.- $\int (2^x-3^x)^0 dx$

1.65.- $\int (2^0-3^0)^n dx$

1.66.- $\int \left(\tau gx-\frac{\operatorname{sen} x}{\cos x}\right)dx$

1.67.- $\int \frac{dx}{3^{-x}}$

1.68.- $\int \sqrt{\frac{3}{4}-x^2}dx$

1.69.- $\int \sqrt{x^2-\frac{3}{4}}dx$

1.70.- $\int \sqrt{x^2+\frac{3}{4}}dx$

1.71.- $\int \frac{dx}{x\sqrt{3-x^2}}$

1.72.- $\int \frac{dx}{x\sqrt{x^2-3}}$

1.73.- $\int \frac{dx}{x\sqrt{x^2+3}}$

1.74.- $\int \operatorname{sen}^{3x} \theta dy$

1.75.- $\int \ell\eta|u|dx$

1.76.- $\int \exp(\ell\eta x)dx$

1.77.- $\int e^{\ell\eta x^2} dx$

1.78.- $\int \frac{\sqrt{x}-\sqrt{2}}{\sqrt{2x}} dx$

1.79.- $\int \sqrt{11-x^2}dx$

1.80.- $\int \sqrt{x^2-11}dx$

1.81.- $\int \sqrt{x^2+11}dx$

1.82.- $\int \ell\eta(e^{\sqrt{x}})dx$

1.83.- $\int \left[\frac{1+\sqrt{x}+\sqrt{x^3}}{1-\sqrt{x}}\right]^0 dx$

1.84.- $\int (\tau g^2 x + \sec^2 x - 1)dx$

1.85.- $\int \frac{dx}{\sqrt{3x^2-1}}$

1.86.- $\int (\mathrm{co}\,\tau g\theta - \mathrm{sen}\,\theta)dx$

1.87.- $\int \frac{dx}{\sqrt{1+3x^2}}$

1.88.- $\int \frac{dx}{\sqrt{1-3x^2}}$

1.89.- $\int \frac{dx}{1+3x^2}$

1.90.- $\int \frac{dx}{3x^2+4}$

1.91.- $\int \frac{dx}{3x^2-1}$

1.92.- $\int \frac{dx}{x\sqrt{3x^2-1}}$

1.93.- $\int \frac{dx}{x\sqrt{1+3x^2}}$

1.94.- $\int \frac{dx}{x\sqrt{1-3x^2}}$

1.95.- $\int \sqrt{1-3x^2}\,dx$

1.96.- $\int \sqrt{1+3x^2}\,dx$

1.97.- $\int \sqrt{3x^2-1}\,dx$

1.98.- $\int (3x^2-1)dx$

1.99.- $\int (3x^2-1)^0 dx$

1.100.- $\int (3x^2-1)^n du$

1.101.- $\int \exp(\ell\eta \frac{\sqrt{x}}{3})dx$

1.102.- $\int \ell\eta(e^{\frac{2x-1}{2}})dx$

1.103.- $\int (e^2+e+1)^x dx$

1.104.- $\int \left(\frac{1+\tau g^2 x}{\sec^2 x} - 1\right)dx$

1.105.- $\int \exp(\ell\eta|1+x|)dx$

1.106.- $\int \sqrt{27-x^2}\,dx$

1.107.- $\int \sqrt{x^2-27}\,dx$

1.108.- $\int \sqrt{x^2+27}\,dx$

1.109.- $\int \frac{dx}{3x\sqrt{x^2-1}}$

1.110.- $\int \frac{dx}{2x\sqrt{1-x^2}}$

1.111.- $\int \frac{dx}{5x\sqrt{x^2+1}}$

1.112.- $\int \frac{dx}{3x\sqrt{9-x^2}}$

1.113.- $\int \frac{dx}{4x\sqrt{x^2+16}}$

1.114.- $\int \frac{dx}{5x\sqrt{x^2-25}}$

1.115.- $\int \frac{(1-\sqrt{x})^2}{x^2}dx$

1.116.- $\int (1+\sqrt{x}+x)^2 dx$

1.117.- $\int (1-\sqrt{x}+x)^2 dx$

1.118.- $\int (1+x)^4 dx$

1.119.- $\int e^{\ell\eta\left|\frac{1-\cos x}{2}\right|}dx$

1.120.- $\int \exp \ell\eta\left(\frac{1+x^2}{x^2}\right)dx$

1.121.- $\int \ell\eta e^{\frac{1-\mathrm{sen}\,x}{3}}dx$

1.122.- $\int (1+\sqrt{x-3x})^0 dx$

1.123.- $\int \ell\eta e^{\frac{(1+x)^2}{2}}dx$

RESPUESTAS

1.32.- $\int 3x^5 dx = 3\int x^5 dx = \frac{3x^{5+1}}{5+1} + c = 3\frac{x^6}{6} + c = \frac{x^6}{2} + c$

1.33.- $\int (1+e)^x dx$

Sea: $a = 1+e,$ Luego: $\int (1+e)^x dx = \int a^x dx = \frac{a^x}{\ell\eta a} + c = \frac{(1+e)^x}{\ell\eta(1+e)} + c$

1.34.- $\int (1+\tau gx)dx = \int dx + \int \tau gx dx = x + \ell\eta|\sec x| + c$

1.35.- $\int \cos^2 \frac{x}{2} dx = \int \frac{1+\cos x}{2}dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos x dx = \frac{1}{2}x + \frac{1}{2}\mathrm{sen}\,x + c$

1.36.- $\int(1+\sqrt{x})^3dx=\int(1+3\sqrt{x}+3(\cancel{\sqrt{x^2}})+\sqrt{x^3})dx=\int dx+3\sqrt{x}+3\int xdx+\int x^{3/2}dx$

$=x+2x^{3/2}+3\frac{x^2}{2}+\frac{2}{5}x^{5/2}+c=x+2x\sqrt{x}+3\frac{x^2}{2}+\frac{2}{5}x^2\sqrt{x}+c$

1.37.- $\int(1+\sqrt{x})^0dx=\int dx=x+c$

1.38.- $\int\frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}}dy=\frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}}\int dy=\frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}}y+c$

1.39.- $\int\frac{dx}{\sqrt{5-x^2}}$

Sea: $a=\sqrt{5}$, Luego: $\int\frac{dx}{\sqrt{5-x^2}}=\int\frac{dx}{\sqrt{(\sqrt{5})^2-x^2}}=\text{arcsen}\frac{x}{\sqrt{5}}+c=\text{arcsen}\frac{\sqrt{5}x}{5}+c$

1.40.- $\int\frac{dx}{\sqrt{x^2-5}}=\int\frac{dx}{\sqrt{x^2-(\sqrt{5})^2}}=\ell\eta\left|x+\sqrt{x^2-5}\right|+c$

1.41.- $\int\frac{dx}{\sqrt{x^2+5}}=\int\frac{dx}{\sqrt{x^2+(\sqrt{5})^2}}=\ell\eta\left|x+\sqrt{x^2+5}\right|+c$

1.42.- $\int\frac{dx}{x^2+5}$

Sea: $a=\sqrt{5}$, Luego: $\int\frac{dx}{x^2+(\sqrt{5})^2}=\frac{1}{\sqrt{5}}\text{arc}\tau g\frac{x}{\sqrt{5}}+c$

$=\frac{\sqrt{5}}{5}\text{arc}\tau g\frac{\sqrt{5}x}{5}+c$

1.43.- $\int\frac{dx}{x^2-5}=\int\frac{dx}{x^2-(\sqrt{5})^2}=\frac{1}{2\sqrt{5}}\ell\eta\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right|+c=\frac{\sqrt{5}}{10}\ell\eta\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right|+c$

1.44.- $\int(\text{sen}^2x+\cos^2x-1)dx=\int(1-1)dx=\int 0dx=c$

1.45.- $\int\sqrt{x}(1-\sqrt{x})dx=\int(\sqrt{x}-x)dx=\int\sqrt{x}dx-\int xdx=\frac{2}{3}x^{3/2}-\frac{x^2}{2}+c$

1.46.- $\int(\tau g^2x+1)dx=\int\sec^2xdx=\tau gx+c$

1.47.- $\int\frac{dx}{x^2-12}=\int\frac{dx}{x^2-(\sqrt{12})^2}=\frac{1}{2\sqrt{12}}\ell\eta\left|\frac{x-\sqrt{12}}{x+\sqrt{12}}\right|+c=\frac{1}{4\sqrt{3}}\ell\eta\left|\frac{x-2\sqrt{3}}{x+2\sqrt{3}}\right|+c$

$=\frac{\sqrt{3}}{12}\ell\eta\left|\frac{x-2\sqrt{3}}{x+2\sqrt{3}}\right|+c$

1.48.- $\int\frac{dx}{x^2+12}$

Sea: $a=\sqrt{12}$, Luego: $\int\frac{dx}{x^2+(\sqrt{12})^2}=\frac{1}{\sqrt{12}}\text{arc}\tau g\frac{x}{\sqrt{12}}+c$

$$=\frac{1}{2\sqrt{3}}\operatorname{arc}\tau g\frac{x}{2\sqrt{3}}+c=\frac{\sqrt{3}}{6}\operatorname{arc}\tau g\frac{\sqrt{3}x}{6}+c$$

1.49.- $\int\frac{dx}{\sqrt{x^2-12}}=\int\frac{dx}{\sqrt{x^2-(\sqrt{12})^2}}=\ell\eta\left|x+\sqrt{x^2-12}\right|+c$

1.50.- $\int\frac{dx}{\sqrt{x^2+12}}=\int\frac{dx}{\sqrt{x^2+(\sqrt{12})^2}}=\ell\eta\left|x+\sqrt{x^2+12}\right|+c$

1.51.- $\int\frac{dx}{\sqrt{12-x^2}}$

Sea: $a=\sqrt{12}$,Luego: $\int\frac{dx}{\sqrt{12-x^2}}=\int\frac{dx}{\sqrt{(\sqrt{12})^2-x^2}}$

$$=\operatorname{arcsen}\frac{x}{\sqrt{12}}+c=\operatorname{arcsen}\frac{x}{2\sqrt{3}}+c=\operatorname{arcsen}\frac{\sqrt{3}x}{6}+c$$

1.52.- $\int\frac{dx}{x\sqrt{x^2-12}}=\int\frac{dx}{x\sqrt{x^2-(\sqrt{12})^2}}=\frac{1}{\sqrt{12}}\operatorname{arcsec}\frac{x}{\sqrt{12}}+c=\frac{1}{2\sqrt{3}}\operatorname{arcsec}\frac{x}{2\sqrt{3}}+c$

$$=\frac{\sqrt{3}}{6}\operatorname{arcsec}\frac{\sqrt{3}x}{6}+c$$

1.53.- $\int\frac{dx}{x\sqrt{12-x^2}}=\int\frac{dx}{x\sqrt{(\sqrt{12})^2-x^2}}=\frac{1}{\sqrt{12}}\ell\eta\left|\frac{x}{\sqrt{12}+\sqrt{12-x^2}}\right|+c$

$$=\frac{\sqrt{3}}{6}\ell\eta\left|\frac{x}{\sqrt{12}+\sqrt{12-x^2}}\right|+c$$

1.54.- $\int\frac{dx}{x\sqrt{12+x^2}}=\frac{\sqrt{3}}{6}\ell\eta\left|\frac{x}{\sqrt{12}+\sqrt{12+x^2}}\right|+c$

1.55.- $\int\frac{dx}{\sqrt{8-2x^2}}=\int\frac{dx}{\sqrt{2(4-x^2)}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{4-x^2}}=\frac{1}{\sqrt{2}}\operatorname{arcsen}\frac{x}{2}+c=\frac{\sqrt{2}}{2}\operatorname{arcsen}\frac{x}{2}+c$

1.56.- $\int\frac{dx}{\sqrt{2x^2-8}}=\int\frac{dx}{\sqrt{2(x^2-4)}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{x^2-4}}=\frac{1}{\sqrt{2}}\ell\eta\left|x+\sqrt{x^2-4}\right|+c$

$$=\frac{\sqrt{2}}{2}\ell\eta\left|x+\sqrt{x^2-4}\right|+c$$

1.57.- $\int\frac{dx}{\sqrt{2x^2+8}}=\int\frac{dx}{\sqrt{2(x^2+4)}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{x^2+4}}=\frac{1}{\sqrt{2}}\ell\eta\left|x+\sqrt{x^2+4}\right|+c$

$$=\frac{\sqrt{2}}{2}\ell\eta\left|x+\sqrt{x^2+4}\right|+c$$

1.58.- $\int\sqrt{x^2-10}dx=\int\sqrt{x^2-(\sqrt{10})^2}dx=\frac{x}{2}\sqrt{x^2-10}-\frac{10}{2}\ell\eta\left|x+\sqrt{x^2-10}\right|+c$

$$=\frac{x}{2}\sqrt{x^2-10}-5\ell\eta\left|x+\sqrt{x^2-10}\right|+c$$

1.59.- $\int\sqrt{x^2+10}dx=\frac{x}{2}\sqrt{x^2+10}+5\ell\eta\left|x+\sqrt{x^2+10}\right|+c$

1.60.- $\int\sqrt{10-x^2}dx=\int\sqrt{(\sqrt{10})^2-x^2}dx=\frac{x}{2}\sqrt{10-x^2}+\frac{10}{2}\operatorname{arcsen}\frac{x}{\sqrt{10}}+c$

$$=\frac{x}{2}\sqrt{10-x^2}+5\operatorname{arcsen}\frac{\sqrt{10}x}{10}+c$$

1.61.- $\int\frac{1-\cos^2 x}{\operatorname{sen}^2 x}dx=\int\frac{\operatorname{sen}^2 x}{\operatorname{sen}^2 x}dx=\int dx=x+c$

1.62.- $\int\sqrt{1-\operatorname{sen}^2 x}dx=\int\sqrt{\cos^2 x}dx=\int\cos xdx=\operatorname{sen}x+c$

1.63.- $\int\sqrt{1-\cos^2 x}dx=\int\sqrt{\operatorname{sen}^2 x}dx=\int\operatorname{sen}xdx=-\cos x+c$

1.64.- $\int(2^x-3^x)^0dx=\int dx=x+c$

1.65.- $\int(2^0-3^0)^n dx=\int(0)^n dx=\int 0dx=c$

1.66.- $\int\left(\tau gx-\frac{\operatorname{sen}x}{\cos x}\right)dx=\int(\tau gx-\tau gx)dx=\int 0dx=c$

1.67.- $\int\frac{dx}{3^{-x}}=\int 3^x dx=\frac{3^x}{\ell\eta 3}+c$

1.68.- $\int\sqrt{\tfrac{3}{4}-x^2}dx=\int\sqrt{(\tfrac{\sqrt{3}}{2})^2-x^2}dx=\frac{x}{2}\sqrt{\tfrac{3}{4}-x^2}+\frac{3/4}{2}\operatorname{arcsen}\frac{x}{\sqrt{3}/2}+c$

$$=\frac{x}{2}\sqrt{\tfrac{3}{4}-x^2}+\frac{3}{8}\operatorname{arcsen}\frac{2x}{\sqrt{3}}+c$$

1.69.- $\int\sqrt{x^2-\tfrac{3}{4}}dx=\int\sqrt{x^2-(\tfrac{\sqrt{3}}{2})^2}dx=\frac{x}{2}\sqrt{x^2-\tfrac{3}{4}}-\frac{3/4}{2}\ell\eta\left|x+\sqrt{x^2-\tfrac{3}{4}}\right|+c$

$$=\frac{x}{2}\sqrt{x^2-\tfrac{3}{4}}-\frac{3}{8}\ell\eta\left|x+\sqrt{x^2-\tfrac{3}{4}}\right|+c$$

1.70.- $\int\sqrt{x^2+\tfrac{3}{4}}dx=\int\sqrt{x^2+(\tfrac{\sqrt{3}}{2})^2}dx=\frac{x}{2}\sqrt{x^2+\tfrac{3}{4}}+\frac{3}{8}\ell\eta\left|x+\sqrt{x^2+\tfrac{3}{4}}\right|+c$

1.71.- $\int\frac{dx}{x\sqrt{3-x^2}}=\int\frac{dx}{x\sqrt{(\sqrt{3})^2-x^2}}=\frac{1}{\sqrt{3}}\ell\eta\left|\frac{x}{\sqrt{3}+\sqrt{3-x^2}}\right|+c$

$$=\frac{\sqrt{3}}{3}\ell\eta\left|\frac{x}{\sqrt{3}+\sqrt{3-x^2}}\right|+c$$

1.72.- $\int\frac{dx}{x\sqrt{x^2-3}}=\frac{1}{\sqrt{3}}\operatorname{arcsec}\frac{x}{\sqrt{3}}+c=\frac{\sqrt{3}}{3}\operatorname{arcsec}\frac{\sqrt{3}x}{3}+c$

1.73.- $\int\frac{dx}{x\sqrt{x^2+3}}=\frac{\sqrt{3}}{3}\ell\eta\left|\frac{x}{\sqrt{3}+\sqrt{x^2+3}}\right|+c$

1.74.- $\int (\operatorname{sen}^{3x}\theta)dy = \operatorname{sen}^{3x}\theta\int dy = (\operatorname{sen}^{3x}\theta)y + c$

1.75.- $\int \ell\eta|u|dx = \ell\eta|u|\int dx = \ell\eta|u|x + c$

1.76.- $\int \exp(\ell\eta x)dx = \int xdx = \frac{x^2}{2} + c$

1.77.- $\int e^{\ell\eta x^2}dx = \int x^2dx = \frac{x^3}{3} + c$

1.78.- $\int \frac{\sqrt{x}-\sqrt{2}}{\sqrt{2x}}dx = \int \frac{\sqrt{x}}{\sqrt{2x}}dx - \int \frac{\sqrt{2}}{\sqrt{2x}}dx = \int \sqrt{\frac{\not x}{2\not x}}dx - \int \sqrt{\frac{\not 2}{\not 2 x}}dx = \frac{1}{\sqrt{2}}\int dx - \int \frac{1}{\sqrt{x}}dx =$

$= \frac{1}{\sqrt{2}}\int dx - \int x^{-1/2}dx = \frac{1}{\sqrt{2}}x - \frac{x^{1/2}}{1/2} + c = \frac{\sqrt{2}}{2}x - 2x^{1/2} + c$

1.79.- $\int \sqrt{11-x^2}dx = \frac{x}{2}\sqrt{11-x^2} + \frac{11}{2}\operatorname{arcsen}\frac{x}{\sqrt{11}} + c = \frac{x}{2}\sqrt{11-x^2} + \frac{11}{2}\operatorname{arcsen}\frac{\sqrt{11}x}{11} + c$

1.80.- $\int \sqrt{x^2-11}dx = \frac{x}{2}\sqrt{x^2-11} - \frac{11}{2}\ell\eta\left|x+\sqrt{x^2-11}\right| + c$

1.81.- $\int \sqrt{x^2+11}dx = \frac{x}{2}\sqrt{x^2+11} + \frac{11}{2}\ell\eta\left|x+\sqrt{x^2+11}\right| + c$

1.82.- $\int \ell\eta(e^{\sqrt{x}})dx = \int \sqrt{x}dx = \int x^{1/2}dx = \frac{x^{3/2}}{3/2} + c = \frac{2}{3}x\sqrt{x} + c$

1.83.- $\int \left[\frac{1+\sqrt{x}+\sqrt{x^3}}{1-\sqrt{x}}\right]^0 dx = \int dx = x + c$

1.84.- $\int (\tau g^2x + \sec^2 x - 1)dx = \int 0dx = c$

1.85.- $\int \frac{dx}{\sqrt{3x^2-1}} = \int \frac{dx}{\sqrt{3}\sqrt{(x^2-1/3)}} = \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{(x^2-1/3)}} = \frac{1}{\sqrt{3}}\ell\eta\left|x+\sqrt{(x^2-1/3)}\right| + c$

$= \frac{\sqrt{3}}{3}\ell\eta\left|x+\sqrt{(x^2-1/3)}\right| + c$

1.86.- $\int (\operatorname{co}\tau g\theta - \operatorname{sen}\theta)dx = (\operatorname{co}\tau g\theta - \operatorname{sen}\theta)\int dx = (\operatorname{co}\tau g\theta - \operatorname{sen}\theta)x + c$

1.87.- $\int \frac{dx}{\sqrt{1+3x^2}} = \int \frac{dx}{\sqrt{3}\sqrt{\frac{1}{3}+x^2}} = \frac{\sqrt{3}}{3}\ell\eta\left|x+\sqrt{\frac{1}{3}+x^2}\right| + c$

1.88.- $\int \frac{dx}{\sqrt{1-3x^2}} = \int \frac{dx}{\sqrt{3}\sqrt{\frac{1}{3}-x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{\frac{1}{3}-x^2}} = \frac{1}{\sqrt{3}}\operatorname{arcsen}\frac{x}{\frac{1}{\sqrt{3}}} + c$

$= \frac{\sqrt{3}}{3}\operatorname{arcsen}\sqrt{3}x + c$

1.89.- $\int \frac{dx}{1+3x^2} = \int \frac{dx}{3(\frac{1}{3}+x^2)} = \frac{1}{3}\int \frac{dx}{\frac{1}{3}+x^2} = \frac{1}{3}\frac{1}{\frac{1}{\sqrt{3}}}\operatorname{arc}\tau g\frac{x}{\frac{1}{\sqrt{3}}} + c = \frac{\sqrt{3}}{3}\operatorname{arc}\tau g\sqrt{3}x + c$

1.90.- $\int \frac{dx}{3x^2+4} = \frac{1}{3}\int \frac{dx}{x^2+\frac{4}{3}} = \frac{1}{3}\frac{1}{\frac{2}{\sqrt{3}}} \operatorname{arc}\tau g \frac{x}{\frac{2}{\sqrt{3}}} + c = \frac{\sqrt{3}}{6} \operatorname{arc}\tau g \frac{\sqrt{3}x}{2} + c$

1.91.- $\int \frac{dx}{3x^2-1} = \frac{1}{3}\int \frac{dx}{x^2-\frac{1}{3}} = \frac{1}{3}\frac{1}{2\frac{1}{\sqrt{3}}} \ell\eta \left| \frac{x-\frac{1}{\sqrt{3}}}{x+\frac{1}{\sqrt{3}}} \right| + c = \frac{\sqrt{3}}{6} \ell\eta \left| \frac{\sqrt{3}x-1}{\sqrt{3}x+1} \right| + c$

1.92.- $\int \frac{dx}{x\sqrt{3x^2-1}} = \int \frac{dx}{\sqrt{3}x\sqrt{x^2-1/3}} = \frac{1}{\sqrt{3}}\int \frac{dx}{x\sqrt{x^2-1/3}} = \frac{1}{\sqrt{3}} \frac{1}{1/\sqrt{3}} \operatorname{arcsec} \frac{x}{1/\sqrt{3}} + c$

$= \operatorname{arcsec} \sqrt{3}x + c$

1.93.- $\int \frac{dx}{x\sqrt{1+3x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{x\sqrt{\frac{1}{3}+x^2}} = \frac{1}{\sqrt{3}} \frac{1}{1/\sqrt{3}} \ell\eta \left| \frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}+x^2}} \right| + c$

$= \ell\eta \left| \frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}+x^2}} \right| + c$

1.94.- $\int \frac{dx}{x\sqrt{1-3x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{x\sqrt{\frac{1}{3}-x^2}} = \ell\eta \left| \frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}-x^2}} \right| + c$

1.95.- $\int \sqrt{1-3x^2}\,dx = \sqrt{3}\int \sqrt{\frac{1}{3}-x^2}\,dx = \sqrt{3}\left[\frac{x}{2}\sqrt{\frac{1}{3}-x^2} + \frac{\frac{1}{3}}{2} \operatorname{arcs}e\operatorname{n} \frac{x}{1/\sqrt{3}} \right] + c$

$= \sqrt{3}\left[\frac{x}{2}\sqrt{\frac{1}{3}-x^2} + \frac{1}{6} \operatorname{arcs}e\operatorname{n} \sqrt{3}x \right] + c$

1.96.- $\int \sqrt{1+3x^2}\,dx = \sqrt{3}\int \sqrt{\frac{1}{3}+x^2}\,dx = \sqrt{3}\left[\frac{x}{2}\sqrt{\frac{1}{3}+x^2} + \frac{\frac{1}{3}}{2} \ell\eta \left| x+\sqrt{\frac{1}{3}+x^2} \right| \right] + c$

$= \sqrt{3}\left[\frac{x}{2}\sqrt{\frac{1}{3}+x^2} + \frac{1}{6} \ell\eta \left| x+\sqrt{\frac{1}{3}+x^2} \right| \right] + c$

1.97.- $\int \sqrt{3x^2-1}\,dx = \sqrt{3}\int \sqrt{x^2-\frac{1}{3}}\,dx = \sqrt{3}\left[\frac{x}{2}\sqrt{x^2-\frac{1}{3}} - \frac{1}{6} \ell\eta \left| x+\sqrt{x^2-\frac{1}{3}} \right| \right] + c$

1.98.- $\int (3x^2-1)dx = 3\int x^2dx - \int dx = x^3 - x + c$

1.99.- $\int (3x^2-1)^0 dx = \int dx = x + c$

1.100.- $\int (3x^2-1)^n du = (3x^2-1)^n \int du = (3x^2-1)^n u + c$

1.101.- $\int \exp(\ell\eta \frac{\sqrt{x}}{3})dx = \int \frac{\sqrt{x}}{3}dx = \frac{1}{3}\int x^{1/2}dx = \frac{1}{3}\frac{x^{3/2}}{3/2} + c = \frac{2}{9}x^{3/2} + c$

1.102.- $\int \ell\eta(e^{\frac{2x-1}{2}})dx = \int \frac{2x-1}{2}dx = \int xdx - \frac{1}{2}\int dx = \frac{x^2}{2} - \frac{1}{2}x + c$

1.103.- $\int (e^2+e+1)^x dx$

Sea: a= (e^2+e+1) , Luego: $\int a^x dx = \frac{a^x}{\ell\eta a} + c = \frac{(e^2+e-1)^x}{\ell\eta(e^2+e-1)} + c$

1.104.- $\int\left(\frac{1+\tau g^2 x}{\sec^2 x} - 1\right)dx = \int (1-1)dx = \int 0dx = c$

1.105.- $\int \exp(\ell\eta|1+x|)dx\int = \int (1+x)dx = \int dx + \int xdx = x + \frac{x^2}{2} + c$

1.106.- $\int \sqrt{27-x^2}dx = \frac{x}{2}\sqrt{27-x^2} + \frac{27}{2}\operatorname{arcsen}\frac{x}{3\sqrt{3}} + c$

1.107.- $\int \sqrt{x^2-27}dx = \frac{x}{2}\sqrt{x^2-27} - \frac{27}{2}\ell\eta\left|x+\sqrt{x^2-27}\right| + c$

1.108.- $\int \sqrt{x^2+27}dx = \frac{x}{2}\sqrt{x^2+27} + \frac{27}{2}\ell\eta\left|x+\sqrt{x^2+27}\right| + c$

1.109.- $\int \frac{dx}{3x\sqrt{x^2-1}} = \frac{1}{3}\int \frac{dx}{x\sqrt{x^2-1}} = \frac{1}{3}\operatorname{arc} secx + c$

1.110.- $\int \frac{dx}{2x\sqrt{1-x^2}} = \frac{1}{2}\int \frac{dx}{x\sqrt{1-x^2}} = \frac{1}{2}\ell\eta\left|\frac{x}{1+\sqrt{1-x^2}}\right| + c$

1.111.- $\int \frac{dx}{5x\sqrt{x^2+1}} = \frac{1}{5}\int \frac{dx}{x\sqrt{x^2+1}} = \frac{1}{5}\ell\eta\left|\frac{x}{1+\sqrt{x^2+1}}\right| + c$

1.112.- $\int \frac{dx}{3x\sqrt{9-x^2}} = \frac{1}{3}\int \frac{dx}{x\sqrt{9-x^2}} = \frac{1}{3}\frac{1}{3}\ell\eta\left|\frac{x}{3+\sqrt{9-x^2}}\right| + c = \frac{1}{9}\ell\eta\left|\frac{x}{3+\sqrt{9-x^2}}\right| + c$

1.113.- $\int \frac{dx}{4x\sqrt{x^2+16}} = \frac{1}{4}\int \frac{dx}{x\sqrt{x^2+16}} = \frac{1}{4}\frac{1}{4}\ell\eta\left|\frac{x}{4+\sqrt{x^2+16}}\right| + c$

$= \frac{1}{16}\ell\eta\left|\frac{x}{4+\sqrt{x^2+16}}\right| + c$

1.114.- $\int \frac{dx}{5x\sqrt{x^2-25}} = \frac{1}{5}\int \frac{dx}{x\sqrt{x^2-25}} = \frac{1}{5}\frac{1}{5}\operatorname{arc} sec\frac{x}{5} + c = \frac{1}{25}\operatorname{arc} sec\frac{x}{5} + c$

1.115.- $\int \frac{(1-\sqrt{x})^2}{x^2}dx = \int \frac{1-2\sqrt{x}+x}{x^2}dx = \int (x^{-2} - 2x^{-3/2} + x^{-1})dx$

$= \int x^{-2}dx - \int 2x^{-3/2}dx + \int x^{-1}dx = -x^{-1} - 2\frac{x^{-1/2}}{-1/2} + \ell\eta|x| + c = -x^{-1} - 2\frac{x^{-1/2}}{-1/2} + \ell\eta|x| + c$

$= -x^{-1} + 4x^{-1/2} + \ell\eta|x| + c = -\frac{1}{x} + \frac{4}{\sqrt{x}} + \ell\eta|x| + c$

1.116.- $\int (1+\sqrt{x}+x)^2 dx = (1+x+x^2+2\sqrt{x}+2x+2x^{3/2})dx$

$= \int (1+2x^{1/2}+3x+2x^{3/2}+x^2)dx = \int dx + 2\int x^{1/2}dx + 3\int xdx + 2\int x^{3/2}dx + \int x^2 dx$

$x + \frac{2x^{3/2}}{3/2} + 3\frac{x^2}{2} + 2\frac{x^{5/2}}{5/2} + \frac{x^3}{3} + c = x + \frac{4x^{3/2}}{3} + 3\frac{x^2}{2} + 4\frac{x^{5/2}}{5} + \frac{x^3}{3} + c$

1.117.- $\int (1-\sqrt{x}+x)^2 dx = \int (1+x+x^2-2\sqrt{x}+2x-2x^{3/2})dx$

$= \int (1-2x^{1/2}+3x-2x^{3/2}+x^2)dx = x-\frac{4x^{3/2}}{3}+3\frac{x^2}{2}-4\frac{x^{5/2}}{5}+\frac{x^3}{3}+c$

1.118.- $\int (1+x)^4 dx = \int (1+4x+6x^2+4x^3+x^4)dx$

$= \int dx + 4\int xdx + 6\int x^2dx + 4\int x^3dx + \int x^4dx = x+2x^2+2x^3+x^4+\frac{1}{5}x^5+c$

1.119.- $\int e^{\ell\eta\left|\frac{1-\cos x}{2}\right|}dx = \int \frac{1-\cos x}{2}dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos xdx = \frac{1}{2}x-\frac{1}{2}\operatorname{sen} xdx$

1.120.- $\int \exp \ell\eta\left(\frac{1+x^2}{x^2}\right)dx = \int \frac{1+x^2}{x^2}dx = \int \frac{1}{x^2}dx + \int dx = \int x^{-2}dx + \int dx = -\frac{1}{x}+x+c$

1.121.- $\int \ell\eta e^{\frac{1-\operatorname{sen} x}{3}}dx = \int \frac{1-\operatorname{sen} x}{3}dx = \frac{1}{3}\int dx - \frac{1}{3}\int \operatorname{sen} xdx = \frac{1}{3}x+\frac{1}{3}\cos x + c$

1.122.- $\int (1+\sqrt{x-3x})^0 dx = \int dx = x+c$

1.123.- $\int \ell\eta e^{\frac{(1+x)^2}{2}}dx = \int \frac{(1+x)^2}{2}dx = \int \frac{1+2x+x^2}{2}dx = \frac{1}{2}\int dx + \int xdx + \frac{1}{2}\int x^2dx$

$= \frac{1}{2}x+\frac{x^2}{2}+\frac{x^3}{6}+c$