

EXAMEN DE MATEMÁTICAS 1º BACHILLERATO.  
LÍMITES

1. Dada la función:

$$y = F(x) = \begin{cases} x^2 - 5 & \text{si } x < 1 \\ 3x + 2 & \text{si } 1 \leq x < 4 \\ 4x - 2 & \text{si } x \geq 4 \end{cases}$$

¿Existe el límite de  $F(x)$  para  $x \rightarrow 1$ ? ¿Y para  $x \rightarrow 4$ ? Razona tu respuesta. Dibuja la gráfica de  $F(x)$ .

2. Calcular  $b$  para que

$$\lim_{x \rightarrow +\infty} \frac{-x^2 + 2x}{bx(1+x)} = \frac{1}{5}$$

3. Calcular los siguientes límites:

$$\text{a) } \lim_{x \rightarrow +\infty} \frac{2x^2 - 5x + 6}{x^3 - x^2 + 8}$$

$$\text{b) } \lim_{x \rightarrow +\infty} \frac{4x^4 - 2x}{x(x^2 - 1)}$$

$$\text{c) } \lim_{x \rightarrow +\infty} \left( \frac{2x^2 + 1}{x^2 + 5} \right)^{\frac{3x+1}{x+2}}$$

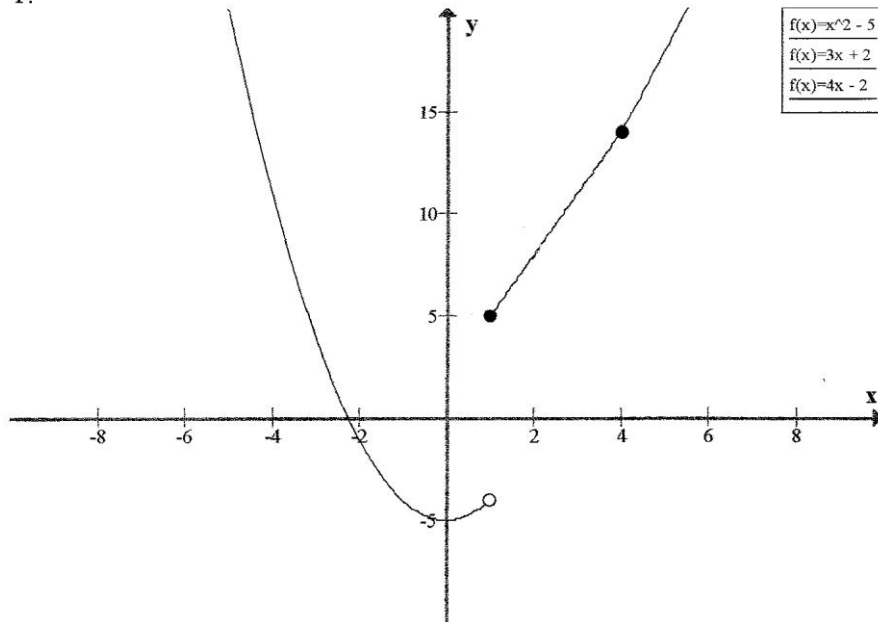
$$\text{d) } \lim_{x \rightarrow +\infty} (\sqrt{x^3 + 2} - 5x)$$

4. Calcular los siguientes límites del número  $e$ :

$$\text{a) } \lim_{x \rightarrow 2} \left( \frac{x+3}{2x+1} \right)^{2/(x-2)}$$

$$\text{b) } \lim_{x \rightarrow +\infty} \left( \frac{3x+1}{3x+4} \right)^{2x}$$

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$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 - 5 = -4 \quad \# \quad \# \text{ limite en } x=1$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 3x + 2 = 5 \quad \text{discontinua de salto finito}$$

$$\lim_{x \rightarrow 4^-} 3x + 2 = 14 = \lim_{x \rightarrow 4^+} 4x - 2 = f(4)$$

$\exists$  límite y es continua en  $x=4$

$$2. \lim_{x \rightarrow \infty} \frac{-x^2 + 2x}{bx(1+x)} = \frac{1}{5}$$

$$\lim_{x \rightarrow \infty} \frac{-x^2 + 2x}{bx(1+x)} = \left[ \frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{-x^2 + 2x}{bx^2 + bx} =$$

$$\lim_{x \rightarrow \infty} \frac{-\frac{x^2}{x^2} + \frac{2x}{x^2}}{\frac{bx^2}{x^2} + \frac{bx}{x^2}} = \lim_{x \rightarrow \infty} \frac{-1 + \frac{2}{x} \rightarrow 0}{b + \frac{b}{x} \rightarrow 0} = \frac{-1}{b} = \frac{1}{5}$$

$$\Downarrow$$

$$\boxed{b = -5}$$

$$3. a) \lim_{x \rightarrow \infty} \frac{2x^2 - 5x + 6}{x^3 - x^2 + 8} = \left[ \frac{\infty}{\infty} \right] =$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{2x^2}{x^3} - \frac{5x}{x^3} + \frac{6}{x^3}}{\frac{x^3}{x^3} - \frac{x^2}{x^3} + \frac{8}{x^3}} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x} - \frac{5}{x^2} + \frac{6}{x^3}}{1 - \frac{1}{x} + \frac{8}{x^3}} =$$

$$= \frac{0}{1} = 0$$

$$b) \lim_{x \rightarrow \infty} \frac{4x^4 - 2x}{x(x^2 - 1)} = \left[ \frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{4x^4 - 2x}{x^3 - x} =$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{4x^4}{x^4} - \frac{2x}{x^4}}{\frac{x^3}{x^4} - \frac{x}{x^4}} = \lim_{x \rightarrow \infty} \frac{4 - \frac{2}{x^3}}{\frac{1}{x} - \frac{1}{x^4}} = \frac{4}{0} = \infty$$

$$c) \lim_{x \rightarrow \infty} \left( \frac{2x^2 + 1}{x^2 + 5} \right)^{\frac{3x+1}{x+2}} = \left( \frac{\infty}{\infty} \right)^{\left( \frac{\infty}{\infty} \right)} =$$

$$= \lim_{x \rightarrow \infty} \left( \frac{2x^2 + 1}{x^2 + 5} \right)^{\frac{3x+1}{x+2}} = 2^3 = 8$$

$$d) \lim_{x \rightarrow \infty} (\sqrt{x^3 + 2x} - 5x) = [\infty - \infty] =$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^3 + 2x} - 5x)(\sqrt{x^3 + 2x} + 5x)}{(\sqrt{x^3 + 2x} + 5x)} =$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^3 + 2x})^2 - (5x)^2}{\sqrt{x^3 + 2x} + 5x} = \lim_{x \rightarrow \infty} \frac{x^3 + 2x - 25x^2}{\sqrt{x^3 + 2x} + 5x} =$$

$$= \lim_{x \rightarrow \infty} \frac{\overset{\text{grad } 3}{x^3 + 2x - 25x^2}}{\overset{\text{grad } \frac{3}{2}}{\sqrt{x^3 + 2x} + 5x}} = \left[ \frac{\infty}{\infty} \right] = \infty$$

$$4. a) \lim_{x \rightarrow 2} \left( \frac{x+3}{2x+1} \right)^{\frac{2}{x-2}} = [1^\infty] =$$

$$= e^{\lim_{x \rightarrow 2} \frac{2}{x-2} \cdot \left( \frac{x+3}{2x+1} - 1 \right)} =$$

$$= e^{\lim_{x \rightarrow 2} \frac{2}{x-2} \cdot \frac{x+3-2x-1}{2x+1}} = e^{\lim_{x \rightarrow 2} \frac{2}{x-2} \cdot \frac{-x+2}{2x+1}} =$$

$$= e^{\lim_{x \rightarrow 2} \frac{2}{x-2} \cdot \frac{-(x-2)}{2x+1}} = e^{\lim_{x \rightarrow 2} \frac{-2}{2x+1}} = e^{-\frac{2}{5}}$$

$$b) \lim_{x \rightarrow \infty} \left( \frac{3x+1}{3x+4} \right)^{2x} = [1^\infty] =$$

$$= e^{\lim_{x \rightarrow \infty} 2x \left( \frac{3x+1}{3x+4} - 1 \right)} =$$

$$= e^{\lim_{x \rightarrow \infty} 2x \left( \frac{3x+1-3x-4}{3x+4} \right)} = e^{\lim_{x \rightarrow \infty} 2x \left( \frac{-3}{3x+4} \right)} =$$

$$= e^{\lim_{x \rightarrow \infty} \frac{-6x}{3x+4}} = e^{-2}$$