INTEGRACIÓN DE FUNCIONES RACIONALES

Mediante el recurso de la descomposición en fracciones simples, el proceso de integración de funciones racionales se puede simplificar notablemente.

EJERCICIOS DESARROLLADOS

7.1.-Encontrar: $\int \frac{dx}{x^2-9}$

Solución.- Descomponiendo el denominador en factores: $x^2-9=(x+3)(x-3)$,
Como los factores son ambos lineales y diferentes se tiene:

$\frac{1}{x^2-9}=\frac{A}{x+3}+\frac{B}{x-3}$, de donde:

$$\frac{1}{x^2-9}=\frac{A}{x+3}+\frac{B}{x-3}\Rightarrow 1=A(x-3)+B(x+3)(*)\Rightarrow 1=(A+B)x+(-3A+3B)$$

Para calcular las constantes A y B, se pueden identificar los coeficientes de igual potencia x en la última expresión, y se resuelve el sistema de ecuaciones dado; obteniendo así los valores de las constantes en referencia (método general) luego:

$$\begin{pmatrix} A+\ \ B=0 \\ -3A+3B=1 \end{pmatrix}\Rightarrow\begin{pmatrix} 3A+3B=0 \\ -3A+3B=1 \end{pmatrix}\Rightarrow 6B=1\Rightarrow B=\frac{1}{6}$$, además:

$$A+B=0\Rightarrow A=-B\Rightarrow A=-\frac{1}{6}$$

También es frecuente usar otro mecanismo, que consiste en la expresión $(*)$
Sustituyendo a x por los valores que anulen los denominadores de las fracciones:

$$x=3\Rightarrow 1=6B\Rightarrow B=\frac{1}{6}$$

$$x=-3\Rightarrow 1=-6A\Rightarrow A=-\frac{1}{6}$$

Usando cualquier método de los señalados anteriormente, se establece que:

$\frac{1}{x^2-9}=\frac{-\frac{1}{6}}{x+3}+\frac{\frac{1}{6}}{x-3}$, Luego se tiene:

$$\int\frac{dx}{x^2-9}=-\frac{1}{6}\int\frac{dx}{x+3}+\frac{1}{6}\int\frac{dx}{x-3}=-\frac{1}{6}\ell\eta|x+3|+\frac{1}{6}\ell\eta|x-3|+c$$

$$=\frac{1}{6}\left(\ell\eta|x-3|-\ell\eta|x+3|\right)+c$$

Respuesta: $\int \frac{dx}{x^2-9}=\frac{1}{6}\ell\eta\left|\frac{x-3}{x+3}\right|+c$

7.2.-Encontrar: $\int \frac{dx}{x^2+7x-6}$

Solución.- Sea: $x^2+7x+6=(x+6)(x+1)$, factores lineales y diferentes; luego:

$\frac{1}{x^2+7x+6}=\frac{A}{x+6}+\frac{B}{x+1}$,

De donde:

$1=A(x+1)+B(x+6)(*)\Rightarrow 1=(A+B)x+(A+6B)$, calculando las constantes A y B por el método general, se tiene: $1=(A+B)x+(A+6B)$

$\begin{pmatrix} A+\quad B=0 \\ A+6B=1 \end{pmatrix}\Rightarrow -\begin{pmatrix} -A-\quad B=0 \\ A+6B=1 \end{pmatrix}\Rightarrow 5B=1\Rightarrow B=1/5$, además:

$A+B=0\Rightarrow A=-B\Rightarrow A=-1/5$

Ahora utilizando el método abreviado se tiene:

$x=-1\Rightarrow 1=5B\Rightarrow B=1/5$

$x=-6\Rightarrow 1=-5A\Rightarrow A=-1/5$

Usando cualquier método se puede establecer:

$\frac{1}{x^2+7x+6}=\frac{-1/5}{x+6}+\frac{1/5}{x+1}$, Luego se tiene:

$\int\frac{dx}{x^2+7x+6}=-\frac{1}{5}\int\frac{dx}{x+6}+\frac{1}{5}\int\frac{dx}{x+1}=-\frac{1}{5}\ell\eta|x+6|+\frac{1}{5}\ell\eta|x+1|+c$

$=\frac{1}{5}\left(\ell\eta|x+1|-\ell\eta|x+6|\right)+c$

Respuesta: $\int\frac{dx}{x^2+7x+6}=\frac{1}{5}\ell\eta\left|\frac{x+1}{x+6}\right|+c$

7.3.-Encontrar: $\int\frac{xdx}{x^2-4x+4}$

Solución.- Sea: $x^2-4x+4=(x-2)^2$, factores lineales con repetición; luego:

$\frac{x}{x^2-x+4}=\frac{A}{x-2}+\frac{B}{(x-2)^2}\Rightarrow\frac{x}{\cancel{x^2-x+4}}=\frac{A(x-2)+B}{\cancel{(x-2)^2}}$,

De donde:

$x=A(x-2)+B(*)$, calculando las constantes A y B por el método general, se tiene: $x=Ax+(-2A+B)$, luego:

$\begin{pmatrix} A\qquad =1 \\ -2A+B=0 \end{pmatrix}\Rightarrow B=2A\Rightarrow B=2(1)\Rightarrow B=2$

Usando el método abreviado, se sustituye en x, el valor que anula el denominador(o los denominadores), y si este no es suficiente se usan para sustituir cualquier valor conveniente de x, esto es: $x=0, x=-1$; luego en $(*)$

$x=2 \Rightarrow 2=B \Rightarrow B=2$

$x=0 \Rightarrow 0=-2A+B \Rightarrow 2A+B \Rightarrow A=B/2 \Rightarrow A=1$

Usando cualquier método se establece:

$$\int \frac{xdx}{x^2-4x+4} = \int \frac{dx}{x-2} + 2\int \frac{dx}{(x-2)^2} = \ell\eta|x-2| - \frac{2}{x-2} + c$$

Respuesta: $\int \frac{xdx}{x^2-4x+4} = \ell\eta|x-2| - \frac{2}{x-2} + c$

7.4.-Encontrar: $\int \frac{(2x^2+3)dx}{x^3-2x^2+x}$

Solución.- Sea: $x^3-2x^2+x = x(x^2-2x+1) = x(x-1)^2$, factores lineales: $x, x-1$; donde este último es con repetición; luego:

$$\frac{2x^2+3}{x^3-2x^2+x} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \Rightarrow \frac{2x^2+3}{\cancel{x^3-2x^2+x}} = \frac{A(x-1)^2+Bx(x-1)+Cx}{\cancel{x(x-1)^2}}$$

De donde:

$2x^2+3 = A(x-1)^2 + Bx(x-1) + Cx(*)$, calculando las constantes A y B por el método general, se tiene: $2x^2+3 = (A+B)x^2 + (-2A-B+C)x + A$, de donde identificando los coeficientes de igual potencia de x se puede obtener el siguiente sistema de ecuaciones:

$$\begin{pmatrix} A+B & =2 \\ -2A-B+C & =0 \\ A & =3 \end{pmatrix} \Rightarrow B=2-A \Rightarrow B=2-3 \Rightarrow B=-1$$, tomando la segunda ecuación del sistema: $C=2A+B \Rightarrow C=2(3)-1 \Rightarrow C=5$, también es posible usar el método abreviado, utilizando para ello la expresión $(*)$ en la cual:

$x=1 \Rightarrow 2(1)+3=C \Rightarrow C=5$

$x=0 \Rightarrow 3=A \Rightarrow A=3$

Usando un valor arbitrario para x, sea este $x=-1$:

$x=-1 \Rightarrow 2(-1)^2+3 = A(-2)^2 + B(-1)(-2) + C(-1) \Rightarrow 5=4A+2B-C$, luego:

$2B=5-4A+C \Rightarrow 2B=5-4(3)+5 \Rightarrow 2B=-2 \Rightarrow B=-1$, S, e establece que:

$$\frac{2x^2+3}{x^3-2x^2+x} = \frac{3}{x} - \frac{1}{x-1} + \frac{5}{(x-1)^2}$$, entonces:

$$\frac{2x^2+3}{x^3-2x^2+x} = 3\int\frac{dx}{x} - \int\frac{dx}{x-1} + 5\int\frac{dx}{(x-1)^2} = 3\ell\eta|x| - \ell\eta|x-1| - \frac{5}{x-1} + c$$

Respuesta: $\int \frac{(2x^2+3)dx}{x^3-2x^2+x} = \ell\eta\left|\frac{x^3}{x-1}\right| - \frac{5}{x-1} + c$

7.5.-Encontrar: $\int \frac{dx}{x^3 - 2x^2 + x}$

Solución.- $x^3 - 2x^2 + x = x(x-1)^2$,factores lineales:

$x, x-1$; donde este último es con repetición; luego:

$$\frac{1}{x^3 - 2x^2 + x} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \Rightarrow \frac{1}{\cancel{x^3 - 2x^2 + x}} = \frac{A(x-1)^2 + Bx(x-1) + Cx}{\cancel{x(x-1)^2}}$$

De donde:

$1 = A(x-1)^2 + Bx(x-1) + Cx(*)$, calculando las constantes A y B por el método general, se tiene: $1 = (A+B)x^2 + (-2A - B + C)x + A$, de donde identificando los coeficientes de igual potencia de x se puede obtener el siguiente sistema de ecuaciones:

$$\begin{pmatrix} A+B & = 0 \\ -2A - B + C = 0 \\ A & = 1 \end{pmatrix} \Rightarrow B = -A \Rightarrow B = -1$$, tomando la segunda ecuación del sistema: $C = 2A + B \Rightarrow C = 2(1) - 1 \Rightarrow C = 1$, a partir de lo cual se tiene:

$$\frac{1}{x^3 - 2x^2 + x} = \frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2}$$

$$\int \frac{dx}{x^3 - 2x^2 + x} = \int \frac{dx}{x} - \int \frac{dx}{x-1} + \int \frac{dx}{(x-1)^2} = \ell\eta|x| - \ell\eta|x-1| - \frac{1}{x-1} + c$$

Respuesta: $\int \frac{dx}{x^3 - 2x^2 + x} = \ell\eta\left|\frac{x}{x-1}\right| - \frac{1}{x-1} + c$

7.6.-Encontrar: $\int \frac{x^4 - 6x^3 + 12x^2 + 6}{x^3 - 6x^2 + 12x - 8}dx$

Solución.- Se sabe que si el grado del polinomio dividendo, es igual o superior al grado del polinomio divisor, previamente conviene efectuar la división de tales polinomios.

$$\begin{array}{l|l} x^4 - 6x^3 + 12x^2 + 0x + 6 & x^3 - 6x^2 + 12x - 8 \\ \hline -x^4 + 6x^3 - 12x^2 + 8x & x \\ \hline \qquad\qquad\qquad \boxed{8x + 6} & \end{array}$$

Luego se tiene: $\int \frac{x^4 - 6x^3 + 12x^2 + 6}{x^3 - 6x^2 + 12x - 8}dx = \int xdx + \int \frac{(8x+6)dx}{x^3 - 6x^2 + 12x - 8}$

La descomposición de: $x^3 - 6x^2 + 12x - 8$:

$$\begin{array}{r|rrrr} & 1 & -6 & 12 & -8 \\ 2 & & 2 & -8 & 8 \\ \hline & 1 & -4 & 4 & \boxed{0} \end{array} \qquad x = 2 \Rightarrow (x-2)$$

$x^2 - 4x + 4 = (x-2)^2$

$x^3 - 6x^2 + 12x - 8 = (x-2)^3$

Esto es factores lineales: $\left[(x-2)\right]$ con repetición por tanto:

$$\frac{8x+6}{x^3-6x^2+12x-8}=\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}$$

$$\frac{8x+6}{x^3-6x^2+12x-8}=\frac{A(x-2)^2+B((x-2)+C}{(x-2)^3}$$

Luego:

$$8x+6=A(x-2)^2+B(x-2)+C\Rightarrow 8x+6=A(x^2-4x+4)+B(x-2)+C$$

$$8x+6=Ax^2+(-4A+B)x+(4A-2B+C)$$

Calculando las constantes A y B por el método general, se tiene:

$$\begin{pmatrix} A & & =0 \\ -4A+ & B & =8 \\ +4A-2B+C=6 & & \end{pmatrix}\Rightarrow B=8+4A\Rightarrow B=8+4(0)\Rightarrow B=8,$$

Resolviendo el sistema: $C=6-4A+2B\Rightarrow C=6-4(0)+2(8)\Rightarrow C=22$, luego:

$$\frac{8x+6}{x^3-6x^2+12x-8}=\frac{0}{x-2}^{0}+\frac{8}{(x-1)^2}+\frac{22}{(x-1)^3}$$, de donde:

$$\int\frac{(8x+6)dx}{x^3-6x^2+12x-8}=8\int\frac{dx}{(x-2)^2}+22\int\frac{dx}{(x-2)^3}$$, o sea:

$$=\int xdx+8\int\frac{dx}{(x-2)^2}+22\int\frac{dx}{(x-2)^3}=\int xdx+8\int(x-2)^{-2}dx+22\int(x-2)^{-3}dx$$

$$\frac{x^2}{2}-\frac{8}{x-2}-\frac{11}{(x-2)^2}+c$$

Respuesta: $\int\frac{x^4-6x^3+12x^2+6}{x^3-6x^2+12x-8}dx=\frac{x^2}{2}-\frac{8}{x-2}-\frac{11}{(x-2)^2}+c$

7.7.-Encontrar: $\int\frac{x^3+x^2+x+3}{x^4+4x^2+3}dx$

Solución.- $x^4+4x^2+3=(x^2+3)(x^2+1)$, la descomposición es en factores cuadráticos sin repetición, por lo tanto:

$$\frac{x^3+x^2+x+3}{x^4+4x^2+3}=\frac{Ax+B}{x^2+3}+\frac{Cx+D}{x^2+1}$$

$$\frac{x^3+x^2+x+3}{x^4+4x^2+3}=\frac{(Ax+B)(x^2+1)+(Cx+D)(x^2+3)}{(x^2+3)(x^2+1)}$$

$$x^3+x^2+x+3=A(x^3+x)+B(x^2+1)+C(x^3+3x)+D(x^2+3)$$

$$x^3+x^2+x+3=(A+C)x^3+(B+D)x^2+(A+3C)x+(B+3D)$$, luego:

$$\begin{array}{l} (1) \\ (2) \\ (3) \\ (4) \end{array}\begin{pmatrix} A & + & C & & =1 \\ & B & & + & D=1 \\ A & & +3C & & =1 \\ & B & & & +3D=3 \end{pmatrix}$$

Con (1) y (3), se tiene: $\begin{pmatrix} A+ & C=1 \\ A+3C=1 \end{pmatrix} \Rightarrow A=1, C=0$

Con (2) y (4), se tiene: $\begin{pmatrix} B+ & D=1 \\ B+3D=3 \end{pmatrix} \Rightarrow B=0, D=1$

Por lo tanto: $\frac{x^3+x^2+x+3}{x^4+4x^2+3}=\frac{x}{x+3}+\frac{1}{x^2+1}$, o sea:

$\int \frac{x^3+x^2+x+3}{x^4+4x^2+3}dx=\int \frac{xdx}{x+3}+\int \frac{dx}{x^2+1}$, sea: $u=x^2+3, du=2xdx$, luego:

$$\int \frac{x^3+x^2+x+3}{x^4+4x^2+3}dx=\frac{1}{2}\int \frac{2xdx}{x+3}+\int \frac{dx}{x^2+1^2}=\frac{1}{2}\int \frac{du}{u}+\int \frac{dx}{x^2+1^2}$$

$$=\frac{1}{2}\ell\eta|u|+\text{arc}\,\tau gx+c=\frac{1}{2}\ell\eta\left|x^2+3\right|+\text{arc}\,\tau gx+c$$

Respuesta: $\int \frac{x^3+x^2+x+3}{x^4+4x^2+3}dx=\frac{1}{2}\ell\eta\left|x^2+3\right|+\text{arc}\,\tau gx+c$

7.8.-Encontrar: $\int \frac{x^4dx}{x^4+2x^2+1}$

Solución.-

$$\begin{array}{l|l} x^4 & x^4+2x^2+1 \\ \hline -x^4-2x^2-1 & 1 \\ \hline \boxed{-2x^2-1} & \end{array}$$

Luego $\int \frac{x^4dx}{x^4+2x^2+1}=\int\left(1-\frac{2x^2+1}{x^4+2x^2+1}\right)dx=\int dx-\int \frac{2x^2+1}{x^4+2x^2+1}dx$

La descomposición del denominador es: $x^4+2x^2+1=(x^2+1)^2$, entonces:

$$\frac{2x^2+1}{x^4+2x^2+1}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2} \Rightarrow \frac{2x^2+1}{x^4+2x^2+1}=\frac{(Ax+B)(x^2+1)(Cx+D)}{(x^2+1)^2}$$

$$2x^2+1=(Ax+B)(x^2+1)+(Cx+D) \Rightarrow 2x^2+1=A(x^3+x)+B(x^2+1)+Cx+D$$

$$2x^2+1=Ax^3+Bx^2+(A+C)x+(B+D)$$

Calculando las constantes por el método general, se tiene:

$$\begin{pmatrix} A & & & =0 \\ & B & & =2 \\ A & & +C & =0 \\ & B & & +D=1 \end{pmatrix}$$

Resolviendo el sistema: $C=-A \Rightarrow A=0 \therefore C=0$, $B+D=1 \Rightarrow D=1-B \Rightarrow D=-1$
luego:

$$\frac{2x^2+1}{x^4+2x^2+1}=\frac{2}{x^2+1}-\frac{1}{(x^2+1)^2}$$, o sea:

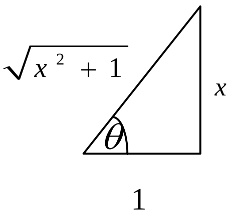
$$\int \frac{2x^2+1}{x^4+2x^2+1}=2\int \frac{dx}{x^2+1^2}-\int \frac{dx}{(x^2+1)^2}=2\int \frac{dx}{x^2+1^2}-\int \frac{dx}{(\sqrt{x^2+1})^4}$$

Sea: $x=\tau g\theta, dx=\sec^2\theta d\theta; \sqrt{x^2+1}=\sec\theta$, luego:

$$=2\operatorname{arc}\tau gx-\int \frac{\sec^2\theta}{\sec^4\theta}d\theta=2\operatorname{arc}\tau gx-\int \frac{d\theta}{\sec^2\theta}=2\operatorname{arc}\tau gx-\int \cos^2\theta$$

$$=2\operatorname{arc}\tau gx-\int \frac{1+\cos 2\theta}{2}d\theta=2\operatorname{arc}\tau gx-\frac{1}{2}\int d\theta-\frac{1}{2}\int \cos 2\theta d\theta$$

$$\operatorname{arc}\tau gx-\frac{1}{2}\theta-\frac{1}{2}\operatorname{sen}2\theta+c=2\operatorname{arc}\tau gx-\frac{1}{2}\theta-\frac{1}{2}\operatorname{sen}\theta\cos\theta+c$$



De la figura se tiene que:

$$\tau g\theta=x, \theta \operatorname{arc}\tau g\theta, \operatorname{sen}\theta=\frac{x}{\sqrt{x^2+1}}, \cos\theta=\frac{1}{\sqrt{x^2+1}}$$

Luego: $=2\operatorname{arc}\tau gx-\frac{1}{2}\operatorname{arc}\tau gx-\frac{1}{2}\frac{x}{\sqrt{x^2+1}}\frac{1}{\sqrt{x^2+1}}+c=2\operatorname{arc}\tau gx-\frac{1}{2}\operatorname{arc}\tau gx-\frac{x}{2(x^2+1)}+c$

Recordando que:

$$\frac{x^4dx}{x^4+2x^2+1}=\int dx-\int \frac{(2x^2+1)dx}{x^4+2x^2+1}=x-2\operatorname{arc}\tau gx+\frac{1}{2}\operatorname{arc}\tau gx+\frac{1}{2}\frac{x}{(x^2+1)}+c$$

Respuesta: $\int \frac{x^4dx}{x^4+2x^2+1}=x-\frac{3}{2}\operatorname{arc}\tau gx+\frac{x}{2(x^2+1)}+c$

7.9.-Encontrar: $\int \frac{x^4dx}{x^4-1}$

Solución.-

$$\begin{array}{l|l} x^4 & x^4-1 \\ -x^4+1 & 1 \\ \hline \boxed{1} & \end{array}$$

Luego:

$$\int \frac{x^4dx}{x^4-1}=\int\left(1+\frac{1}{x^4-1}\right)dx=\int dx+\int \frac{dx}{x^4-1}$$

Descomponiendo en factores el denominador:
$x^4-1=(x^2-1)(x^2+1)=(x^2+1)(x+1)(x-1)$, es decir factores lineales y cuadráticos sin repetición por tanto:

$$\frac{1}{x^4-1}=\frac{Ax+B}{x^2+1}+\frac{C}{x+1}+\frac{D}{x-1}$$

$$\frac{1}{\cancel{x^4-1}}=\frac{(Ax+B)(x^2-1)+C(x^2+1)(x-1)+D(x+1)(x^2+1)}{\cancel{(x^2+1)(x+1)(x+1)}}$$

$$1=A(x^3-x)+B(x^2+1)+C(x^3-x^2+x-1)+D(x^3+x^2+x+1)$$

$$1=(A+C+D)x^3+(B-C+D)x^2+(-A+C+D)x+(-B-C+D)$$

Luego:

$$\begin{matrix}(1)\\(2)\\(3)\\(4)\end{matrix}\begin{pmatrix}A & & +C+D=0\\ & B & -C+D=0\\ -A & & +C+D=0\\ & -B & -C+D=1\end{pmatrix}$$

Con (1) y (3), se tiene: $\begin{pmatrix}A+C+D=0\\-A+C+D=0\end{pmatrix}\Rightarrow 2C+2D=0$ **(5)**

Con (2) y (4), se tiene: $\begin{pmatrix}B-C+D=0\\-B-C+D=1\end{pmatrix}\Rightarrow -2C+2D=1$ **(6)**

Con (5) y (6), se tiene: $\begin{pmatrix}2C+2D=0\\-2C+2D=1\end{pmatrix}\Rightarrow C=-\frac{1}{4}, D=\frac{1}{4}$

Además: $A=0, B=-\frac{1}{2}$, luego:

$\frac{1}{x^4-1}=-\frac{1}{2(x^2+1)}-\frac{1}{4(x+1)}+\frac{1}{4(x-1)}$, con lo cual:

$$\int\frac{dx}{x^4-1}=-\frac{1}{2}\int\frac{dx}{(x^2+1)}-\frac{1}{4}\int\frac{dx}{(x+1)}+\frac{1}{4}\int\frac{dx}{(x-1)}$$

$$=-\frac{1}{2}\operatorname{arc}\tau gx-\frac{1}{4}\ell\eta|x+1|+\frac{1}{4}\ell\eta|x-1|+c$$

Dado que: $\int\frac{x^4dx}{x^4-1}=\int dx+\int\frac{dx}{x^4-1}=x-\frac{1}{2}\operatorname{arc}\tau gx+\frac{1}{4}\ell\eta\left|\frac{x-1}{x+1}\right|+c$, entonces:

Respuesta: $\int\frac{1}{x^4-1}=x-\frac{1}{2}\operatorname{arc}\tau gx+\frac{1}{4}\ell\eta\left|\frac{x-1}{x+1}\right|+c$

7.10.-Encontrar: $\int\frac{x^4-2x^3+3x^2-x+3}{x^3-2x^2+3x}dx$

Solución.-

$$\begin{array}{l|l} x^4-2x^3+3x^2-x+3 & x^3-2x^2+3x \\ \cline{2-2} \underline{-x^4+2x^3-3x^2\qquad} & x \\ \qquad\qquad\boxed{-x+3} & \end{array}$$

Luego:

$$\int \frac{x^4-2x^3+3x^2-x+3}{x^3-2x^2+3x}dx = \int\left(x-\frac{x-3}{x^3-2x^2+3x}\right)dx = \int xdx - \int \frac{x-3}{x^3-2x^2+3x}dx$$

Descomponiendo en factores el denominador:

$x^3-2x^2+3x = x(x^2-2x+3)$, es decir un factor lineal y uno cuadrático; por lo cual:

$$\frac{x-3}{x^3-2x^2+3x} = \frac{A}{x}+\frac{Bx+C}{x^2-2x+3} \Rightarrow \frac{x-3}{\cancel{x^3-2x^2+3x}} = \frac{A(x^2-2x+3)+(Bx+C)x}{\cancel{x(x^2-2x+3)}}$$

$$x-3 = A(x^2-2x+3)+(Bx+C)x \Rightarrow x-3 = (A+B)x^2+(-2A+C)x+3A$$

De donde:

$$\begin{pmatrix} A+B & & = & 0 \\ -2A & +C & = & 1 \\ 3A & & = & -3 \end{pmatrix} \Rightarrow \begin{cases} A=-1 \\ B=-A \Rightarrow B=1 \\ C=1+2A \Rightarrow C=-1 \end{cases}$$

Luego:

$\frac{x-3}{x^3-2x^2+3x} = -\frac{1}{x}+\frac{x-1}{x^2-2x+3}$, de donde:

$$\int \frac{x-3}{x^3-2x^2+3x}dx = -\int\frac{dx}{x}+\int\frac{x-1}{x^2-2x+3}dx = -\ell\eta|x| + \int\frac{x-1}{x^2-2x+3}dx$$

$$\int \frac{x^4-2x^3+3x^2-x+3}{x^3-2x^2+3x}dx = \int xdx + \ell\eta|x| - \int\frac{x-1}{x^2-2x+3}dx$$

$$= \frac{x^2}{2}+\ell\eta|x| - \int\frac{x-1}{x^2-2x+3}dx = \frac{x^2}{2}+\ell\eta|x| - \frac{1}{2}\int\frac{2(x-1)dx}{x^2-2x+3}$$

Sea: $u = x^2-2x+3, du = (2x-2)dx \Rightarrow du = 2(x-1)dx$

$$= \frac{x^2}{2}+\ell\eta|x| - \frac{1}{2}\int\frac{du}{u} = \frac{x^2}{2}+\ell\eta|x| - \frac{1}{2}\ell\eta\left|x^2-2x+3\right|+c$$

Respuesta: $\int \frac{x^4-2x^3+3x^2-x+3}{x^3-2x^2+3x}dx = \frac{x^2}{2}+\ell\eta\left|\frac{x}{\sqrt{x^2-2x+3}}\right|+c$

EJERCICICOS PROPUESTOS

Usando La técnica de descomposición en fracciones simples parciales, calcular las siguientes integrales:

7.11.- $\int\frac{(x^5+2)dx}{x^2-1}$ **7.12.-** $\int\frac{xdx}{(x+1)^2}$ **7.13.-** $\int\frac{x^3dx}{x^2-2x-3}$

7.14.- $\int\frac{(3x+7)dx}{(x-1)(x-2)(x-3)}$ **7.15.-** $\int\frac{dx}{x^3+1}dx$ **7.16.-** $\int\frac{(x+5)dx}{x^2-x+6}$

7.17.- $\int\frac{(x^2+1)dx}{x^3+1}$ **7.18.-** $\int\frac{(x^2+6)dx}{(x-1)^2(x-2)}$ **7.19.-** $\int\frac{(x^2-1)dx}{(x^2+1)(x-2)}$

7.20.- $\int \frac{xdx}{x^2-4x-5}$

7.21.- $\int \frac{xdx}{x^2-2x-3}$

7.22.- $\int \frac{(x+1)dx}{x^2+4x-5}$

7.23.- $\int \frac{x^2dx}{x^2+2x+1}$

7.24.- $\int \frac{dx}{x(x+1)^2}$

7.25.- $\int \frac{dx}{(x+1)(x^2+1)}$

7.26.- $\int \frac{dx}{x(x^2+x+1)}$

7.27.- $\int \frac{2x^2+5x-1}{x^3+x^2-2x}dx$

7.28.- $\int \frac{(x^2+2x+3)dx}{(x-1)(x+1)^2}$

7.29.- $\int \frac{3x^2+2x-2}{x^3-1}dx$

7.30.- $\int \frac{x^4-x^3+2x^2-x+2}{(x-1)(x^2+2)^2}dx$

7.31.- $\int \frac{(2x^2-7x-1)dx}{x^3+x^2-x-1}$

7.32.- $\int \frac{3x^2+3x+1}{x^3+2x^2+2x+1}dx$

7.33.- $\int \frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^2}dx$

7.34.- $\int \frac{2xdx}{(x^2+x+1)^2}$

7.35.- $\int \frac{x^2+2x+3}{x^3-x}dx$

7.36.- $\int \frac{(2x^2-3x+5)dx}{(x+2)(x-1)(x-3)}$

7.37.- $\int \frac{(3x^2+x-2)dx}{(x-1)(x^2+1)}$

7.38.- $\int \frac{(x+5)dx}{x^3-3x+2}$

7.39.- $\int \frac{2x^3+3x^2+x-1}{(x+1)(x^2+2x+2)^2}dx$

7.40.- $\int \frac{(2x+1)dx}{3x^3+2x-1}$

7.41.- $\int \frac{(2x^2+3x-1)dx}{x^3+2x^2+4x+2}$

7.42.- $\int \frac{x^4-2x^2+3x+4}{(x-1)^3(x^2+2x+2)}dx$

7.43.- $\int \frac{e^t dt}{e^{2t}+3e^t+2}$

7.44.- $\int \frac{\operatorname{sen}\theta d\theta}{\cos^2\theta+\cos\theta-2}$

7.45.- $\int \frac{4x^4-2x^3-x^2+3x+1}{(x^3+x^2-x-1)}dx$

7.46.- $\int \frac{3x^4dx}{(x^2+1)^2}$

7.47.- $\int \frac{(2x^2+41x-91)dx}{x^3-2x^2-11x+12}$

7.48.- $\int \frac{(2x^4+3x^3-x-1)dx}{(x-1)(x^2+2x+2)^2}$

7.49.- $\int \frac{dx}{e^{2x}+e^x-2}$

7.50.- $\int \frac{\operatorname{sen} xdx}{\cos x(1+\cos^2 x)}$

7.51.- $\int \frac{(2+\tau g^2\theta)\sec^2\theta d\theta}{1+\tau g^3\theta}$

7.52.- $\int \frac{(5x^3+2)dx}{x^3-5x^2+4x}$

7.53.- $\int \frac{x^5dx}{(x^3+1)(x^3+8)}$

RESPUESTAS

7.11.- $\int \frac{(x^5+2)dx}{x^2-1}$

Solución.-

$$\int \frac{(x^5+2)dx}{x^2-1} = \int\left(x^3+x+\frac{x+2}{x^2-1}\right)dx = \int x^3dx + \int xdx + \int \frac{x+2}{x^2-1}dx$$

$$= \frac{x^4}{4}+\frac{x^2}{2}+\int \frac{(x+2)dx}{(x+1)(x-1)} \ (*)$$, luego:

$$\frac{x+2}{x^2-1} = \frac{A}{x+1}+\frac{B}{x-1} \Rightarrow x+2 = A(x-1)+B(x+1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 3=2B \Rightarrow B=3/2 \\ x=-1 \Rightarrow 1=-2A \Rightarrow A=-1/2 \end{cases}$$

$$(*) = \frac{x^4}{4}+\frac{x^2}{2}-\frac{1}{2}\int\frac{dx}{x+1}+\frac{3}{2}\int\frac{dx}{x-1} = \frac{x^4}{4}+\frac{x^2}{2}-\frac{1}{2}\ell\eta|x+1|+\frac{3}{2}\ell\eta|x-1|+c$$

$$= \frac{x^4}{4}+\frac{x^2}{2}+\eta\left|\frac{(x-1)^{3/2}}{\sqrt{x+1}}\right|+c$$

7.12.- $\int\frac{xdx}{(x+1)^2}$

Solución.-

$\int\frac{xdx}{(x+1)^2} = \int\frac{Adx}{x+1}+\int\frac{Bdx}{(x+1)^2}$ $(*)$, luego:

$$\frac{x}{(x+1)^2} = \frac{A}{x+1}+\frac{B}{(x+1)^2} \Rightarrow x = A(x+1)+B$$

$$\therefore \begin{cases} x=-1 \Rightarrow -1=B \\ x=0 \Rightarrow 0=A+B \Rightarrow A=-B \Rightarrow A=-1 \end{cases}$$

$$(*)\ \int\frac{dx}{x+1}-\int\frac{dx}{(x+1)^2} = \ell\eta|x+1|+(x+1)^{-1}+c = \ell\eta|x+1|+\frac{1}{x+1}+c$$

7.13.- $\int\frac{x^3dx}{x^2-2x-3}$

Solución.-

$$\int\frac{x^3dx}{x^2-2x-3} = \int\left(x+2+\frac{7x+6}{x^2-2x-3}\right)dx = \int xdx+2\int dx+\int\frac{(7x+6)dx}{x^2-2x-3}$$

$= \frac{x^2}{2}+2x+\int\frac{(7x+6)dx}{(x-3)(x+1)}$ $(*)$, luego:

$$\frac{(7x+6)}{(x-3)(x+1)} = \frac{A}{x-3}+\frac{B}{x+1} \Rightarrow 7x+6 = A(x+1)+B(x-3)$$

$$\therefore \begin{cases} x=3 \Rightarrow 27=4A \Rightarrow A=27/4 \\ x=-1 \Rightarrow -1=-4B \Rightarrow B=1/4 \end{cases}$$

$$(*) = \frac{x^2}{2}+2x+\frac{27}{4}\int\frac{dx}{x-3}+\frac{1}{4}\int\frac{dx}{x+1} = \frac{x^2}{2}+2x+\frac{27}{4}\ell\eta|x-3|+\frac{1}{4}\ell\eta|x+1|+c$$

$$= \frac{x^2}{2}+2x+\frac{1}{4}\ell\eta|(x-3)^{27}(x+1)|+c$$

7.14.- $\int\frac{(3x+7)dx}{(x-1)(x-2)(x-3)}$

Solución.-

$$\int\frac{(3x+7)dx}{(x-1)(x-2)(x-3)} = \int\frac{Adx}{x-1}+\int\frac{Bdx}{x-2}+\int\frac{Cdx}{x-3}\ (*)$$

$$\frac{(3x+7)}{(x-1)(x-2)(x-3)}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$$

$3x-7=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$, luego:

$$\therefore\begin{cases}x=1\Rightarrow -4=2A\Rightarrow A=-2\\ x=2\Rightarrow -1=-B\Rightarrow B=1\\ x=3\Rightarrow 2=2C\Rightarrow C=1\end{cases}$$

$$(*)=-2\int\frac{dx}{x-1}+\int\frac{dx}{x-2}+\int\frac{dx}{x-3}=-2\ell\eta\left|x-1\right|+\ell\eta\left|x-2\right|+\ell\eta\left|x-3\right|+c$$

$$=\ell\eta\left|\frac{(x-2)(x-3)}{(x-1)^2}\right|+c$$

7.15.- $\int\frac{dx}{x^3+1}dx$

Solución.-

$\int\frac{dx}{x^3+1}dx=\int\frac{dx}{(x+1)(x^2-x+1)}=\int\frac{Adx}{x+1}+\int\frac{(Bx+C)dx}{(x^2-x+1)}$ $(*)$, luego:

$$\frac{1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{(Bx+C)}{(x^2-x+1)}\Rightarrow 1=A(x^2-x+1)+(Bx+C)(x+1)$$

$$\therefore\begin{cases}x=-1\Rightarrow 1=3A\Rightarrow A=1/3\\ x=0\Rightarrow 1=A+C\Rightarrow C=1-A\Rightarrow C=2/3\\ x=1\Rightarrow 1=A+(B+C)2\Rightarrow 1=1/3+2B+2C\Rightarrow 1/3=B+C\Rightarrow B=1/3-C\end{cases}$$

$$\Rightarrow B=-1/3$$

$$(*)=\frac{1}{3}\int\frac{dx}{x+1}+\int\frac{(-1/3\,x+2/3)dx}{(x^2-x+1)}=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{3}\int\frac{(x-2)dx}{x^2-x+1}$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\int\frac{(2x-4)dx}{x^2-x+1}=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\int\frac{(2x-1-3)dx}{x^2-x+1}$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\int\frac{(2x-1)dx}{x^2-x+1}+\frac{1}{2}\int\frac{dx}{x^2-x+1}$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\ell\eta\left|x^2-x+1\right|+\frac{1}{2}\int\frac{dx}{(x^2-x+1/4)+3/4}$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\ell\eta\left|x^2-x+1\right|+\frac{1}{2}\int\frac{dx}{(x-1/2)^2+(\sqrt{3}/2)^2}$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\ell\eta\left|x^2-x+1\right|+\frac{1}{2}\frac{1}{\sqrt{3}/2}\operatorname{arc}\tau g\,\frac{x-1/2}{\sqrt{3}/2}+c$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\ell\eta\left|x^2-x+1\right|+\frac{\sqrt{3}}{3}\operatorname{arc}\tau g\,\frac{2x-1}{\sqrt{3}}+c$$

$$= \ell\eta\left|\frac{\sqrt[3]{x+1}}{\sqrt[6]{x^2-x+1}}\right| + \frac{\sqrt{3}}{3}\operatorname{arc}\tau g\,\frac{2x-1}{\sqrt{3}} + c$$

7.16.- $\int \frac{(x+5)dx}{x^2-x+6}$

Solución.-

$\int \frac{(x+5)dx}{x^2-x+6} = \int \frac{(x+5)dx}{(x+3)(x-2)} = \int \frac{Adx}{(x+3)} + \int \frac{Bdx}{(x-2)}$ $(*)$, luego:

$$\frac{(x+5)}{(x^2+x-6)} = \frac{A}{(x+3)} + \frac{B}{(x-2)} \Rightarrow x+5 = A(x-2) + B(x+3)$$

$$\therefore \begin{cases} x = 2 \Rightarrow 7 = 5B \Rightarrow B = 7/5 \\ x = -3 \Rightarrow 2 = -5A \Rightarrow A = -2/5 \end{cases}$$

$$(*) = -\frac{2}{5}\int \frac{dx}{x+3} + \frac{7}{5}\int \frac{dx}{x-2} = -\frac{2}{5}\ell\eta|x+3| + \frac{2}{5}\ell\eta|x-2| + c = \frac{1}{5}\ell\eta\left|\frac{(x-2)^7}{(x+3)^2}\right| + c$$

7.17.- $\int \frac{(x^2+1)dx}{x^3+1}$

Solución.-

$\int \frac{(x^2+1)dx}{x^3+1} = \int \frac{(x^2+1)dx}{(x+1)(x^2-x+1)} = \int \frac{Adx}{(x+1)} + \int \frac{(Bx+C)dx}{(x^2-x+1)}$ $(*)$, luego:

$$\frac{(x^2+1)}{x^3+1} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2-x+1)} \Rightarrow x^2+1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$\therefore \begin{cases} x = -1 \Rightarrow 2 = 3A \Rightarrow A = 2/3 \\ x = 0 \Rightarrow 1 = A + C \Rightarrow C = 1/3 \\ x = 1 \Rightarrow 2 = A + (B+C)2 \Rightarrow B = 1/3 \end{cases}$$

$$(*)\ \int \frac{(x^2+1)dx}{x^3+1} = \int \frac{(x^2+1)dx}{(x+1)(x^2-x+1)} = \frac{2}{3}\int \frac{dx}{(x+1)} + \frac{1}{3}\int \frac{(x+1)dx}{(x^2-x+1)}$$

$$= \frac{2}{3}\ell\eta|x+1| + \frac{1}{3}\int \frac{\left[1/2\,(2x-1) + 2/3\right]dx}{(x^2-x+1)} = \frac{2}{3}\ell\eta|x+1| + \frac{1}{6}\int \frac{(2x-1)dx}{(x^2-x+1)} + \frac{1}{2}\int \frac{dx}{(x^2-x+1)}$$

$$= \frac{2}{3}\ell\eta|x+1| + \frac{1}{6}\ell\eta|x^2-x+1| + \frac{1}{2}\int \frac{dx}{(x^2-x+1)}$$

$$= \frac{2}{3}\ell\eta|x+1| + \frac{1}{6}\ell\eta|x^2-x+1| + \frac{1}{2}\int \frac{dx}{(x^2-x+1/4) + 3/4}$$

$$= \frac{4}{6}\ell\eta|x+1| + \frac{1}{6}\ell\eta|x^2-x+1| + \frac{1}{2}\int \frac{dx}{(x-1/2)^2 + (\sqrt{3}/2)^2}$$

$$=\frac{1}{6}\ell\eta\left|(x+1)^4(x^2-x+1)\right|+\frac{1}{2}\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arc}\tau g\,\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}+c$$

$$=\frac{1}{6}\ell\eta\left|(x+1)^4(x^2-x+1)\right|+\frac{\sqrt{3}}{3}\operatorname{arc}\tau g\,\frac{2x-1}{\sqrt{3}}+c$$

7.18.- $\int\frac{(x^2+6)dx}{(x-1)^2(x-2)}$

Solución.-

$\int\frac{(x^2+6)dx}{(x-1)^2(x-2)}=\int\frac{Adx}{(x+1)}+\int\frac{Bdx}{(x-1)^2}+\int\frac{Cdx}{(x+2)}$ $(*)$, luego:

$$\frac{(x^2+6)}{(x-1)^2(x-2)}=\frac{A}{(x+1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+2)}$$

$$x^2+6=A(x+1)+(x+2)+B(x+2)+C(x-1)^2$$

$$\therefore\begin{cases}x=1\Rightarrow 7=3B\Rightarrow B=\frac{7}{3}\\ x=-2\Rightarrow 10=9C\Rightarrow C=\frac{10}{9}\\ x=0\Rightarrow 6=-2A+B+C\Rightarrow A=-\frac{1}{9}\end{cases}$$

$$(*)=-\frac{1}{9}\int\frac{dx}{(x+1)}+\frac{7}{3}\int\frac{dx}{(x-1)^2}+\frac{10}{9}\int\frac{dx}{(x+2)}=-\frac{1}{9}\ell\eta|x-1|-\frac{7}{3}\frac{1}{x-1}+\frac{10}{9}\ell\eta|x+2|+c$$

$$=\frac{1}{9}\ell\eta\left|\frac{(x+2)^{10}}{x-1}\right|-\frac{7}{3(x-1)}+c$$

7.19.- $\int\frac{(x^2-1)dx}{(x^2+1)(x-2)}$

Solución.-

$\int\frac{(x^2-1)dx}{(x^2+1)(x-2)}=\int\frac{Ax+B}{(x^2+1)}dx+\int\frac{Cdx}{(x-2)}$ $(*)$, luego:

$$\frac{(x^2-1)}{(x^2+1)(x-2)}=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x-2)}\Rightarrow x^2-1=(Ax+B)(x-2)+C(x^2+1)$$

$$\therefore\begin{cases}x=2\Rightarrow 3=5C\Rightarrow C=\frac{3}{5}\\ x=0\Rightarrow -1=-2B+C\Rightarrow B=\frac{4}{5}\\ x=1\Rightarrow 0=-(A+B)+2C\Rightarrow A=\frac{2}{5}\end{cases}$$

$$(*)=\int\frac{(\frac{2}{5}x+\frac{4}{5})dx}{(x^2+1)}+\int\frac{\frac{3}{5}dx}{(x-2)}=\frac{1}{5}\int\frac{2xdx}{(x^2+1)}+\frac{4}{5}\int\frac{dx}{(x^2+1)}+\frac{3}{5}\int\frac{dx}{x-2}$$

$$=\frac{1}{5}\ell\eta\left|x^2+1\right|+\frac{4}{5}\operatorname{arc}x+\frac{3}{5}\ell\eta|x-2|+c=\frac{1}{5}\ell\eta\left|(x^2+1)(x-2)^3\right|+\frac{4}{5}\operatorname{arc}x+c$$

7.20.- $\int \frac{xdx}{x^2-4x-5}$

Solución.-

$$\int \frac{xdx}{x^2-4x-5} = \int \frac{xdx}{(x+5)(x-1)} = \int \frac{Adx}{(x+5)} + \int \frac{Bdx}{(x-1)}$$ $(*)$, luego:

$$\frac{x}{(x+5)(x-1)} = \frac{A}{(x+5)} + \frac{B}{(x-1)} \Rightarrow x = A(x-1) + B(x+5)$$

$$\therefore \begin{cases} x = 1 \Rightarrow 1 = 6B \Rightarrow B = 1/6 \\ x = -5 \Rightarrow -5 = -6A \Rightarrow A = 5/6 \end{cases}$$

$$(*) = \frac{5}{6}\int \frac{dx}{(x+5)} + \frac{1}{6}\int \frac{dx}{(x-1)} = \frac{5}{6}\ell\eta|x+5| + \frac{1}{6}\ell\eta|x-1| + c = \frac{5}{6}\ell\eta\left|(x+5)^5(x-1)\right| + c$$

7.21.- $\int \frac{xdx}{x^2-2x-3}$

Solución.-

$$\int \frac{xdx}{x^2-2x-3} = \int \frac{xdx}{(x-3)(x+1)} = \int \frac{Adx}{(x-3)} + \int \frac{Bdx}{(x+1)}$$ $(*)$, luego:

$$\frac{x}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)} \Rightarrow x = A(x+1) + B(x-3)$$

$$\therefore \begin{cases} x = -1 \Rightarrow -1 = -4B \Rightarrow B = 1/4 \\ x = 3 \Rightarrow 3 = 4A \Rightarrow A = 3/4 \end{cases}$$

$$(*) = \frac{3}{4}\int \frac{dx}{(x-3)} + \frac{1}{4}\int \frac{B}{(x+1)} = \frac{3}{4}\ell\eta|x-3| + \frac{1}{4}\ell\eta|x+1| + c = \frac{1}{4}\ell\eta\left|(x-3)^3(x+1)\right| + c$$

7.22.- $\int \frac{(x+1)dx}{x^2+4x-5}$

Solución.-

$$\int \frac{(x+1)dx}{x^2+4x-5} = \int \frac{(x+1)dx}{(x+5)(x-1)} = \int \frac{Adx}{(x+5)} + \int \frac{Bdx}{(x-1)}$$ $(*)$, luego:

$$\frac{x+1}{(x^2+4x-5)} = \frac{A}{(x+5)} + \frac{B}{(x-1)} \Rightarrow x+1 = A(x-1) + B(x+5)$$

$$\therefore \begin{cases} x = 1 \Rightarrow 2 = 6B \Rightarrow B = 1/3 \\ x = -5 \Rightarrow 3 = -4A \Rightarrow -6A = 2/3 \end{cases}$$

$$(*) = \frac{2}{3}\int \frac{dx}{(x+5)} + \frac{1}{3}\int \frac{B}{(x-1)} = \frac{2}{3}\ell\eta|x+5| + \frac{1}{3}\ell\eta|x-1| + c = \frac{1}{3}\ell\eta\left|(x+5)^2(x-1)\right| + c$$

7.23.- $\int \frac{x^2dx}{x^2+2x+1}$

Solución.-

$$\int\frac{x^2dx}{x^2+2x+1}=\int\left(1-\frac{2x+1}{x^2+2x+1}\right)dx=\int dx-\int\frac{(2x+1)dx}{x^2+2x+1}=\int dx-\int\frac{(2x+1)dx}{(x+1)^2}$$

$= x-\left[\int\frac{Adx}{(x+1)}+\int\frac{Bdx}{(x+1)^2}\right]$ $(*)$, luego:

$$\frac{2x+1}{(x+1)^2}=\frac{A}{(x+1)}+\frac{B}{(x+1)^2}\Rightarrow 2x+1=A(x+1)+B$$

$$\therefore\begin{cases}x=-1\Rightarrow -1=B\Rightarrow B=-1\\ x=0\Rightarrow 1=A+B\Rightarrow A=2\end{cases}$$

$$(*)=x-\left[2\int\frac{dx}{(x+1)}-\int\frac{dx}{(x+1)^2}\right]=x-\left[2\ell\eta|x+1|+\frac{1}{x+5}\right]+c=x-2\ell\eta|x+1|-\frac{1}{x+5}+c$$

7.24.- $\int\frac{dx}{x(x+1)^2}$

Solución.-

$\int\frac{dx}{x(x+1)^2}=\int\frac{Adx}{x}+\int\frac{Bdx}{(x+1)}+\int\frac{Cdx}{(x+1)^2}$ $(*)$, luego:

$$\frac{1}{x(x+1)^2}=\frac{A}{x}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}\Rightarrow 1=A(x+1)^2+Bx(x+1)+Cx$$

$$\therefore\begin{cases}x=-1\Rightarrow 1=-C\Rightarrow C=-1\\ x=0\Rightarrow 1=A\Rightarrow A=1\\ x=1\Rightarrow 1=4A+2B+C\Rightarrow B=-1\end{cases}$$

$$(*)=\int\frac{dx}{x}-\int\frac{dx}{(x+1)}-\int\frac{dx}{(x+1)^2}=\ell\eta|x|-\ell\eta|x+1|+\frac{1}{x+1}+c=\ell\eta\left|\frac{x}{x+1}\right|+\frac{1}{x+1}+c$$

7.25.- $\int\frac{dx}{(x+1)(x^2+1)}$

Solución.-

$\int\frac{dx}{(x+1)(x+1)^2}=\int\frac{Adx}{x+1}+\int\frac{Bx+C}{(x^2+1)}dx$ $(*)$, luego:

$$\frac{1}{(x+1)(x^2+1)}=\frac{A}{x+1}+\frac{Bx+C}{(x^2+1)}\Rightarrow 1=A(x^2+1)+(Bx+C)(x+1)$$

$$\therefore\begin{cases}x=-1\Rightarrow 1=2A\Rightarrow A=1/2\\ x=0\Rightarrow 1=A+C\Rightarrow C=1/2\\ x=1\Rightarrow 1=2A+(B+C)2\Rightarrow B=-1/2\end{cases}$$

$$(*)\quad =\frac{1}{2}\int\frac{dx}{(x+1)}+\int\frac{(-1/2\,x+1/2)dx}{(x^2+1)}=\frac{1}{2}\ell\eta|x+1|-\frac{1}{2}\int\frac{x-1}{(x^2+1)}dx$$

$$=\frac{1}{2}\ell\eta|x+1|-\frac{1}{4}\int\frac{2xdx}{(x^2+1)}+\frac{1}{2}\int\frac{dx}{(x^2+1)}=\frac{1}{2}\ell\eta|x+1|-\frac{1}{4}\ell\eta|x^2+1|+\frac{1}{2}\operatorname{arc}\tau gx+c$$

$$= \frac{1}{4}\ell\eta\left|\frac{(x+1)^2}{x^2+1}\right| + \frac{1}{2}\operatorname{arc}\tau g x + c$$

7.26.- $\int \frac{dx}{x(x^2+x+1)}$

Solución.-

$\int \frac{dx}{x(x^2+x+1)} = \int \frac{Adx}{x} + \int \frac{Bx+C}{(x^2+x+1)}dx \ (*)$, luego:

$$\frac{1}{x(x^2+x+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+x+1)} \Rightarrow 1 = A(x^2+x+1) + (Bx+C)x$$

$$\therefore \begin{cases} x=0 \Rightarrow 1 = A \Rightarrow A = 1 \\ x=1 \Rightarrow 1 = 3A+B+C \Rightarrow B+C = -2 \\ x=-1 \Rightarrow 1 = A+B-C \Rightarrow B-C = 0 \end{cases}$$

$$(*) \quad = \int \frac{dx}{x} - \int \frac{(x+1)dx}{(x^2+x+1)} = \ell\eta|x+1| - \frac{1}{2}\int \frac{(2x+2)dx}{(x^2+x+1)}$$

$$= \ell\eta|x| - \frac{1}{2}\int \frac{(2x+1)+1}{(x^2+x+1)}dx = \ell\eta|x| - \frac{1}{2}\int \frac{(2x+1)dx}{(x^2+x+1)} - \frac{1}{2}\int \frac{dx}{(x^2+x+1)}$$

$$= \ell\eta|x| - \frac{1}{2}\ell\eta|x^2+x+1| - \frac{1}{2}\int \frac{dx}{(x^2+x+\frac{1}{4})+\frac{3}{4}}$$

$$= \ell\eta|x| - \frac{1}{2}\ell\eta|x^2+x+1| - \frac{1}{2}\int \frac{dx}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

$$= \ell\eta|x| - \frac{1}{2}\ell\eta|x^2+x+1| - \frac{1}{2}\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arc}\tau g\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}} + c$$

$$= \ell\eta|x| - \frac{1}{2}\ell\eta|x^2+x+1| - \frac{\sqrt{3}}{3}\operatorname{arc}\tau g\frac{2x+1}{\sqrt{3}} + c$$

7.27.- $\int \frac{2x^2+5x-1}{x^3+x^2-2x}dx$

Solución.-

$\int \frac{(2x^2+5x-1)dx}{(x^3+x^2-2x)} = \int \frac{Adx}{x} + \int \frac{Bdx}{(x-1)} + \int \frac{Cdx}{(x+2)} \ (*)$, luego:

$$\frac{2x^2+5x-1}{(x^3+x^2-2x)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x+2)}$$

$$2x^2+5x-1 = A(x-1)(x+2) + Bx(x+2) + Cx(x-1)$$

$$\therefore \begin{cases} x=0 \Rightarrow -1 = -2A \Rightarrow A = \frac{1}{2} \\ x=1 \Rightarrow 6 = 3B \Rightarrow B = 2 \\ x=-2 \Rightarrow -3 = 6C \Rightarrow C = -\frac{1}{2} \end{cases}$$

$$(*) = \frac{1}{2}\int \frac{dx}{x} + 2\int \frac{dx}{(x-1)} - \frac{1}{2}\int \frac{dx}{(x+2)} = \frac{1}{2}\ell\eta|x| + 2\ell\eta|x-1| - \frac{1}{2}\ell\eta|x+2| + c$$

7.28.- $\int \frac{x^2+2x+3}{(x-1)(x+1)^2}dx$

Solución.-

$\int \frac{x^2+2x+3}{(x-1)(x+1)^2}dx = \int \frac{Adx}{(x-1)} + \int \frac{Bdx}{(x+1)} + \int \frac{Cdx}{(x+1)^2}$ $(*)$, luego:

$$\frac{x^2+2x+3}{(x-1)(x+1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)} + \frac{C}{(x+1)^2}$$

$$x^2+2x+3 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 6 = 4A \Rightarrow A = 3/2 \\ x=-1 \Rightarrow 2 = -2C \Rightarrow C = -1 \\ x=0 \Rightarrow 3 = A-B-C \Rightarrow B = -1/2 \end{cases}$$

$$(*) = \frac{3}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1} - \int \frac{dx}{(x+1)^2} = \frac{3}{2}\ell\eta|x-1| - \frac{1}{2}\ell\eta|x+1| + \frac{1}{x+1} + c$$

$$= \frac{1}{2}\ell\eta\left|\frac{(x-1)^3}{x+1}\right| + \frac{1}{x+1} + c$$

7.29.- $\int \frac{3x^2+2x-2}{x^3-1}dx$

Solución.-

$\int \frac{3x^2+2x-2}{x^3-1}dx = \int \frac{3x^2+2x-2}{(x-1)(x^2+x+1)}dx = \int \frac{Adx}{x-1} + \int \frac{(Bx+C)dx}{(x^2+x+1)}$ $(*)$, luego:

$$\frac{3x^2+2x-2}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+x+1)}$$

$$3x^2+2x-2 = A(x^2+x+1) + (Bx+C)(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 3 = 3A \Rightarrow A = 1 \\ x=0 \Rightarrow -2 = A-C \Rightarrow C = 3 \\ x=-1 \Rightarrow -1 = A + (-B+C)(-2) \Rightarrow B = 2 \end{cases}$$

$$(*) = \int \frac{dx}{x-1} + \int \frac{(2x+3)dx}{(x^2+x+1)} = \ell\eta|x-1| + \int \frac{(2x+1)+2}{(x^2+x+1)}dx$$

$$= \ell\eta|x-1| + \int \frac{(2x+1)dx}{(x^2+x+1)} + 2\int \frac{dx}{(x^2+x+1)}$$

$$= \ell\eta|x-1| + \ell\eta|x^2+x+1| + 2\int \frac{dx}{(x+1/2)^2 + (\sqrt{3}/2)^2}$$

$$= \ell\eta\left|(x-1)(x^2+x+1)\right| + 2\frac{1}{\sqrt{3}/2}\operatorname{arc}\tau g\frac{x+1/2}{\sqrt{3}} + c$$

$$= \ell\eta\left|(x-1)(x^2+x+1)\right| + \frac{4\sqrt{3}}{3}\operatorname{arc}\tau g\frac{2x+1}{\sqrt{3}} + c$$

7.30.- $\int \frac{x^4-x^3+2x^2-x+2}{(x-1)(x^2+2)^2}dx$

Solución.-

$\int \frac{x^4-x^3+2x^2-x+2}{(x-1)(x^2+2)^2}dx = \int \frac{Adx}{x-1} + \int \frac{(Bx+C)dx}{(x^2+2)} + \int \frac{(Dx+E)dx}{(x^2+2)^2}$ $(*)$, luego:

$$\frac{x^4-x^3+2x^2-x+2}{(x-1)(x^2+2)^2} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+2)} + \frac{Dx+E}{(x^2+2)^2}$$

$$x^4-x^3+2x^2-x+2 = A(x^2+2)^2 + (Bx+C)(x-1)(x^2+2) + (Dx+E)(x-1)$$

$$= A(x^4+4x^2+4) + (Bx+C)(x^3+2x-x^2-2) + Dx^2 - Dx + Ex - E$$

$$= Ax^4 + 4Ax^2 + 4A + Bx^4 + 2Bx^2 - Bx^3 - 2Bx + Cx^3 + 2Cx - Cx^2 - 2C$$

$$\Rightarrow +Dx^2 - Dx + Ex - E$$

$$= (A+B)x^4 + (C-B)x^3 + (4A-C+2B+D)x^2 + (-2B+2C-D+E)x + (4A-2C-E)$$

Igualando coeficientes, se tiene:

$$\begin{pmatrix} A+B & & & & =1 \\ -B+C & & & & =-1 \\ 4A+2B-C+D & & & & =2 \\ -2B+2C-D+E & & & & =-1 \\ 4A-2C-E & & & & =2 \end{pmatrix} \therefore A = 1/3, B = 2/3, C = -1/3, D = -1, E = 0$$

$$(*) = \frac{1}{3}\int\frac{dx}{x-1} + \int\frac{(2/3\,x - 1/3)dx}{(x^2+2)} - \int\frac{xdx}{(x^2+2)^2}$$

$$= \frac{1}{3}\int\frac{dx}{x-1} + \frac{1}{3}\int\frac{2xdx}{(x^2+2)} - \frac{1}{3}\int\frac{dx}{(x^2+2)} - \frac{1}{2}\int\frac{2xdx}{(x^2+2)^2}$$

$$= \frac{1}{3}\ell\eta|x-1| + \frac{1}{3}\ell\eta\left|x^2+2\right| - \frac{\sqrt{2}}{6}\operatorname{arc}\tau g\frac{x}{\sqrt{2}} + \frac{1}{2}\frac{1}{x^2+2} + c$$

$$= \frac{1}{3}\ell\eta\left|(x-1)(x^2+2)\right| - \frac{\sqrt{2}}{6}\operatorname{arc}\tau g\frac{x}{\sqrt{2}} + \frac{1}{2(x^2+2)} + c$$

7.31.- $\int \frac{2x^2-7x-1}{x^3+x^2-x-1}dx$

Solución.-

$\int \frac{2x^2-7x-1}{x^3+x^2-x-1}dx = \int \frac{2x^2-7x-1}{(x-1)(x+1)^2}dx = \int \frac{Adx}{x-1} + \int \frac{Bdx}{(x+1)} + \int \frac{Cdx}{(x+1)^2}$ $(*)$, luego:

$$\frac{2x^2-7x-1}{(x^3+x^2-x-1)}=\frac{A}{x-1}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$$

$$2x^2-7x-1=A(x+1)^2+B(x-1)(x+1)+C(x-1)$$

$$\therefore\begin{cases} x=-1\Rightarrow 8=-2C\Rightarrow C=-4 \\ x=1\Rightarrow -6=4A\Rightarrow A=-3/2 \\ x=0\Rightarrow -1=A-B-C\Rightarrow B=7/2 \end{cases}$$

$$(*)=-\frac{3}{2}\int\frac{dx}{x-1}+\frac{7}{2}\int\frac{dx}{x+1}-4\int\frac{dx}{(x+1)^2}=-\frac{3}{2}\ell\eta\left|x-1\right|+\frac{7}{2}\ell\eta\left|x+1\right|+\frac{4}{x+1}+c$$

$$=-\frac{1}{2}\ell\eta\left|\frac{(x+1)^7}{(x-1)^3}\right|+\frac{4}{x+1}+c$$

7.32.- $\int\frac{3x^2+3x+1}{x^3+2x^2+2x+1}dx$

Solución.-

$\int\frac{3x^2+3x+1}{x^3+2x^2+2x+1}dx=\int\frac{(3x^2+3x+1)dx}{(x+1)(x^2+x+1)}=\int\frac{Adx}{x+1}+\int\frac{(Bx+C)dx}{(x^2+x+1)}$ $(*)$, luego:

$$\frac{3x^2+3x+1}{(x+1)(x^2+x+1)}=\frac{A}{x+1}+\frac{Bx+C}{(x^2+x+1)}$$

$$3x^2+3x+1=A(x^2+x+1)+(Bx+C)(x+1)$$

$$\therefore\begin{cases} x=-1\Rightarrow A=1 \\ x=0\Rightarrow 1=A+C\Rightarrow C=0 \\ x=1\Rightarrow 7=3A+(B+C)(2)\Rightarrow B=2 \end{cases}$$

$$(*)=\int\frac{dx}{x+1}+\int\frac{2xdx}{(x^2+x+1)}=\ell\eta\left|x+1\right|+\int\frac{(2x+1)-1}{(x^2+x+1)}dx$$

$$=\ell\eta\left|x+1\right|+\int\frac{(2x+1)dx}{(x^2+x+1)}-\int\frac{dx}{(x^2+x+1)}$$

$$=\ell\eta\left|x+1\right|+\ell\eta\left|x^2+x+1\right|-\int\frac{dx}{(x^2+x+1/4)+(\sqrt{3}/2)^2}$$

$$=\ell\eta\left|x+1\right|+\ell\eta\left|x^2+x+1\right|-\frac{1}{\sqrt{3}/2}\operatorname{arc}\tau g\,\frac{x+1/2}{\sqrt{3}/2}+c$$

$$=\ell\eta\left|(x+1)(x^2+x+1)\right|-\frac{2\sqrt{3}}{3}\operatorname{arc}\tau g\,\frac{2x+1}{\sqrt{3}}+c$$

7.33.- $\int\frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^2}dx$

Solución.-

$$\int \frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^3}dx = \int \frac{Adx}{x-1} + \int \frac{Bdx}{(x-1)^2} + \int \frac{Cdx}{(x+1)} + \int \frac{Ddx}{(x+1)^2} + \int \frac{Edx}{(x+1)^3}$$ $(*)$, luego:

$$\frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{E}{(x+1)^3}$$

$$x^3+7x^2-5x+5 = A(x-1)(x+1)^3 + B(x+1)^3 + C(x-1)^2(x+1)^2$$

$$\Rightarrow +D(x-1)^2(x+1) + E(x-1)^2$$

$$= Ax^4 + 2Ax^3 - 2Ax - A + Bx^3 + 3Bx^2 + 3Bx + B + Cx^4 - 2Cx^2 + C$$

$$\Rightarrow +Dx^3 - Dx^2 - Dx + D + Ex^2 - 2Ex + E$$

$$= (A+C)x^4 + (2A+B+D)x^3 + (3B-2C-D+E)x^2$$

$$\Rightarrow +(-2A+3B-D-2E)x + (-A+B+C+D+E)$$

Igualando coeficientes, se tiene:

$$\begin{pmatrix} A & & +C & & & = 0 \\ 2A & + B & & +D & & = 1 \\ & +3B & -2C & -D & +E & = 7 \\ -2A & +3B & & -D & -2E & = -5 \\ -A & +B & +C & +D & +E & = 2 \end{pmatrix} \therefore A=0, B=1, C=0, D=0, E=4$$

$$(*) = \int \frac{dx}{(x-1)^2} + 4\int \frac{dx}{(x+1)^3} = -\frac{1}{x-1} - \frac{2}{(x+1)^2} + c = -\frac{x^2-4x-1}{(x-1)(x+1)^2} + c$$

7.34.- $\int \frac{2xdx}{(x^2+x+1)^2}$

Solución.-

$$\int \frac{2xdx}{(x^2+x+1)^2} = \int \frac{(Ax+B)dx}{x^2+x+1} + \int \frac{(Cx+D)dx}{(x^2+x+1)^2}$$ $(*)$, luego:

$$\frac{2x}{(x^2+x+1)^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2}$$

$$2x = (Ax+B)(x^2+x+1) + Cx + D \Rightarrow 2x = Ax^3 + Ax^2 + Ax + Bx^2 + Bx + B + Cx + D$$

$= Ax^3 + (A+B)x^2 + (A+B+C)x + B + D$, igualando coeficientes se tiene:

$$\begin{pmatrix} A & & & & = 0 \\ A+B & & & & = 0 \\ A+B & +C & & & = 2 \\ & & & +D & = 0 \end{pmatrix}$$

$\therefore A=0, B=0, C=2, D=0$

$(*) = \int \frac{2xdx}{(x^2+x+1)}$, de donde el método sugerido pierde aplicabilidad; tal como se había planteado la técnica trabajada debe ser sustituida por otra:

$$\int \frac{2xdx}{(x^2+x+1)} = \int \frac{(2x+1)dx}{(x^2+x+1)} - \int \frac{dx}{(x^2+x+1)^2}$$

$$= \int \frac{(2x+1)dx}{(x^2+x+1)} - \frac{16}{9}\int \frac{dx}{\left\{\left[\frac{2}{\sqrt{3}}(x+\frac{1}{2})\right]^2+1\right\}}(**)$$

sea: $u = \frac{2}{\sqrt{3}}(x+\frac{1}{2}), dx = \frac{\sqrt{3}}{2}du$, entonces:

$(**) -\frac{1}{x^2+x+1} - \frac{16}{9}\frac{\sqrt{3}}{2}\int \frac{du}{(u^2+1)^2}$, trabajando la integral sustituyendo trigonométricamente:

$= -\frac{1}{x^2+x+1} - \frac{8\sqrt{3}}{9}\int \frac{\sec^2\theta d\theta}{\sec^4\theta}$, ya que: $u = \tau g\theta, du = \sec^2\theta d\theta$

$$= -\frac{1}{x^2+x+1} - \frac{8\sqrt{3}}{9}\left[\frac{1}{2}\operatorname{arc}\tau g u + \frac{1}{2}\frac{u}{(u^2+1)}\right]$$

$$= -\frac{1}{x^2+x+1} - \frac{8\sqrt{3}}{9}\left\{\frac{1}{2}\operatorname{arc}\tau g \frac{2}{\sqrt{3}}(x+\frac{1}{2}) + \frac{\frac{2}{\sqrt{3}}(x+\frac{1}{2})}{2\left[\frac{4}{3}(x+\frac{1}{2})^2+1\right]}\right\}+c$$

$$= -\frac{1}{x^2+x+1} - \frac{8\sqrt{3}}{9}\left\{\frac{1}{2}\operatorname{arc}\tau g \frac{2}{\sqrt{3}}(x+\frac{1}{2}) + \frac{x+\frac{1}{2}}{\sqrt{3}\left[\frac{4}{3}(x+\frac{1}{2})^2+1\right]}\right\}+c$$

$$= -\frac{1}{x^2+x+1} - \frac{4\sqrt{3}}{9}\operatorname{arc}\tau g \frac{2}{\sqrt{3}}(x+\frac{1}{2}) - \frac{8}{9}\frac{(x+\frac{1}{2})}{\left[\frac{4}{3}(x+\frac{1}{2})^2+1\right]}+c$$

7.35.- $\int \frac{x^2+2x+3}{x^3-x}dx$

Solución.-

$\int \frac{x^2+2x+3}{x^3-x}dx = \int \frac{x^2+2x+3}{x(x-1)(x+1)}dx = \int \frac{Adx}{x} + \int \frac{Bdx}{(x-1)} + \int \frac{Cdx}{(x+1)}$ $(*)$, luego:

$$\frac{x^2+2x+3}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x+1)}$$

$$x^2+2x+3 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$$

$$\therefore \begin{cases} x=0 \Rightarrow 3=-A \Rightarrow A=-3 \\ x=-1 \Rightarrow 2=2C \Rightarrow C=1 \\ x=1 \Rightarrow 6=2B \Rightarrow B=3 \end{cases}$$

$$(*) = -3\int \frac{dx}{x} + 3\int \frac{dx}{(x-1)} + \int \frac{dx}{(x+1)} = -3\ell\eta|x| + 3\ell\eta|x-1| + \ell\eta|x+1| + c$$

$$= \ell\eta\left|\frac{(x-1)^3(x+1)}{x^3}\right| + c$$

7.36.- $\int \frac{(2x^2-3x+5)dx}{(x+2)(x-1)(x-3)}$

Solución.-

$\int \frac{2x^2-3x+5}{(x+2)(x-1)(x-3)}dx = \int \frac{Adx}{(x+2)} + \int \frac{Bdx}{(x-1)} + \int \frac{Cdx}{(x-3)}$ $(*)$, luego:

$$\frac{2x^2-3x+5}{(x+2)(x-1)(x-3)} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{x-3}$$

$$2x^2-3x+5 = A(x-1)(x-3) + B(x+2)(x-3) + C(x+2)(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 4=-6B \Rightarrow B=-2/3 \\ x=3 \Rightarrow 14=10C \Rightarrow C=7/5 \\ x=-2 \Rightarrow 19=15A \Rightarrow A=19/15 \end{cases}$$

$$(*) = \frac{19}{15}\int \frac{dx}{x+2} - \frac{2}{3}\int \frac{dx}{x-1} + \frac{7}{5}\int \frac{dx}{x-3} = \frac{19}{15}\ell\eta\left|x+2\right| - \frac{2}{3}\ell\eta\left|x-1\right| + \frac{7}{5}\ell\eta\left|x-3\right| + c$$

7.37.- $\int \frac{3x^2+x-2}{(x-1)(x^2+1)}dx$

Solución.-

$\int \frac{3x^2+x-2}{(x-1)(x^2+1)}dx = \int \frac{Adx}{(x-1)} + \int \frac{(Bx+C)dx}{(x^2+1)}$ $(*)$, luego:

$$\frac{3x^2+x-2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$3x^2+x-2 = A(x^2+1) + (Bx+C)(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 2=2A \Rightarrow A=1 \\ x=0 \Rightarrow -2=A-C \Rightarrow C=3 \\ x=2 \Rightarrow 12=5A+2B+C \Rightarrow B=2 \end{cases}$$

$$(*) = \int \frac{dx}{x-1} + \int \frac{(2x+3)dx}{x^2+1} = \int \frac{dx}{x-1} + \int \frac{2xdx}{x^2+1} + 3\int \frac{dx}{x^2+1}$$

$$= \ell\eta\left|x-1\right| + \ell\eta\left|x^2+1\right| + 3\,\mathrm{arc}\,\tau gx + c = \ell\eta\left|(x-1)(x^2+1)\right| + 3\,\mathrm{arc}\,\tau gx + c$$

7.38.- $\int \frac{(x+5)dx}{x^3-3x+2}$

Solución.-

$\int \frac{(x+5)dx}{x^3-3x+2} = \int \frac{(x+5)dx}{(x-1)^2(x+2)} = \int \frac{Adx}{(x-1)} + \int \frac{Bdx}{(x-1)^2} + \int \frac{Cdx}{(x+2)}$ $(*)$, luego:

$$\frac{x+5}{x^3-3x+2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$x+5 = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

$$\therefore \begin{cases} x=1 \Rightarrow 6=3B \Rightarrow B=2 \\ x=-2 \Rightarrow 3=9C \Rightarrow C=1/3 \\ x=0 \Rightarrow 5=-2A+B+C \Rightarrow A=-1/3 \end{cases}$$

$$(*) = -\frac{1}{3}\int\frac{dx}{(x-1)}+2\int\frac{dx}{(x-1)^2}+\frac{1}{3}\int\frac{dx}{(x+2)} = -\frac{1}{3}\ell\eta|x-1|-\frac{2}{x-1}+\frac{1}{3}\ell\eta|x+2|+c$$

$$= \frac{1}{3}\ell\eta\left|\frac{x+2}{x-1}\right|-\frac{2}{x-1}+c$$

7.39.- $\int\frac{2x^3+3x^2+x-1}{(x+1)(x^2+2x+2)^2}dx$

Solución.-

$\int\frac{(2x^3+3x^2+x-1)dx}{(x+1)(x^2+2x+2)^2} = \int\frac{Adx}{x+1}+\int\frac{(Bx+C)dx}{(x^2+2x+2)}+\int\frac{(Dx+E)dx}{(x^2+2x+2)^2}$ $(*)$, luego:

$$\frac{2x^3+3x^2+x-1}{(x+1)(x^2+2x+2)^2} = \frac{A}{x+1}+\frac{Bx+C}{(x^2+2x+2)}+\frac{Dx+E}{(x^2+2x+2)^2}$$

$$2x^3+3x^2+x-1 = A(x^2+2x+2)^2+(Bx+C)(x^2+2x+2)(x+1)+(Dx+E)(x+1)$$

$$= Ax^4+4Ax^3+8Ax^2+8Ax+4A+Bx^4+3Bx^3+4Bx^2+2Bx+Cx^3+3Cx^2+4Cx$$

$$\Rightarrow +2C+Dx^2+Dx+Ex+E$$

$$= (A+B)x^4+(4A+3B+C)x^3+(+8A+4B+3C+D)x^2$$

$$\Rightarrow +(8A+2B+4C+D+E)x+(4A+2C+E)$$

Igualando coeficientes, se tiene:

$$\begin{pmatrix} A & +B & & & & = 0 \\ 4A & +3B & +C & & & = 2 \\ 8A & +4B & +3C & +D & & = 3 \\ 8A & +2B & +4C & +D & +E & = 1 \\ 4A & & +2C & & +E & = -1 \end{pmatrix} \therefore A=-1, B=1, C=3, D=-2, E=-3$$

$$(*) = -\int\frac{dx}{x+1}+\int\frac{(x+3)dx}{(x^2+2x+2)}-\int\frac{(2x+3)dx}{(x^2+2x+2)^2}$$

$$= -\ell\eta|x-1|+\frac{1}{2}\int\frac{(2x+6)dx}{(x^2+2x+2)}-\int\frac{(2x+2)+1dx}{(x^2+2x+2)^2}$$

$$= -\ell\eta|x-1|+\frac{1}{2}\int\frac{(2x+2)+4}{(x^2+2x+2)}dx-\int\frac{(2x+2)dx}{(x^2+2x+2)^2}-\int\frac{dx}{(x^2+2x+2)^2}$$

$$= -\ell\eta|x-1|+\frac{1}{2}\int\frac{(2x+2)dx}{(x^2+2x+2)}+2\int\frac{dx}{(x^2+2x+2)}-\int\frac{(2x+2)dx}{(x^2+2x+2)^2}-\int\frac{dx}{(x^2+2x+2)^2}$$

$$= -\ell\eta|x-1|+\frac{1}{2}\ell\eta|x^2+2x+2|+2\int\frac{dx}{(x+1)^2+1}+\frac{1}{2}\frac{1}{x^2+2x+2}-\int\frac{dx}{\left[(x+1)^2+1\right]^2}$$

$$= -\ell\eta\left|x-1\right| + \frac{1}{2}\ell\eta\left|x^2+2x+2\right| + 2\,\text{arc}\,\tau g(x+1)$$

$$\Rightarrow +\frac{1}{2}\frac{1}{x^2+2x+2} - \frac{1}{2}\frac{x+1}{x^2+2x+2} - \frac{1}{2}\text{arc}\,\tau g(x+1) + c$$

$$= \ell\eta\left|\frac{\sqrt{x^2+2x+2}}{x+1}\right| + \frac{3}{2}\text{arc}\,\tau g(x+1) - \frac{1}{2}\frac{x}{x^2+2x+2} + c$$

7.40.- $\int \frac{(2x^2+3x-1)dx}{x^3+2x^2+4x+2}$

Solución.-

$$\int \frac{(2x^2+3x-1)dx}{x^3+2x^2+4x+2} = \int \frac{(2x^2+3x-1)dx}{(x+1)(x^2+2x+2)} = \int \frac{Adx}{(x+1)} + \int \frac{(Bx+C)dx}{(x^2+2x+2)}\ (*)\text{, luego:}$$

$$\frac{(2x^2+3x-1)}{(x+1)(x^2+2x+2)} = \frac{A}{(x+1)} + \frac{(Bx+C)}{(x^2+2x+2)}$$

$$2x^2+3x-1 = A(x^2+2x+2) + (Bx+C)(x+1)$$

$$\therefore \begin{cases} x=-1 \Rightarrow -2 = A \Rightarrow A = -2 \\ x=0 \Rightarrow -1 = 2A + C \Rightarrow C = 3 \\ x=1 \Rightarrow 4 = 5A + (B+C)(2) \Rightarrow B = 4 \end{cases}$$

$$(*) = -2\int \frac{dx}{(x+1)} + \int \frac{(4x+3)dx}{x^2+2x+2} = -2\ell\eta\left|x+1\right| + 2\int \frac{(2x+2)-1}{x^2+2x+2}dx$$

$$= -2\ell\eta\left|x+1\right| + 2\int \frac{(2x+2)dx}{x^2+2x+2} - 2\int \frac{dx}{x^2+2x+2}$$

$$= -2\ell\eta\left|x+1\right| + 2\ell\eta\left|x^2+2x+2\right| - 2\,\text{arc}\,\tau g(x+1) + c$$

7.41.- $\int \frac{(2x+1)dx}{3x^3+2x-1}$

Solución.-

$$\int \frac{(2x+1)dx}{3x^3-2x-1} = \int \frac{(2x+1)dx}{(x-1)(3x^2+3x+1)} = \int \frac{Adx}{(x-1)} + \int \frac{(Bx+C)dx}{(3x^2+3x+1)}\ (*)\text{, luego:}$$

$$\frac{(2x+1)}{(3x^3-2x-1)} = \frac{A}{(x-1)} + \frac{(Bx+C)}{(3x^2+3x+1)}$$

$$2x+1 = A(3x^2+3x+1) + (Bx+C)(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 3 = 7A \Rightarrow A = 3/7 \\ x=0 \Rightarrow 1 = A - C \Rightarrow C = -4/7 \\ x=-1 \Rightarrow -1 = A + (-B+C)(-2) \Rightarrow B = -9/7 \end{cases}$$

$$(*) = \frac{3}{7}\int \frac{dx}{(x-1)} - \frac{1}{7}\int \frac{(9x+4)dx}{3x^2+3x+1} = \frac{3}{7}\ell\eta\left|x-1\right| - \frac{1}{7}\frac{9}{6}\int \frac{(6x+3-1/3)dx}{3x^2+3x+1}$$

$$=\frac{3}{7}\ell\eta|x-1|-\frac{3}{14}\int\frac{(6x+3)dx}{3x^2+3x+1}+\frac{1}{14}\int\frac{dx}{3x^2+3x+1}$$

$$=\frac{3}{7}\ell\eta|x-1|-\frac{3}{14}\ell\eta|3x^2+3x+1|+\frac{1}{14}\int\frac{dx}{3(x+1/2)^2+1/4}$$

$$=\frac{3}{7}\ell\eta|x-1|-\frac{3}{14}\ell\eta|3x^2+3x+1|+\frac{2}{7}\int\frac{dx}{12(x+1/2)^2+1}$$

$$=\frac{3}{7}\ell\eta|x-1|-\frac{3}{14}\ell\eta|3x^2+3x+1|+\frac{\sqrt{3}}{21}\operatorname{arc}\tau g\,2\sqrt{3}(x+1/2)+c$$

7.42.- $\int\frac{x^4-2x^2+3x+4}{(x-1)^3(x^2+2x+2)}dx$

Solución.-

$\int\frac{x^4-2x^2+3x+4}{(x-1)^3(x^2+2x+2)}dx=\int\frac{Adx}{(x-1)}+\int\frac{Bdx}{(x-1)^2}+\int\frac{Cdx}{(x-1)^3}+\int\frac{(Dx+E)dx}{(x^2+2x+2)}\ (*)$, luego:

$$\frac{x^4-2x^2+3x+4}{(x-1)^3(x^2+2x+2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}+\frac{Dx+E}{(x^2+2x+2)}$$

$$x^4-2x^2+3x+4=A(x-1)^2(x^2+2x+2)+B(x-1)(x^2+2x+2)$$

$$\Rightarrow +C(x^2+2x+2)+(Dx+E)(x-1)^3$$

$$x^4-2x^2+3x+4=A(x^2-2x+1)(x^2+2x+2)+B(x^3+2x^2+2x-x^2-2x-2)$$

$$\Rightarrow +C(x^2+2x+2)+(Dx+E)(x^3-3x^2+3x-1)$$

$$x^4-2x^2+3x+4=Ax^4-Ax^2-2Ax+2A+Bx^3+Bx^2-2B+Cx^2+2Cx+2C$$

$$\Rightarrow +Dx^4-3Dx^3+3Dx^2-Dx+Ex^3-3Ex^2+3Ex-E$$

$$x^4-2x^2+3x+4=(A+D)x^4+(B-3D+E)x^3+(-A+B+C+3D-3E)x^2$$

$$\Rightarrow +(-2A+2C-D+3E)x+(-2A-2B+2C-E)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A & & & +D & & = & 1\\ & B & & -3D & +\ E & = & 0\\ -\ A & +\ B & +\ C & +3D & -3E & = & -2\\ -2A & & +2C & -D & +3E & = & 3\\ 2A & -2B & +2C & & -E & = & 4 \end{pmatrix}$$

$$\therefore A=106/125,\ B=9/25,\ C=6/5,\ D=19/125,\ E=102/125$$

$$(*)=\frac{106}{125}\int\frac{dx}{x+1}-\frac{9}{25}\int\frac{dx}{(x-1)^2}+\frac{6}{5}\int\frac{dx}{(x-1)^3}+\frac{1}{125}\int\frac{(19x+102)dx}{(x^2+2x+2)}$$

$$=\frac{106}{125}\ell\eta|x-1|+\frac{9}{25}\frac{1}{x-1}+\frac{6}{5}\frac{1}{(-2)(x-1)^2}+\frac{19}{125}\int\frac{(x+102/19)dx}{(x^2+2x+2)}$$

$$=\frac{106}{125}\ell\eta|x-1|+\frac{9}{25(x-1)}-\frac{3}{5(x-1)^2}+\frac{19}{250}\int\frac{(2x+2)+8\frac{14}{19}}{(x^2+2x+2)}dx$$

$$=\frac{106}{125}\ell\eta|x-1|+\frac{9}{25(x-1)}-\frac{3}{5(x-1)^2}+\frac{19}{250}\ell\eta\left|x^2+2x+2\right|+\frac{\not{19}}{250}\frac{166}{\not{19}}\int\frac{dx}{(x^2+2x+1)+1}$$

$$=\frac{106}{125}\ell\eta|x-1|+\frac{9}{25(x-1)}-\frac{3}{5(x-1)^2}+\frac{19}{250}\ell\eta\left|x^2+2x+2\right|+\frac{166}{250}\int\frac{dx}{(x+1)^2+1}$$

$$=\frac{106}{125}\ell\eta|x-1|+\frac{9}{25(x-1)}-\frac{3}{5(x-1)^2}+\frac{19}{250}\ell\eta\left|x^2+2x+2\right|+\frac{166}{250}\operatorname{arc}\tau g(x+1)+c$$

7.43.- $\int\frac{e^t dt}{e^{2t}+3e^t+2}$

Solución.-

$\int\frac{e^t dt}{e^{2t}+3e^t+2}=\int\frac{e^t dt}{(e^t+2)(e^t+2)}\ (*)$, Sea: $u=e^t+1, du=e^t dt; e^t+2=u+1$

Luego:

$$(*)\ \int\frac{du}{(u+1)u}=\int\frac{Adu}{(u+1)}+\int\frac{Bdu}{u}\ (**)$$

$$\frac{1}{(u+1)u}=\frac{A}{(u+1)}+\frac{B}{u}\Rightarrow 1=Au+B(u+1)$$

$$\therefore\begin{cases}u=0\Rightarrow 1=B\Rightarrow B=1\\ u=-1\Rightarrow 1=-A\Rightarrow A=-1\end{cases}$$

$$(**)=-\int\frac{du}{(u+1)}+\int\frac{du}{u}=-\ell\eta|u+1|+\ell\eta|u|+c=-\ell\eta\left|e^t+2\right|+\ell\eta\left|e^t+1\right|+c$$

$$=\ell\eta\left|\frac{e^t+1}{e^t+2}\right|+c$$

7.44.- $\int\frac{\operatorname{sen}\theta d\theta}{\cos^2\theta+\cos\theta-2}$

Solución.-

$\int\frac{\operatorname{sen}\theta d\theta}{\cos^2\theta+\cos\theta-2}=\int\frac{\operatorname{sen}\theta d\theta}{(\cos\theta+2)(\cos\theta-1)}\ (*)$,

Sea: $u=\cos\theta-1, du=-\operatorname{sen}\theta d\theta, \cos\theta+2=u+3$

Luego:

$$(*)\ \int\frac{-du}{(u+3)u}=-\int\frac{du}{u(u+3)}=-\int\frac{Adu}{u}-\int\frac{Bdu}{u+3}\ (**)$$

$$\frac{1}{u(u+3)}=\frac{A}{u}+\frac{B}{u+3}\Rightarrow 1=A(u+3)+Bu$$

$$\therefore\begin{cases}u=0\Rightarrow 1=3A\Rightarrow A=\frac{1}{3}\\ u=-3\Rightarrow 1=-3B\Rightarrow B=-\frac{1}{3}\end{cases}$$

$$(**)=-\frac{1}{3}\int\frac{du}{u}+\frac{1}{3}\int\frac{du}{(u+3)}=-\frac{1}{3}\ell\eta|u|+\frac{1}{3}\ell\eta|u+3|+c$$

$=-\frac{1}{3}\ell\eta|\cos\theta-1|+\frac{1}{3}\ell\eta|\cos\theta+2|+c$, Como: $|\cos\theta|<1$, se tiene:

$$=-\frac{1}{3}\ell\eta|1-\cos\theta|+\frac{1}{3}\ell\eta|2+\cos\theta|+c=\frac{1}{3}\ell\eta\left|\frac{2+\cos\theta}{1-\cos\theta}\right|+c$$

7.45.- $\int\frac{4x^4-2x^3-x^2+3x+1}{(x^3+x^2-x-1)}dx$

Solución.-

$$\int\frac{4x^4-2x^3-x^2+3x+1}{(x^3+x^2-x-1)}dx=\int\left(4x-6+\frac{9x^2+x-5}{x^3+x^2-x-1}\right)dx$$

$$=\int 4dx-\int 6dx+\int\frac{(9x^2+x-5)dx}{x^3+x^2-x-1}=2x^2-6x+\int\frac{(9x^2+x-5)dx}{x^3+x^2-x-1}\ (*)$$

Trabajando sólo la integral resultante:

$\int\frac{(9x^2+x-5)dx}{x^3+x^2-x-1}=\int\frac{(9x^2+x-5)dx}{(x+1)^2(x-1)}=\int\frac{Adx}{(x+1)}+\int\frac{Bdx}{(x+1)^2}+\int\frac{Cdx}{(x-1)}\ (**)$, luego:

$$\frac{(9x^2+x-5)}{(x^3+x^2-x-1)}=\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{x-1}$$

$$=9x^2+x-5=A(x+1)(x-1)+B(x-1)+C(x+1)^2$$

$$\therefore\begin{cases}x=1\Rightarrow 5=4C\Rightarrow C=5/4\\ x=-1\Rightarrow 3=-2B\Rightarrow B=-3/2\\ x=0\Rightarrow -5=-A-B+C\Rightarrow A=31/4\end{cases}$$

$$(**)=\frac{31}{4}\int\frac{dx}{(x+1)}-\frac{3}{2}\int\frac{dx}{(x+1)^2}+\frac{5}{4}\int\frac{dx}{(x-1)}=\frac{31}{4}\ell\eta|x+1|+\frac{3}{2(x+1)}+\frac{5}{4}\ell\eta|x-1|+c$$

$$(*)=2x^2-6x+\frac{31}{4}\ell\eta|x+1|+\frac{3}{2(x+1)}+\frac{5}{4}\ell\eta|x-1|+c$$

7.46.- $\int\frac{3x^4dx}{(x^2+1)^2}$

Solución.-

$$\int\frac{3x^4dx}{(x^2+1)^2}=\int\frac{3x^4dx}{(x^4+2x^2+1)}=3\int\left[1-\frac{2x^2+1}{(x^2+1)^2}\right]dx=3\int dx-3\int\frac{2x^2+1}{(x^2+1)^2}dx$$

$$=3x-3\int\frac{2x^2+1}{(x^2+1)^2}dx\ (*)$$

Trabajando sólo la integral resultante:

$\int\frac{(2x^2+1)dx}{(x^2+1)^2}=\int\frac{(Ax+B)dx}{(x^2+1)}+\int\frac{(Cx+D)dx}{(x^2+1)^2}\ (**)$, luego:

$$\frac{(2x^2+1)}{(x^2+1)^2}=\frac{Ax+B}{(x^2+1)}+\frac{Cx+D}{(x^2+1)^2}\Rightarrow 2x^2+1=(Ax+B)(x^2+1)+Cx+D$$

$$\Rightarrow 2x^2+1=Ax^3+Ax+Bx^2+B+Cx+D\Rightarrow 2x^2+1=Ax^3+Bx^2+(A+C)x+(B+D)$$

Igualando coeficientes: $A=0, B=2, A+C=0\Rightarrow C=0, B+D=1\Rightarrow D=-1$

$$(**)=2\int\frac{dx}{(x^2+1)}-\int\frac{dx}{(x^2+1)^2}=2\operatorname{arc}\tau gx-\frac{1}{2}\left(\operatorname{arc}\tau gx+\frac{x}{1+x^2}\right)+c$$

$$=\frac{3}{2}\operatorname{arc}\tau gx-\frac{x}{2(1+x^2)}+c$$

$$(*)=3x-\frac{9}{2}\operatorname{arc}\tau gx-\frac{x}{2(1+x^2)}+c$$

7.47.- $\int\frac{(2x^2+41x-91)dx}{x^3-2x^2-11x+12}$

Solución.-

$$\int\frac{(2x^2+41x-91)dx}{x^3-2x^2-11x+12}=\int\frac{(2x^2+41x-91)dx}{(x-1)(x+3)(x-4)}$$

$$=\int\frac{(2x^2+41x-91)dx}{(x-1)(x+3)(x-4)}=\int\frac{Adx}{x-1}+\int\frac{Bdx}{x+3}+\int\frac{Cdx}{x-4}\ (*)$$

$$\frac{(2x^2+41x-91)}{(x-1)(x+3)(x-4)}=\frac{A}{x-1}+\frac{B}{x+3}+\frac{C}{x-4}$$

$$(2x^2+41x-91)=A(x+3)(x-4)+B(x-1)(x-4)+C(x-1)(x+3)$$

$$\therefore\begin{cases}x=-3\Rightarrow 18-123-91=B(-4)(-7)\Rightarrow B=-7\\ x=4\Rightarrow 32+164-91=C(3)(7)\Rightarrow C=5\\ x=1\Rightarrow 2+41-91=A(4)(-3)\Rightarrow A=4\end{cases}$$

$$(*)=4\int\frac{dx}{(x-1)}-7\int\frac{dx}{(x+3)}+5\int\frac{dx}{(x-4)}=4\ell\eta|x-1|-7\ell\eta|x+3|+5\ell\eta|x-4|+c$$

$$=\ell\eta\left|\frac{(x-1)^4(x-4)^5}{(x+3)^7}\right|+c$$

7.48.- $\int\frac{(2x^4+3x^3-x-1)dx}{(x-1)(x^2+2x+2)^2}$

Solución.-

$$\int\frac{2x^4+3x^3-x-1}{(x-1)(x^2+2x+2)^2}dx=\int\frac{Adx}{(x-1)}+\int\frac{(Bx+C)dx}{(x^2+2x+2)}+\int\frac{(Dx+E)dx}{(x^2+2x+2)^2}\ (*)$$, luego:

$$\frac{2x^4+3x^2-x-1}{(x-1)(x^2+2x+2)^2}=\frac{A}{(x-1)}+\frac{Bx+C}{(x^2+2x+2)}+\frac{Dx+E}{(x^2+2x+2)^2}$$

$$2x^4+3x^3-x-1=A(x^2+2x+2)^2+(Bx+C)(x-1)(x^2+2x+2)+(Dx+E)(x-1)$$

$$2x^4+3x^3-x-1=A(x^4+4x^2+4+4x^3+4x^2+8x)+B(x^4+2x^3+2x^2-x^3-2x^2-2x)$$

$$\Rightarrow +C(x^3+2x^2+2x-x^2-2x-2)+D(x^2-x)+E(x-1)$$

$$2x^4+3x^3-x-1=(A+B)x^4+(4A+B+C)x^3+(8A+C+D)x^2$$
$$\Rightarrow +(8A-2B-D+E)x+(4A-2C-E)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A & + & B & & & & & = & 2 \\ 4A & + & B & + & C & & & = & 3 \\ 8A & & & + & C & +D & & = & 0 \\ 8A & & -2B & & & -D & +E & = & -1 \\ 4A & & & & -2C & & -E & = & -1 \end{pmatrix}$$

$$\therefore A=3/25, B=47/25, C=16/25, D=-8/5, E=1/5$$

$$(*)=\frac{3}{25}\int\frac{dx}{x-1}+\frac{1}{25}\int\frac{(47x+16)dx}{(x^2+2x+2)}-\frac{1}{5}\int\frac{(8x-1)dx}{(x^2+2x+2)^2}$$

$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{25}\int\frac{(x+16/47)dx}{(x^2+2x+2)}-\frac{8}{5}\int\frac{(x-1/8)dx}{(x^2+2x+2)^2}$$

$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{50}\int\frac{(2x+2)-62/47}{(x^2+2x+2)}dx-\frac{4}{5}\int\frac{(2x+2)-9/4}{(x^2+2x+2)^2}dx$$

$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{50}\int\frac{(2x+2)dx}{(x^2+2x+2)}-\frac{62}{50}\int\frac{dx}{(x^2+2x+2)}-\frac{4}{5}\int\frac{(2x+2)dx}{(x^2+2x+2)^2}$$

$$\Rightarrow +\frac{9}{5}\int\frac{dx}{(x^2+2x+2)^2}$$

$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{50}\ell\eta\left|x^2+2x+2\right|-\frac{62}{50}\int\frac{dx}{(x+1)^2+1}+\frac{4}{5}\int\frac{1}{(x^2+2x+2)}$$

$$\Rightarrow +\frac{9}{5}\int\frac{dx}{\left[(x+1)^2+1\right]^2}$$

$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{50}\ell\eta\left|x^2+2x+2\right|-\frac{62}{50}\operatorname{arc}\tau g(x+1)+\frac{4}{5(x^2+2x+2)}$$

$$\Rightarrow +\frac{9}{5}\left[\frac{1}{2}\operatorname{arc}\tau g(x+1)+\frac{1}{2}\frac{x+1}{x^2+2x+2}\right]+c$$

$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{50}\ell\eta\left|x^2+2x+2\right|-\frac{17}{50}\operatorname{arc}\tau g(x+1)+\frac{9x+17}{10(x^2+2x+2)}+c$$

7.49.- $\displaystyle\int\frac{dx}{e^{2x}+e^x-2}$

Solución.-

$$\int\frac{dx}{e^{2x}+e^x-2}=\int\frac{dx}{(e^x)^2+e^x-2}=\int\frac{dx}{\left[(e^x)^2+e^x+1/4\right]-2-1/4}$$

$$=\int\frac{dx}{\left[e^x+\frac{1}{2}\right]^2-(\frac{3}{2})^2}\ (*)$$, Sea: $u=e^x+\frac{1}{2}, du=e^x dx\Rightarrow dx=\frac{du}{u-\frac{1}{2}}$

Luego:

$$(*)\int\frac{\frac{du}{u-\frac{1}{2}}}{u^2-(\frac{3}{2})^2}=\int\frac{du}{(u-\frac{1}{2})(u+\frac{3}{2})(u-\frac{3}{2})}=\int\frac{Adu}{u-\frac{1}{2}}-\int\frac{Bdu}{(u+\frac{3}{2})}+\int\frac{Cdu}{(u-\frac{3}{2})}\ (**)$$

$$\frac{1}{(u-\frac{1}{2})(u+\frac{3}{2})(u-\frac{3}{2})}=\frac{A}{(u-\frac{1}{2})}-\frac{B}{(u+\frac{3}{2})}+\frac{C}{(u-\frac{3}{2})}$$

$$1=A(u+\frac{3}{2})(u-\frac{3}{2})-B(u-\frac{1}{2})(u-\frac{3}{2})+C(u-\frac{1}{2})(u+\frac{3}{2})$$

$$\therefore\begin{cases}u=\frac{1}{2}\Rightarrow 1=A(2)(-1)\Rightarrow A=-\frac{1}{2}\\ u=-\frac{3}{2}\Rightarrow 1=B(-2)(-3)\Rightarrow B=\frac{1}{6}\\ u=\frac{3}{2}\Rightarrow 1=C(1)(3)\Rightarrow C=\frac{1}{3}\end{cases}$$

$$(**)=-\frac{1}{2}\int\frac{du}{(u-\frac{1}{2})}+\frac{1}{6}\int\frac{du}{(u+\frac{3}{2})}+\frac{1}{3}\int\frac{du}{(u-\frac{3}{2})}$$

$$=-\frac{1}{2}\ell\eta\left|(u-\frac{1}{2})\right|+\frac{1}{6}\ell\eta\left|(u+\frac{3}{2})\right|+\frac{1}{3}\ell\eta\left|(u-\frac{3}{2})\right|+c$$

$$=\frac{1}{6}\ell\eta\left|\frac{(u+\frac{3}{2})(u-\frac{3}{2})^2}{(u-\frac{1}{2})^3}\right|+c=\frac{1}{6}\ell\eta\left|\frac{(e^x+2)(e^x-1)^2}{(e^x)^3}\right|+c=\frac{1}{6}\ell\eta\left|\frac{(e^x+2)(e^x-1)^2}{e^{3x}}\right|+c$$

7.50.- $\int\frac{\operatorname{sen} xdx}{\cos x(1+\cos^2 x)}$

Solución.-

$$\int\frac{\operatorname{sen} xdx}{\cos x(1+\cos^2 x)}=\int\frac{-\operatorname{sen} xdx}{\cos x(1+\cos^2 x)}=-\int\frac{du}{u(1+u^2)}=-\int\frac{Adu}{u}-\int\frac{(Bu+C)du}{(1+u^2)}\ (*)$$

Sea: $u=\cos x, du=-\operatorname{sen} xdx$

$$\frac{1}{u(1+u^2)}=\frac{A}{u}+\frac{(Bu+C)}{(1+u^2)}\Rightarrow 1=A(1+u^2)+(Bu+C)u$$

$$1=A+Au^2+Bu^2+Cu\Rightarrow 1=(A+B)u^2+Cu+A$$

Igualando Coeficientes se tiene:

$$\therefore\begin{cases}A+B=0\Rightarrow B=-A\Rightarrow B=-(1)\Rightarrow B=-1\\ C=0,\\ A=1\end{cases}$$

$$(*)=-\int\frac{du}{u}+\int\frac{udu}{1+u^2}=-\ell\eta|u|+\ell\eta\left|\sqrt{1+u^2}\right|+c=-\ell\eta|\cos x|+\ell\eta\left|\sqrt{1+(\cos x)^2}\right|+c$$

$$= \ell\eta\left|\frac{\sqrt{1+(\cos x)^2}}{\cos x}\right| + c$$

7.51.- $\int \frac{(2+\tau g^2\theta)\sec^2\theta d\theta}{1+\tau g^3\theta}$

Solución.-

$$\int \frac{(2+\tau g^2\theta)\sec^2\theta d\theta}{1+\tau g^3\theta} = \int \frac{(2+u^2)du}{(1+u^3)} = \int \frac{(2+u^2)du}{(1+u)(u^2-u+1)} \ (*)$$

Sea: $u = \tau g\theta, du = -\sec^2\theta d\theta$

$\int \frac{(2+u^2)du}{(1+u^3)} = \int \frac{Adu}{(1+u)} + \int \frac{Bu+C}{(u^2-u+1)}$, luego:

$$\frac{(2+u^2)}{(1+u^3)} = \frac{A}{(1+u)} + \frac{Bu+C}{(u^2-u+1)} \Rightarrow (2+u^2) = A(u^2-u+1) + (Bu+C)(1+u)$$

$$(2+u^2) = Au^2 - Au + A + Bu^2 + Bu + C + Cu$$

$$(2+u^2) = (A+B)u^2 + (-A+B+C)u + A + C$$

Igualando Coeficientes se tiene:

$$\begin{pmatrix} A+B & & =1 \\ -A+B & +C & =0 \\ A & +C & =2 \end{pmatrix} \therefore A=1, B=0, C=1$$

$$(*) = \int \frac{du}{1+u} + \int \frac{du}{u^2-u+1} = \int \frac{du}{1+u} + \int \frac{du}{(u-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$= \ell\eta|1+u| + \frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arc}\tau g\frac{u-\frac{1}{2}}{\frac{\sqrt{3}}{2}} + c = \ell\eta|1+u| + \frac{2}{\sqrt{3}}\operatorname{arc}\tau g\frac{2u-1}{\sqrt{3}} + c$$

$$= \ell\eta|1+\tau g\theta| + \frac{2}{\sqrt{3}}\operatorname{arc}\tau g\frac{(2\tau g\theta - 1)}{\sqrt{3}} + c$$

7.52.- $\int \frac{(5x^3+2)dx}{x^3-5x^2+4x}$

Solución.-

$$\int \frac{(5x^3+2)dx}{x^3-5x^2+4x} = \int \frac{(5x^3+2)dx}{x(x-1)(x-4)} = \int \frac{Adx}{x} + \int \frac{Bdx}{(x-1)} + \int \frac{Cdx}{(x-4)} \ (*)$$

$\frac{(5x^3+2)}{x(x-1)(x-4)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-4)}$, Luego:

$$(5x^3+2) = A(x-1)(x-4) + Bx(x-4) + Cx(x-1)$$

Igualando Coeficientes se tiene:

$$\therefore \begin{cases} x=0 \Rightarrow 2=4A \Rightarrow A=\frac{1}{2} \\ x=1 \Rightarrow 7=-3B \Rightarrow B=-\frac{7}{3} \\ x=4 \Rightarrow 322=12C \Rightarrow C=\frac{161}{6} \end{cases}$$

$$(*)=\frac{1}{2}\int\frac{dx}{x}-\frac{7}{3}\int\frac{dx}{x-1}+\frac{161}{6}\int\frac{dx}{x-4}=\frac{1}{2}\ell\eta|x|-\frac{7}{3}\ell\eta|x-1|+\frac{161}{6}\ell\eta|x-4|+c$$

$$=\frac{3}{6}\ell\eta|x|-\frac{14}{3}\ell\eta|x-1|+\frac{161}{6}\ell\eta|x-4|+c=\frac{1}{6}\ell\eta\left|\frac{x^3(x-4)^{161}}{(x-1)^{14}}\right|+c$$

7.53.- $\int\frac{x^5dx}{(x^3+1)(x^3+8)}$

Solución.-

$$\int\frac{x^5dx}{(x^3+1)(x^3+8)}=\int\frac{x^5dx}{(x+1)(x^2-x+1)(x+2)(x^2-2x+4)}$$

$$=\int\frac{Adx}{(x+1)}+\int\frac{Bdx}{(x+2)}+\int\frac{(Cx+D)dx}{(x^2-x+1)}+\int\frac{(Ex+F)dx}{(x^2-2x+4)}\ (*)$$, luego:

$$\frac{x^5}{(x^3+1)(x^3+8)}=\frac{A}{(x+1)}+\frac{B}{(x+2)}+\frac{Cx+D}{(x^2-x+1)}+\frac{Ex+F}{(x^2-2x+4)}$$, luego:

$$x^5=A(x+2)(x^2-x+1)(x^2-2x+4)+B(x+1)(x^2-x+1)(x^2-2x+4)$$

$$\Rightarrow +(Cx+D)(x+1)(x+2)(x^2-2x+4)+(Ex+F)(x+1)(x+1)(x^2-x+1)$$

$$x^5=A(x^5+8x^2-x^4-8x+x^3+8)+B(x^5-2x^4+4x^3+x^2-2x+4)$$

$$\Rightarrow +(Cx+D)(x^4+8x+x^3+8)+(Ex+F)(x^4+2x^3+x+2)$$

$$x^5=(A+B+C+E)x^5+(-A-2B+C+D+2E+F)x^4+(A+4B+D+2F)x^3$$

$$\Rightarrow +(8A+B+8C+E)x^2+(-8A-2B+8C+8D+2E+F)x+(8A+4B+8D+2F)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A & +B & +C & & +E & & =1 \\ -A & -2B & +C & +D & +2E & +F & =0 \\ A & +4B & & +D & & +2F & =0 \\ 8A & +B & +8C & & +E & & =0 \\ 8A & -2B & +8C & +8D & +2E & +F & =0 \\ 8A & +4B & & +8D & & +2F & =0 \end{pmatrix}$$

$$\therefore A=-\frac{1}{21},B=\frac{8}{21},C=-\frac{2}{21},D=\frac{1}{21},E=\frac{16}{21},F=-\frac{16}{21}$$

$$(*)=-\frac{1}{21}\int\frac{dx}{x+1}+\frac{8}{21}\int\frac{dx}{(x+2)}-\frac{1}{21}\int\frac{(2x-1)dx}{(x^2-x+1)}+\frac{16}{21}\int\frac{(x-1)dx}{(x^2-2x+4)}$$

$$=-\frac{1}{21}\ell\eta|x+1|+\frac{8}{21}\ell\eta|x+2|-\frac{1}{21}\ell\eta|x^2-x+1|+\frac{8}{21}\int\frac{(2x-2)dx}{x^2-2x+4}$$

$$= -\frac{1}{21}\ell\eta\left|x+1\right| + \frac{8}{21}\ell\eta\left|x+2\right| - \frac{1}{21}\ell\eta\left|x^2-x+1\right| - \frac{8}{21}\ell\eta\left|x^2-2x+4\right| + c$$

$$= \frac{1}{21}\ell\eta\left|\frac{\left[(x+2)(x^2-2x+4)\right]^8}{(x+1)(x^2-x+1)}\right| + c$$